ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1993
Seria: MATEMATYKA-FIZYKA Z. 68 Nr kol. 1147

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CONSTRUCTION OF THE SYSTEM OF LINEAR INDEPENDENT INTEGRALS FOR CERTAIN CLASS OF ITERATED ORDINARY DIFFERENTIAL EQUATION OF THE (m·n) ORDER

Summary. In the paper a construction of the fundamental system of integrals for the equation (1) $L_m^n u(x) = 0$, is given using the fundamental system of the integrals for the equation (2) $L_m u(x)= 0$. To the construction the convenient Cauchy function H^j are applied. In Lemma 2 the linear independence of the system Z is proved.

1. INTRODUCTION

The subject of the paper is a construction of the fundamental system of the integrals for the equation

$$L_m^n u(x) = 0, \quad L_m = D_x^m + \sum_{i=1}^{m} p_i(x) D_x^{m-i}, \quad L^n = L(L^{n-1}), \quad x \in (a,b), \tag{1}$$

$$p_i \in C^{m \cdot n}([a,b]), \quad i=1,\dots,m$$

2. NOTATIONS

Let $S = \{u_1(x),\dots,u_m(x)\}$ denote the fundamental system of the integrals for the equation

$$L_m u(x) = 0 \tag{2}$$

Let $H(x,s)$ denote the Cauchy function for equation (2). By [1], p. 114, the function H is of the form

$$H(x,s) = W^{-1}(x,s) \det M(x,s), \tag{3}$$

where W is the Wronski determinant of the system (S) and

$$M(x,s) = \begin{bmatrix} u_1(s),\dots,u_m(s) \\ \dots\dots\dots\dots \\ D_s^{m-2}u_1(s),\dots D_s^{m-2}u_m(s) \\ u_1(x),\dots,u_m(x) \end{bmatrix}$$

The function $H(x,s)$ satisfies the following conditions;

1^o $H \in C^{m-1}((a,b) \cdot (a,b))$,

2^o $H \in C^m((a,b) \cdot (a,b))$ for $x \neq s$,

3^o $D_x^i H(x,x) = 0$, $i=0,1,\dots,m-2$, $D^{m-1}H(x,x) = 1$ for $x \in (a,b)$,

4^o $L_x H(x,s) = 0$ for $x \neq s$, $(x,s) \in (a,b) \times (a,b)$,

5^o if $v(x) \in C([a,b])$,

then $L(\int_a^x H(x,s)\, v(s)ds) = v(x)$ for every $x \in (a,b)$.

Let $H^o(x,s) = H(x,s)$, $H^j(x,s) = \int_a^x H(x,s_1)H^{j-1}(s_1,s)ds_1$, $j=1,\dots n-1$.

Let $S^j = \{u_1^j,\dots,u_m^j\}$ denote the system of the linearly indepedent integrals of the equations

$$L^{j+2}u(x) = 0, \tag{4}$$

for $j=0,1,\dots,n-2$ respectively.

3. PROPERTIES OF THE SYSTEM S^j

Let $u_i^j(x) = \int_a^x H^j(x,s)u_i(s)ds$, $i=1,\dots,m$, $j=0,1,\dots,n-2$

Lemma 1. If $p_i \in C^{m\cdot n}([a,b])$, $i=1,\dots,m$, then the functions u_i^j, $i=1,\dots,m$, $j=0,1,\dots,n-2$ satisfy equations (4).

Proof. For $j=0$ we have

$$u_i^o(x) = \int_a^x H(x,s)u_i(s)ds, \quad i=1,\dots,m$$

and, using the properties of the Cauchy function, we obtain

$$Lu_i^o(x) = u_i(x)+ \int_a^x L_x H(x,s)u(s))ds = u_i(x), \quad i=1,\dots,m$$

$$L^2u_i^o(x) = Lu_i(x)= 0, \quad i=1,\dots,m, \quad L^j u_i^o(x)= 0, \quad j=3,4,\dots,n, \quad i=1,\dots,m$$

Similarly we obtain

$$Lu_i^j(x)=u_i^{j-1}(x), \quad j=1,2,\dots,n-2, \quad L^n u_i^{n-2}(x) = Lu_i(x) = 0, \quad i=1,\dots,m.$$

Lemma 2. If $p_i \in C^{m\cdot n}([a,b])$, then the functions

$$u_1,\dots,u_m, \quad u_1^o,\dots,u_m^o,\dots\dots,u_1^{n-2}, \quad u_2^{n-2},\dots,u_m^{n-2} \tag{5}$$

are linearly independent for $x \in (a,b)$.

Proof. Let

$$C_1,\dots,C_m, \quad C_1^o,\dots,C_m^o, \quad \dots\dots\dots\dots,C_1^{n-2},\dots\dots,C_m^{n-2},$$

be arbitrary constants. And put

$$V(x) = \sum_{i=1}^m C_i u_i(x) + \sum_{i=1}^m C_i^o u_i^o(x) + \dots\dots + \sum_{i=1}^m C_i^{n-2} u_i^{n-2}(x) = 0.$$

We have

$$L^{n-1}V(x) = \sum_{i=1}^m C_i^{n-2} u_i(x) = 0 \Longrightarrow C_i^{n-2} = 0, \quad i=1,\dots,m,$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$L^2V(x) = \sum_{i=1}^m C_i^1 u_i(x) = 0 \Longrightarrow C_i^1 = 0, \quad i=1,\dots,m,$$

$$LV(x) = \sum_{i=1}^{m} C_i^o u_i(x) = 0 \Longrightarrow C_i^o = 0, \quad i=1,\dots,m,$$

$$V(x) = \sum_{i=1}^{m} C_i u_i(x) = 0 \Longrightarrow C_i = 0, \quad i=1,\dots,m.$$

4. FUNDAMENTAL THEOREM

Let

$$Z = S \cup \bigcup_{j=0}^{n-2} S^j.$$

By chice lemmas 1 and 2 we obtain

Theorem. If $p_i \in C^{m\cdot n}([a,b])$, then the system Z is the fundamental system of the integrals for the equation (1).

REFERENCES

[1] **Rabczuk R.:** Elementy nierówności różniczkowych. Warszawa 1976.

KONSTRUKCJA CAŁEK LINIOWO-NIEZALEŻNYCH DLA PEWNEJ KLASY RÓWNAŃ RÓŻNICZKOWYCH ZWYCZAJNYCH ITEROWANYCH RZĘDU (m·n)

S t r e s z c z e n i e

Przedmiotem pracy jest konstrukcja układu całek liniowo niezależnych dla równania (1) $L_m^n u(x) = 0$ na podstawie znanego układu podstawowego dla równania $L_m u = 0$. Do konstrukcji stosuje się funkcję Cauchy'ego (3) dla równania (2). W lemacie 2 sprawdzona jest niezależność liniowa całek układu Z.

КОНСТРУКЦЯ ИНТЕГРАЛОВ ЛИНЕЙНО НЕЗАВИСИМЫХ ДЛЯ НЕКОТОРОГО КЛАССА ИТЕРОВАНЫХ ОБИКНОВЕННЫХ ДИФФЕРЕНЦЕЛЬНЫХ УРАВНЕНИЙ ПОРЯДКИ $(M \cdot n)$

Резюме. В работе дается конструкця фундаментальной системи интеприлов для уравнения (1) $L_m^n u(x) = 0$ при помощи интегрослов уравненя (2) $L_m u(x) = 0$. Сисмему Z строимся применяя функцию Коши H^-. В лемме 2 проверена линейная независимость интегралов системы Z.