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## REDUCTION OF THE OCULAR DISTANCE ON THE SLOPE PROJECTION PLANE

Summary. Oryginal method for reduction of the ocular distance on the slope projection plane is introduced.

## REDUKCJA GEEBOKOŚCI TŁOWEJ NA NACHYLONEJ PŁASZCZYŹNIE TEOWEJ

Streszczenie. Przedstawiono oryginalną metodę redukcji głębokości tlowej na nachylonej plaszczyźnie tlowej.

## REDUKTION DES DISTANZKREISRADIUS AUF DER GENEIGTEN BILDEBENE

Zusammenfassung. Eine originalle Methode für the Reduktion des Distanzkreisradius auf der geneigten Bildbene ist eingeführt.

The three points projection (in which the three basic directions in space, each perpendicular to other two, have proper convergence points) is here interpreted as the central projection on the slope projection plane. The methods of three points perspective enable the drawing of the perspective on the slope projection plane. Orginal methods for the projections of vertical segments and for reduction of the ocular distance on the slope projection plane are introduced.

Let (Fig. la) be given: vertical projection plane $\tau$, projection center $0 \notin \tau$ with its perpendicular projection $0^{\tau}$ on $\tau$ and a plane $\pi$ constituting an angle $90^{\circ}-\varphi$ with the vertical direction.

The plane $\pi$ and the plane $\zeta$ passing through 0 parallel to $\pi$ intersect $\tau$ at the traces $t_{\pi}$ and $z_{\pi}$ of $\pi$.

A figure $F$ standing on $\pi$ has $F^{\prime}$ as its three points projection from 0 onto $\tau$.
Now revolving all the introduced figures about the axis $t_{\pi}$ through the angle $\varphi$ we obtain (Fig. 1b): the plane $\pi$ in horizontal position as the base plane with the figure $F$ standing on it, the horizontal plane $\zeta$ passing through 0 , the slope projection plane $\tau$ constituting the angle $\varphi$ with the vertical direction and including (without any changes in comparison with Figure 1a) the traces $t_{\pi}$ and $z_{\pi}$, the perpendicular projection $0^{\tau}$ of 0 and projection $F^{s}$ of $F$.


Fig. 1.

The projecting plane $\tau$ we call now the slope projecting plane, the traces $t_{\pi}$ and $z_{\pi}$ — the base line $p$ and the horizon line $h$, respectively, and the projection $F^{s}$ of the figure $F$ - the projection onto the slope projection plane.

Placing the slope projection plane $\tau$ on the drawning sheet (Fig. 2) we draw the distance circle with center $0^{r}$ and the radius equal to the ocular distance $\delta$ and the parallel base and horizon lines $p$ and $h$ ( $h$ does not pass through $0^{r}$ ).

The line $l^{s}$ passing through $0^{r}$ perpendicular to $h$ intersects the lines $p$ and $h$ at the points $T_{0}=l^{s} \cap p$ and $Z_{0}=l^{s} \cap h$. We notice that if the radius $0^{r} 0^{x}$ of the distance circle is parallel to $h$ then the angle $\varphi\left(h, Z_{0} 0^{x}\right)$ is equal to the angle $\varphi$ between the projection plane $\tau$ and the vertical direction in the space.

As in three points projection the point $0^{0} \in l^{s}$ (where $Z_{0} 0^{0}=Z_{0} 0^{x}$ ) is the rabatment of the center 0 of projection onto $\tau$ and $Z_{n}=z^{x} \cap l^{s}$ (where $z^{x}$ passes through $0^{x}$ perpendicularly to $Z_{0} 0^{x}$ ) is the convergence point for the lines perpendicular to the base plane $\pi$.


Fig. 2.

The point $Z_{0}=l^{s} \cap h$ (and not $0^{r}$ as in the three points projection) we take as the center of similarity between the projection on the slope projection plane and its reduced map caused by the reduction of the ocular distance $\delta$.

So we have obtained the base for the projection onto the slope projection plane. We can notice that the parameters of this base are the following values: the distance of the parallel base and horizon lines $p$ and $h$, the angle $p\left(z_{q}^{x}, h\right)$ between the projection plane $\tau$ and the vertical direction in space and the ocular distance $\delta=\left|0^{\top} 0^{x}\right|$.

Example. Given the perpendicular projections of some cuboids (Fig. 3a) draw the perspective of them on the slope projection plane.

We draw (Fig. 3b): the parallel base and the horizon lines $p \| h$, the axis $\|^{s} \perp h$ intersecting them at $T_{0}=l^{s} \cap p$ and $Z_{0}=l^{s} \cap h$, the line $z_{q}^{x}$ through $Z_{0}$ at the angle $\varphi$ to $h$ and the line $q^{x} \| z_{q}^{x}$ through $T_{0}$.

We cut the plan of the cuboids on Figure $3 a$ with a line $p_{0}$ constituting the angle $\psi$ with the line perpendicular to the $x$ axis being the intersection line of the reference planes of the given horizontal and vertical projections of the cuboids.

On Figure 3b, we draw a line $z_{1}^{o 1 / 5}$ at the angle $\psi$ to $h$ and denote its intersections with $h$ and $l^{s}$ by $Z_{1}^{o 1 / 5}$ and $0_{1}^{o 1 / 5}$, respectively. The number $1 / 5$ is here the reduction factor chosen to reach on the paper sheet the convergence point $Z_{1} \in h$ such that $Z_{0} Z_{1}=$ $5 Z_{0} Z_{1}^{1 / 5}$.

