Romuald BOLEJKO[1]

3. CALCULATION OF SOUND ABSORPTION OF HELMHOLTZ RESONATORS BY NUMERICAL METHODS

3.1. Introduction

Helmholtz sound absorbers are commonly used in room acoustics and noise protection. The classic Helmholtz resonator consists of a closed chamber and a neck. The chamber is acoustic compliance, while the air in the neck of the resonator is the acoustic mass. The resonator chamber and neck may also contain lossy elements, e.g. fibrous or porous materials. Therefore, such a system is a vibration system with one degree of freedom, which is characterized by a single resonance (Fig. 1). In resonance, the acoustic mass in the neck of the resonator vibrates with high amplitude, and as a result of losses in the system, the energy of the acoustic wave is converted into heat energy. Thus, near the resonant frequency of Helmholtz resonator can show significant sound absorption. This means that such Helmholtz absorbers are useful when you need to suppress noise in a limited frequency band. In particular, such resonators are often used to absorb sound in the low frequency range, where the effectiveness of fibrous and porous materials is limited.

The properties of Helmholtz absorbers can be determined analytically only for their simple structures. For example, an analytical solution exists for systems with perfectly rigid walls and simple geometry of the resonator neck: a neck with a circular cross-section or in the form of a slit. The shape of the resonator chamber is not taken into account in such calculations and the geometry of the resonator neck is approximated by a cylinder for circular holes or a rectangular for slots [6]. Besides, there are difficulties in identifying losses in the resonator, which in turn have a strong effect on the quality factor Q at resonance, and thus determine the useful frequency range of the absorber. In addition, other authors have shown, e.g. in the works [1, 2], that the shape of the neck can have a major impact on the sound absorption of the resonator. Thus, there is a need to calculate the sound-absorbing properties of Helmholtz resonators taking

[1] Department of Acoustics, Multimedia and Signal Processing, Wrocław University of Science and Technology, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland, romuald.bolejko@pwr.edu.pl

into account their chamber and neck geometry and atypical losses (e.g. in the form of acoustically soft chamber walls).

Helmholtz sound absorbers are often used to control reverberation time and eliminate sound coloration in small rooms in the low frequency range. The use of resonators together with typical sound-absorbing materials made of fibrous and porous materials makes it possible to obtain the desired acoustic parameters of the room throughout the wide frequency band and, above all, to control the acoustics of the room in the low frequency range.

In noise protection, Helmholtz's sound absorbers are used to control narrowband noise, including tonal noise. Also, due to their design (they are resistant to harmful external conditions), such solutions are commonly used for road and rail noise control.

This work was part of a larger project on the use of photovoltaic (PV) panels on railway acoustic screens. Due to the placement of acoustically hard, i.e. highly reflective glass elements such as PV panels on a sound-absorbing acoustic screen, there will be a significant reduction in its noise absorption capacity. This occurs when the acoustically soft surface of the screen is covered by low-sound-absorbing elements (PV panels). To prevent this, it was proposed to design a way of attaching and arranging PV panels on the screen to create a sound-absorbing resonance system with a strong sound absorption capacity, the so-called Helmholtz-based sound-absorbing system.

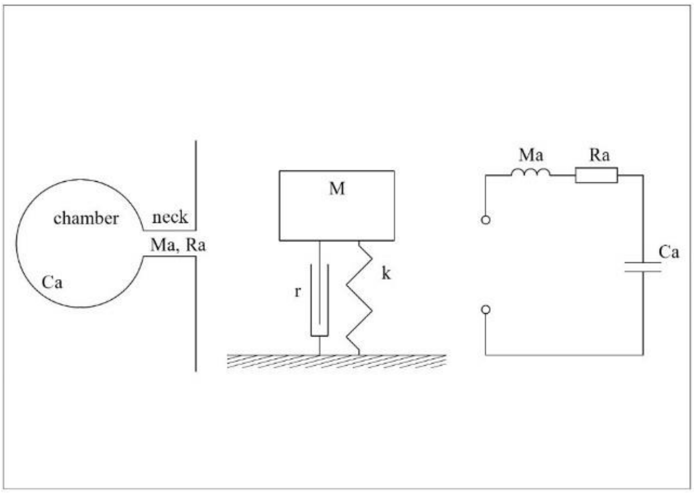

Fig. 1. Helmholtz resonator, its mechanical and electrical equivalent models
Rys. 1. Rezonator Helmholtza w ujęciu analogii elektro-mechano-akustycznej

