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DEDICATED TO PROFESSOR MIECZYSŁAW KUCHARZEWSKI
WITH BEST WISHES ON HIS 70TH BIRTHDAY

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ON SOME PROBLEM IN GEOMETRY FOR CONNOISSEURS

In the beautiful and rare book of Petersen [1] there is the following problem is given:

Let us consider an arbitrary quadrilateral, the intersection point S of its diagonals and four perpendiculars S1, S2, S3 and S4 to the corresponding sides, cf. Fig.1.

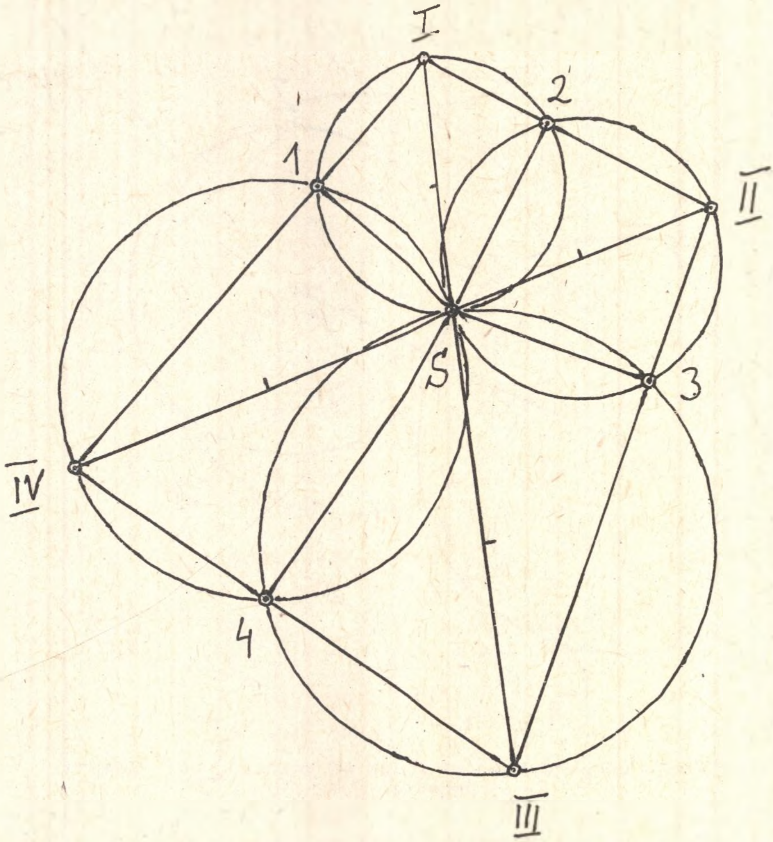

Fig. 1. Analysis of the problem

Given the points 1,2,3,4 there is the problem of reconstruction of the quadrilateral. The intersection point S is not given.

The solution of this problem without any mathematical formula may be of interest (the analytical method leads to the result that the point $S(x_0,y_0)$ is the intersection point of two curves of the fourth degree in x and y).

Analysis of the problem

At the point S two circles 4 IV 1 S and 2 II 3 S are externally tangent. The centers of these circles lie at the midpoints of the segments S IV and S II.

The circles 1 I 2 S and 3 III 4 s possess a similar property.

Construction

Let us draw the axes fo symmetry of the segment 41 and of the segment 23, cf. Fig. 2. Denote these axes by sym 41 and sym 23.

