ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1995
Seria: MATEMATYKA-FIZYKA z. 74 Nr kol. 1285

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PERSPECTIVE OF VERTICAL SEGMENTS ON THE SLOPE PROJECTION PLANE

Summary. A simple construction for measurement of vertical segments on the slope projection plane is presented.

PERSPEKTYWA PIONOWYCH ODCINKÓW NA NACHYLONEJ PŁASZCZYŹNIE TŁOWEJ

Streszczenie. Przedstawiona jest tu prosta konstrukcja mierzenia pionowych odcinków na nachylonej płaszczyźnie tłowej.

PERSPEKTIV DER VERTIKALEN SEGMENTS AUF DER GENEIGTEN BILDEBENE

Zusammenfassung. Eine einfache Konstruktion der vertikalen Segments auf der geneigten Bildebene is eigeführt.

1. Introduction

In this paper a simple construction for the measurement of vertical segments on the slope projection plane is presented. We shall see that this construction resembles that one on the vertical projection plane. On the slope projection plane we have also a parallel translation which must be only completed by an appropriate rotation.

2. Construction

Let (Fig. 1) be given: the parallel basis and the horizon lines $p \parallel h$ (cf. [1]), the symetrical axis $l^s \perp p$, the orthogonal projection $0^\tau \in l^s$ of the projection center 0 (lying below the horizon line h) and the segment $0^\tau 0^x \perp l^s$ whose measure is the ocular distance δ.

We join 0^x with $Z_0 = h \cap l^s$ and draw through 0^x the line $z_n^x \perp 0^x Z_0$. The point $Z_n = z_n^x \cap l^s$ is the convergence point for the vertical lines perpendicular to the basis plane π.

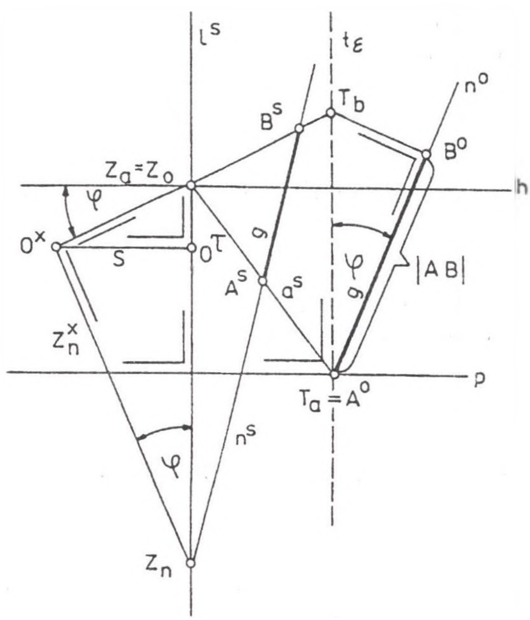

Fig. 1.

The equal angles $\varphi(h, Z_0 0^x)$ and $\varphi(z_n^x, l^s)$ represent the angle between the projection plane τ and the vertical direction V^∞.

The perspective A^s of a point $A \in \pi$ is lying on the line $a^s(T_a, Z_a)$, where T_a is some point of p and $Z_a = Z_0$. The line $n^s(Z_n, A^s)$ is the perspective of the line n which passes through A perpendicularly to the basis plane π. If B^s is a point of n^s different from A^s, then $A^s B^s$ is the perspective of a segment $AB \perp \pi$ with the end A on π.

Through T_a we draw two lines: t_ε perpendicular to p and n^0 under the angle φ to t_ε.

Now we project with Z_0 as the centre A^sB^s onto $T_aT_b \in t_\varepsilon$ and then we project T_aT_b orthogonally onto $A^0B^0 \subset n^0$. The obtained segment A^0B^0 is equal to the segment $AB \perp \pi$ (with $A \in \pi$) having the given projection A^sB^s.

3. Proof

On Figure 2 (as on Fig. 1) we have the base of projection: $p \parallel h$, $l^s \perp p$, $0^\tau \in l^s$ (lying below h) and $0^\tau 0^x \parallel h$. We construct the rectangular triangle $Z_0 0_x Z_n$ with $Z_0 = h \cap l^s$ and $Z_n \in l^s$.