3.2. Methods

The sound absorption coefficient shall quantifially determine the sound absorption capacity of the test surfaces. Theoretically, the properties of each surface are clearly determined by its acoustic impedance. Based on acoustic impedance, therefore it is possible to determine its sound absorption coefficient. A certain difficulty is a fact that the absorption of sound also depends on the angle of the wave incident. The sound absorption at a specified angle of the acoustic wave incidence on the test surface shall be determined by the physical sound absorption coefficient. This coefficient for the flat wave and normal incidence is determined from the equation:

$$a_f = 1 - \left|\frac{Z_{aj} - \rho_0 c}{Z_{aj} + \rho_0 c}\right|^2, \quad (1)$$

where: Z_{aj} – specific acoustic impedance of the surface, $\rho_0 c$ – characteristic acoustic impedance of medium, for air at 20° $\rho_0 c$= 415 rayl.

Based on the acoustic impedance of the surface, it is also possible to determine the reverberant absorption coefficient, which determines the sound-absorbing properties of the elements placed in the reverberation field [6].

The acoustic impedance of the sound-absorbing system may be determined analytically from the proper equations defined for porous and fibrous materials or a limited number of resonant absorbers. Analytical solutions, however, exist only for homogeneous materials and for simple structures of sound-absorbing systems. Theoretically, Z_{aj} of systems of any structure can be designated using FEM and BEM numerical methods.

In the case of numerical methods, acoustic impedance may be determined on the base of acoustic velocity and sound pressure distributions on the surface of the system. Based on pressure p and acoustic volume velocity V distributions, acoustic impedance is calculated from:

$$Z_{aj} = Z_a S_u = \frac{p}{V} S_u = \frac{p}{v}, \quad (2)$$

where: Z_a – acoustic impedance in rayl/m^2, p – pressure in Pa, V – volume acoustic velocity in m^3/s, v - acoustic velocity in m/s, S_u – area of the top surface of the system in m^2. For perfectly hard perforated panels, it is sufficient to count the acoustic impedance of only the holes (for the hard surfaces $v = 0$).

However, the possibility of calculating acoustic impedance for determining the sound absorption coefficient of resonant systems based on pressure and acoustic velocity distributions requires further verification.

3.2.1. Analytical method

The sound-absorbing properties of the Helmholtz resonator, expressed by the sound absorption coefficient α_f, can only be determined analytically for a resonator with perfectly hard inner walls and simple internal loss cases. The physical sound absorption coefficient for the perpendicular incidence of the flat wave is determined from (1) using the following equation of the acoustic impedance of the system:

$$Z_{aj} = R_{aj} + \mathrm{i}X_{aj} \quad (3)$$

$$X_{aj} = \omega M_{aj} - \frac{1}{\omega C_{aj}},$$

where: R_{aj} – acoustic resistance in rayl, M_{aj} – acoustic mass in kg/m^2, C_{aj} – acoustic compliance in m/Pa, ω – angular frequency.

The acoustic mass and acoustic compliance are determined from the geometric dimensions of the resonator, while the acoustic resistance is determined on the base of properties of the fabric in a gap or fibrous material filling the resonator chamber.