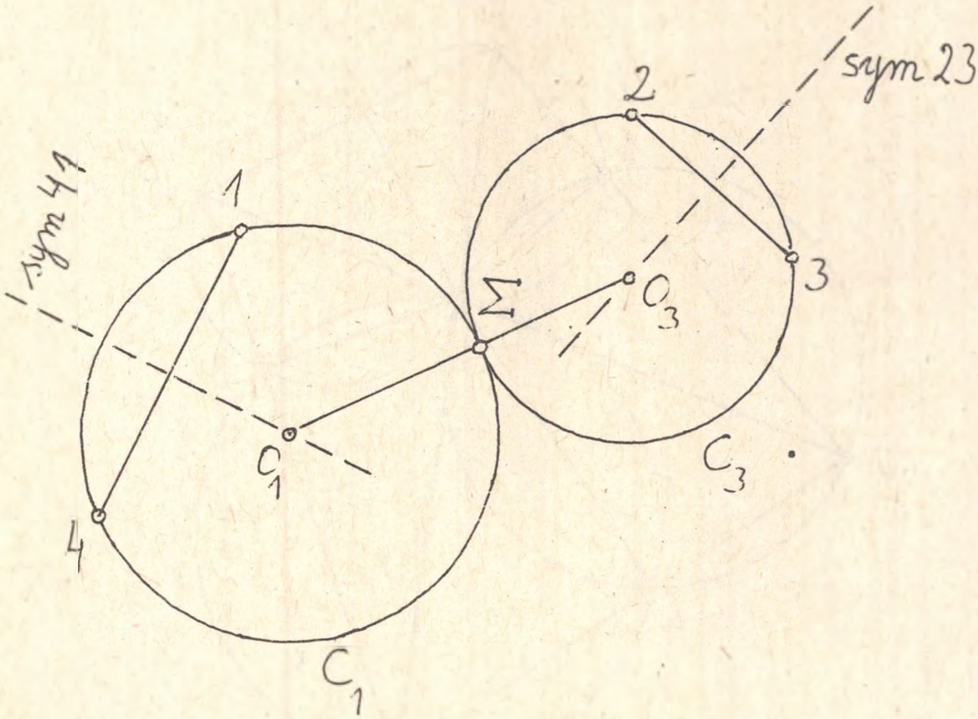

Fig. 2. First step: construction of locus of points $\sum$ of external tangency

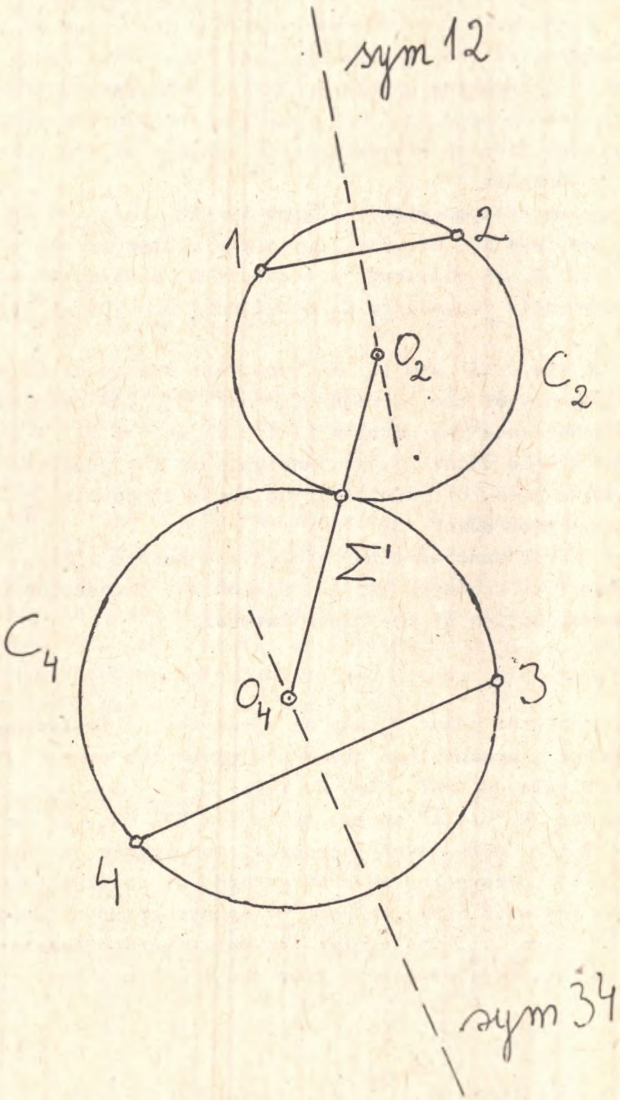

Fig. 3. Second step: construction of locus of points $\sum'$ of external tangency

Any point O_1 on sym 41 is equidistant from the end points 4 and 1, therefore O_1 is the center of the circle C_1 passing through 4 and 1. We construct the circle C_3 passing through the points 2 and 3 and externally tangent to the cricle C_1 at the point denoted by Σ.

Choosing O_1 on the axis sym 41 we construct the locus of points Σ of external tangency of the circles C_1 and C_3. This locus contains the desired point S since the circle 4 IV 1 S passess through the points 4 and 1, the circle 2 II 3 S passess through the points 2 and 3 and these both circles are externally tangent at the point S, cf. Analysis of the problem.

In a similar way we proceed with the axes sym 12 and sym 34.

Any point O_2 on sym 12, cf. Fig. 3, is the center of the circle C_2 passing through 1 and 2. We construct the circle C_4 passing through the points 3 and 4 externally tangent to C_2 and find the point Σ' of external tangency.

Choosing O_2 on the axis sym 12 we construct the locus of points Σ' of external tangency of the circles C_2 and C_4. The desired point S belongs to this locus since the circles 1 I 2 S and 3 III 4 S are externally tangent at the point S, cf. Analysis of the problem.

Thus both curves : locus of points Σ and locus of points Σ' intersect each other at the desired point S.

With the point S at hand we draw through the points 1,2,3,4 the lines perpendicular to the segments S1, S2, S3 and S4, respectively, and this ends the reconstruction of the quadrilateral.

Remarks

The construction of the point Σ may be given in the following way:

Let us suppose for a moment that there are given the center O_1 and radius r of the circle C_1, cf. Fig. 2. Then $O_1O_3 - O_32 = r$. This means that the center O_3 is 1° on the axis sym 23 and 2° on the hyperbola $O_1P - P2 = r$ with given focuses at the points 2 and O_1 and given axis $2a = r$. There are several methods of construction such a hyperbola. There are also other methods of construction of externally tangent circles C_1 and C_3. Obviously, the construction possesses the theoretical and aesthetic value rather than the practical one.

REFERENCES

[1] Petersen J.: Methoden und Theorien zur Lösung geometrischer Konstruktionsaufgaben, Kopenhagen 1879.

[2] Petersen J.: Metody i teorie rozwiązywania zadań geometrycznych konstrukcyjnych, tłumaczył K. Hertz, Warszawa 1881 (jedyne wydanie).