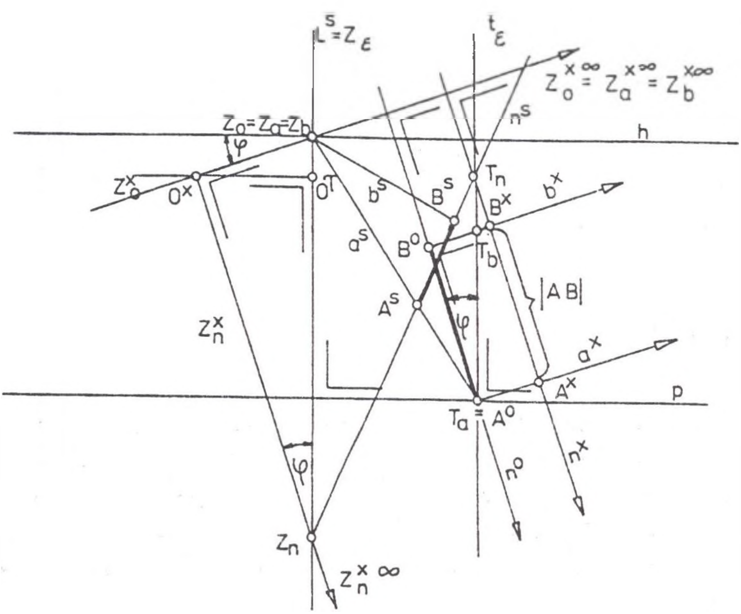

Fig. 2.

We assume the projection A^sB^s of a segment $AB \perp \pi$ with $A \in \pi$, on the line $n^s(Z_n, A^s)$; A^s is here putting on the line $a^s(T_a, Z_a)$, where T_a is a point of p and $Z_a = Z_0$.

Now we consider the plane ε of the lines n and a which intersect each other at the point A. Its convergence line $z_\varepsilon(Z_n, Z_a) = l^s$ and its trace t_ε on the projection plane passes through T_a parallelly to l^s. The line b passing through B parallelly to a has the projection $b^s(Z_b, B^s)$, where $Z_b = Z_a = Z_0$, and the trace $T_b = b^s \cap t_\varepsilon$ on the projection plane. Similarly, the trace of the line n is $T_n = n^s \cap t_\varepsilon$.

We construct the rabatment of the plane ε:

1. the maps of the points Z_0 and Z_n are perpendicular directions $Z_0^{x\infty} \perp Z_n^{x\infty}$ of the lines $z_0^x(0^x, Z_0)$ and $z_n^x(0^x, Z_n)$,
2. The map n^x of the line n^s passes through T_n and has the direction $Z_n^{x\infty}$,
3. the maps a^x and b^x of the lines a^s and b^s pass through T_a and T_b, respectively, and have the same direction $Z_a^{x\infty} = Z_b^{x\infty} = Z_n^{x\infty}$,
4. the maps of the points A^s and B^s are: $A^x = n^x \cap a^x$ and $B^x = n^x \cap b^x$; the map of the segment A^sB^S is then the segment A^0B^0 having the measure $|AB|$ of the segment AB projecting onto A^sB^s.

At the end we draw through T_a the line n^0 at the angle φ to the line t_ε; of course $n^0 \parallel z_n^x$ and so $n^0 \parallel n^x$. It is easy to see that the lines a^x and b^x project orthogonally to n^x the segment A^xB^x and the segment T_aT_b as well onto $A^0B^0 \subset n^0$, where $A^0 = T_a$. As the quadrangle $A^0B^0B^xA^x$ is a rectangle we have $A^xB^x = A^0B^0$ and so the construction of the Figure 1 is proved.

4. Remarks

Conversely, knowing the measure of AB we can build on Fig. 1 the triangle $A^0B^0T_n$ and obtain the perspective A^sB^s of AB. The construction is valid also in the case of a low horizon (when the orthogonal projection 0^r of the projection center 0 lies above the horizon line h) and of vertical projection plane as well, where the basis and horizon lines are interpreted as the traces on the projection plane of a plane inclined to the horizontal basis plane.

References

[1] B. Grochowski, *Reduction of the ocular distance on the slope projection plane*, Demonstratio Mathematica (to appear).

Recenzent: Dr hab. Jerzy Mroczkowski

Wpłynęło do redakcji 07.06.1995 r.

Streszczenie

W perspektywie pionowej odcinki pionowe odmierza się po ich równoległym przesunięciu na płaszczyznę tłową. Podana metoda odmierzania odcinków pionowych w perspektywie na nachylone tło polega także na ich równoległym przesunięciu, tak aby ich dolne końce znalazły sią na płaszczyźnie tła, a następnie na ich kładach przez obroty o kąt płaszczyzny tła z pionem. Metoda jest prosta w stosowaniu i przystępna w dydaktyce.