The structure of the actual Helmholtz absorber usually consists of a chamber partially filled with sound-absorbing material and a resonator neck. Such a system can therefore be considered as a layered system consisting of a layer of fibrous material placed on a hard surface, a layer of air and a perforated element (Fig. 2). It is most convenient to use the transfer matrix method to determine the acoustic impedance of such a system. In this method, the acoustic impedance of the current layer is determined taking into account its wave properties and the acoustic impedance of the previous layer. Thus, it is possible to calculate the actual Helmholtz absorber based on the acoustic impedance of three layers: fibrous material d_1 thickness, air layer d_2 thick and acoustic impedance Z_{pj} of a panel with perforation [6]:

$$\begin{gathered} Z_{aj} = Z_{pj} + Z_{2j} \\ Z_{2j} = \frac{-Z_{1j}\mathrm{i}\rho_0 c\cot(kd_2) + {\rho_0}^2 c^2}{Z_{1j} - \mathrm{i}\rho_0 c\cot(kd_2)}, \\ Z_{1j} = -\mathrm{i}z_0\cot(k_0 d_1) \end{gathered} \tag{4}$$

where: z_0, k_0 – specific impedance and wavenumber of fibrous porous material, k – wavenumber in the air.

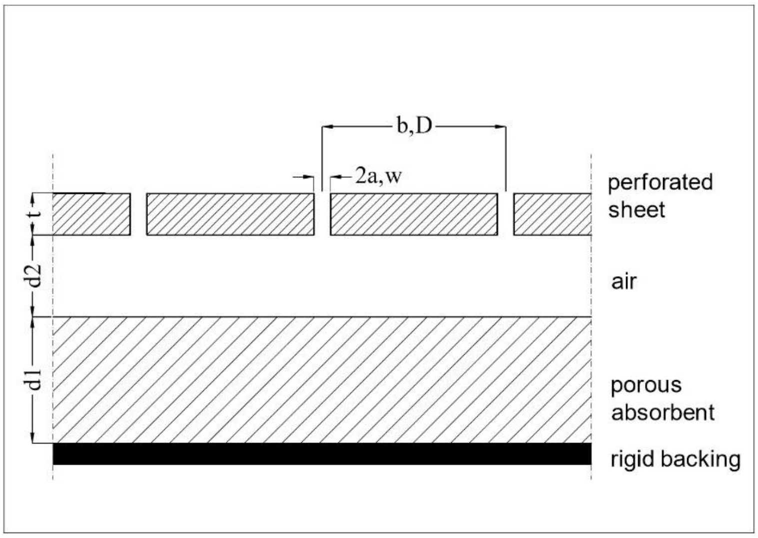

Fig. 2. Simple Helmholtz resonator with 3 layers
Rys. 2. Rezonator Helmholtza jako układ warstwowy

The characteristic impedance z_0 and the wave number k_0 of fibrous/porous material can be determined using existing analytical models for homogeneous materials, for example the Delany-Bazley model [3]. The acoustic impedance of the Z_{pj} perforated sheet can be determined analytically for selected perforation methods of the panel.

The acoustic impedance of a perforated panel without a resonance chamber is defined from equation (3) as the sum of the acoustic resistance R_{aj} and the acoustic mass M_{aj}. However, for systems in which fibrous material is present, R_{aj} holes/perforations are much less than the losses determined by the fibrous material placed in the resonator chamber. Thus, the unit impedance of the perforated panel can only be determined by the mass M_{aj}.

For panels with evenly spaced circular holes, the acoustic mass M_{aj} can be calculated from the equation [4]:

$$M_{aj} = \frac{\rho_0}{\varepsilon}\left[t + 2 \cdot 0.85a\left(1 - \frac{a}{b}\right)\right], \tag{5}$$

where: ρ_0 – air density in kg/m^3, a – hole radius in m, b – distance between holes in m, t – sheet thickness in m, ε – fraction of open space.

The second term in square brackets is an end correction factor for a circular hole. In the references [4, 5, 6] you can find several dependencies on the parameter, but one given above is used most often.

In the case of slotted panels, there is a certain difficulty in determining the end correction factor for the slit. Measurements of such systems, on the other hand, indicate a large impact of this on their sound-absorbing properties. There are two basic equations on the acoustic mass M_{aj} of narrow slit. The first mentioned by Beranek for a single narrow slip [4]:

$$M_{aj} = \frac{6}{5}\frac{\rho_0}{\varepsilon}t, \tag{6}$$

where: t – thickness of sheet in m, ε – a fraction of open space. Whereby the end correction factor is factor 6/5.