The perpendicular $z_{2}^{o 1 / 5}$ to $z_{1}^{o 1 / 5}$ passing through $0^{01 / 5}$ intersects $h$ at $Z_{2}^{o 1 / 5}=z_{2}^{o 1 / 5} \cap h$.

So we have the convergence point $Z_{1}$ for the lines perpendicular to the $x$ axis on Figure 3 a and the reduced convergence point $Z_{2}^{o 1 / 5}$ for the lines parallel to this axis. Finally, we construct the convergence scales on the parallel lines passing through $Z_{0}$ and $Z_{2}^{o 1 / 5}$, respectively, with the unit segments satisfying the ratio $j: j^{1 / 5}=5: 4$.

On Figure 3a we denote the intersection points of $p_{0}$ with the lines perpendicular to the $x$ axis by odd numbers and those with the lines parallel to the $x$ axis by even numbers. Then we mark all these points on the edge of the paper sheet which we shift to contact with the base line $p$ on Figure 3b.

Joining $Z_{1}$ with odd numbered points of the sheet edge and $Z_{2}$ with the even numbered (by means of the constructed convergence scales) we obtain a net of lines which cuts out the perspective of the plan of the given cuboids.

We construct now the convergence scales for the convergence point $Z_{n}$ for the lines perpendicular to the base plane as follows:

- the line passing through $0^{x 1 / 5}$ perpendicular to $z_{q}^{x}\left(Z_{0}, 0^{x^{1 / 5}}\right)$ intersects $l^{s}$ at $Z_{n}^{1 / 5}$,
- the base lines for the scales are: the line $h$ and the line passing through $Z_{n}^{1 / 5}$ and parallel to $h$,
- their unit segments satisfies the ratio $j: j^{1 / 5}=5: 4$.

By means of the constructed scales for the convergence point $Z_{n}$ we draw through the vertices of the plan the perspectives of the lines perpendicular to the base plan $\pi$. To obtain the projections of the side edges of the cuboids we draw (compare [1]) the line $z_{o}^{x}$ perpendicular to the line $q^{T}$ in a free place of Figure 3b.

The lenghts of the side edges given on the elevation of the cuboids on Figure 3a we measure off on the line $z_{0}^{x}$ from the point of its intersection with $q^{x}$ obtaining the segments $A^{x} B^{x}$ and $C^{x} D^{x}$.

Then we proceed as follows:

- we project $A^{x} B^{x}$ and $C^{x} D^{x}$ parallel to the direction of the line $z_{q}^{x 1 / 5}\left(Z_{0}, 0^{x 1 / 5}\right)$ on the line $l^{3}$ into the segments $A_{1} B_{1}$ and $C_{1} D_{1}$, where $A_{1}=C_{1}=T_{0}$,
- we shift $A_{1} B_{1}$ and $C_{1} D_{1}$ parallel to the line $p$ into the segments $A_{0} B_{0}$ and $C_{0} D_{0}$, where $A_{0}=p \cap Z_{0} A^{s}$ and $C_{0}=p \cap Z_{0} C^{s}$,
- we project $A_{0} B_{0}$ and $C_{0} D_{0}$ from $Z_{0}$ on the lines $Z_{n} A^{s}$ and $Z_{0} C^{s}$ obtaining the projections $A^{s} B^{s}$ and $C^{s} D^{s}$ of the side edges $A B$ and $C D$ of the cuboids.

The projections of the remaining horizontal edges of the cuboids are drawn perspective parallel to the sides of their bases by means of the convergence points $Z_{1}$ and $Z_{2}$.

Rys. 3.

Remark 1. On Figure 3a, we intersect at least three of the lines parallel to the $x$ axis and make use of the perspective harp which join its arbitrarily chosen vertex with the intersection points of the lines parallel to the $x$ axis with a line perpendicular to it.

The suitable intersection of the harp shifted to Figure 3b gives for example the point denoted by ${ }^{\prime}+"$ which is then joined with $Z_{2}$ by means of the convergence scales.

Remark 2. Choosing the reduced perpendicular projection of the projection center 0 above the horizon line $h$ we obtain the convergence of the side edges of the cuboids to the upper side of the picture. Such a perspective is applied for considerably high buildings viewing from a low horizon.

Remark 3. It has to be stressed that the perspective on Figure $3 b$ allows a great variety of the parameters of the perspective base. Picking suitable values for the distance of the basic lines $p$ and $h$, the angle $\varphi$ of inclination of the projection plane, the ocular distance $\delta$ and its reduction factor and suitably place on the basic line $p$ the point set of the sheet edge marked on the Figure $3 a$ we can obtain various perspective views. Choosing the best of them is the matter of experience not limited in the presented method by the drawing rules.

## References

[1] B. Grochowski, Central projection of a segment perpendicular to the given plane, Demonstratio Mathematica XXII, 2 (1989).

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## Streszczenie

Redukcja głębokości tlowej na nachylonej plaszczyźnie tlowej jest tu oparta na przyjęciu punktu zbiegu prostych spadu plaszczyzny podstawy (prostopadłych do jej śladu tlowego) jako środka podobieństwa kreślonej perspektywy i jej zredukowanego obrazu. Wprowadzona redukcja, w połączeniu z oryginalną metodą odmierzania odcinków pionowych, pozwala na kreślenie, szczególnie metodami pośrednimi, nieprzerysowanej perspektywy zespołu budynków (osiedla) oraz budynków wysokich.