The second most commonly used dependency is determined by Smits and Kosten [5]. This relationship gives a better approximation for slotted systems with wide slots:

$$M_{aj} = \frac{\rho_0}{\varepsilon}(t + 2\delta w) \tag{7}$$

$$\delta = -\frac{1}{\pi}\ln\left[\sin\left(\frac{1}{2}\pi\varepsilon\right)\right],$$

where w – width of slit in m, δ – end correction factor for a slit.

For slotted systems made up of plank elements with a width similar to or several times the width of the gap, both dependencies give similar results. On the other hand, the results of calculations of large-slot systems are only consistent with the measurements for the dependency specified by Smits and Kosten.

In this work, analytical models of a circular perforated resonant absorber and slotted systems were used to verify the results of calculations obtained by numerical methods and to assess the accuracy of the proposed method for determining the sound-absorbing properties of Helmholtz resonance systems by numerical methods.

3.2.2. Numerical methods

Numerical methods, unlike analytical methods, allow the calculation of acoustic field parameters for any geometry of the modeled object and all possible boundary conditions, e.g. acoustically soft resonator walls. In acoustics, two numeric methods are most commonly used: the Boundary Elements Method (BEM) and the Finite Elements Method (FEM). In these methods, the actual, continuous, modeled area or its edge shall be replaced by a set of discrete elements of small dimensions in relation to the sound wavelength. Based on the values in the nodes, using an approximation of acoustic parameter values within a finite element, it is possible to create a system of N equations with N unknowns. The values in the mesh nodes are obtained by solving the wave equation for the given boundary conditions. N specifies the number of nodes in the discrete model.

BEM method

In the BEM method, only the edge of the modeled closed area is discretized. The sound pressure and velocity inside or outside the enclosed space shall be determined based on the pressure distribution and the acoustic velocity only at its edge. Fig. 3a illustrates an example of a boundary mesh for a classic Helmholtz resonator.

The sound pressure at any point X is determined by the distribution of the sound pressure $p(Y)$ and its derivative $\partial_n p(Y)$ in the normal direction to the boundary element specified on the surface S of the boundary mesh:

$$p(X) = \int_S \left(p(Y)\partial_n G(X,Y) - G(X,Y)\partial_n p(Y)\right)\mathrm{d}S(Y), \tag{8}$$

where: $G(X,Y) = \frac{\mathrm{e}^{-ikr}}{4\pi r}$ Green function, $r = |Y - X|$, k – wave number.

In turn, the distribution of the sound pressure $p(Y)$ and its derivative $\partial_n p(Y)$ in the normal direction to the boundary element is determined based on the boundary conditions specified in the model.

a) b)

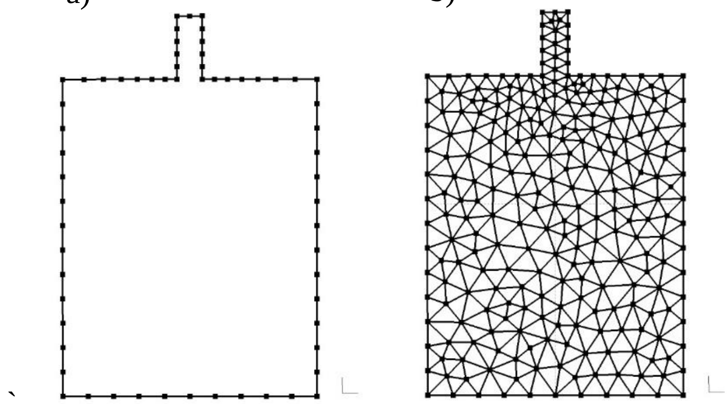

Fig. 3. 2D boundary elements (a) and finite elements (b) mesh of Helmholtz resonator
Rys. 3. Siatka 2D elementów brzegowych (a) i elementów skończonych (b) rezonatora Helmholtza

The BEM has difficulty modeling the acoustic field inside and outside the partially open area, as there is a discontinuity of the boundary edge. In the case of resonant absorber modeling, this limitation of the method is very important, since it does not allow to take into account the radiation impedance of the resonator neck, nor the mutual impedance of the radiation of the resonator group. Therefore, the usefulness of the BEM in determining the sound-absorbing properties of Helmholtz absorbers appears to be very limited.

FEM method

In FEM, the entire modeled area is discretized (Fig. 3b). Therefore, this method is especially suitable for modeling an acoustic field inside a limited space. Analysis of radiation problems in the free field requires an appropriate "acoustically infinite" end to the modeled space. For this purpose, so-called 'infinite' elements or free field boundary condition shall be used. Fig. 4 illustrates the two-dimensional FEM model of the discrete group 3 Helmholtz resonators, which for such a formulation of computational problem are slotted resonators (the neck of the resonator is a slit, not a hole). On the selected edges of the elements, on the semi-circle, the radiation condition should be set to free field conditions.

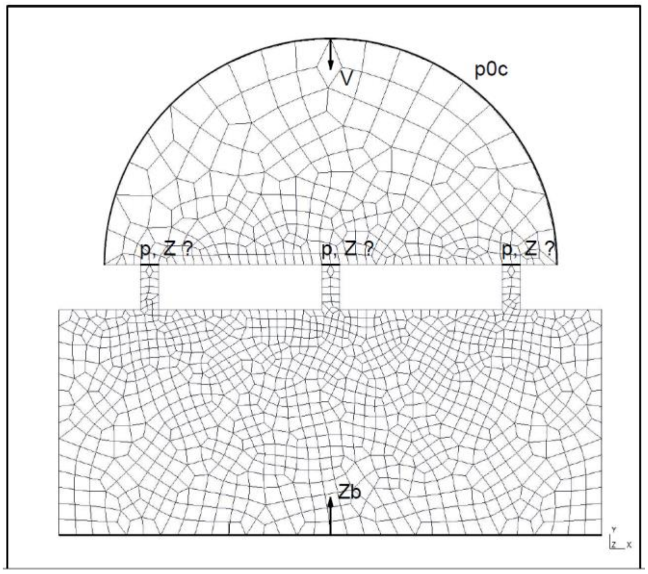

Fig. 4. 2D FEM model of slotted Helmholtz resonator with three slots and free field termination
Rys. 4. Siatka 2D elementów skończonych szczelinowego rezonatora Helmholtza

The sound pressure in FEM models is determined by the N system solution of equations under the set boundary conditions, which can be recorded in matrix form:

$$([K] + i\rho\omega[C] - \omega^2[M])\{p\} = -i\rho\omega\{F\}, \tag{9}$$

where: $[K]$ – acoustic stiffness matrix, $[C]$ – matrix of losses, $[M]$ – acoustic mass matrix, $\{p\}$ – vector of sound pressure at nodes, $\{F\}$ – vector of acoustic load at nodes.

The acoustic velocity in the FEM is determined by the differential of sound pressure along with the selected directions in the individual nodes of the discrete mesh.

The FEM is available in many commercial computer programs and generally produces reliable calculation results. Therefore, in this work, the verification of the possibility of the calculation of Helmholtz absorbers by numerical methods was mainly based on the FEM.

3.3. Results and discussion

In order to verify the possibility of determining the sound absorption coefficient using the methodology presented, calculations were carried out for two Helmholtz resonant systems: the round-necked system and the slotted one. The first system consisted of a closed cylinder-shaped chamber and a resonator neck of a set length with a circular cross-section. The dimensions and geometry of the resonator are shown in Fig. 5. The fraction of the open space of the system was equal to ε = 16%. An impedance boundary condition was defined on the lower wall of the chamber, while the other walls of the system were perfectly rigid. The specified boundary condition corresponded to the acoustic impedance of mineral wool with a flow resistance of 3000 Pa·s/m and a thickness of 10 cm placed on a rigid wall. An analytical model Delany-Bazley [3] specified for fibrous and porous materials was used to calculate the acoustic impedance. Fig. 6 provides impedance calculated using this analytical model. It was the boundary condition of the lower wall of the resonator chamber in numerical models.

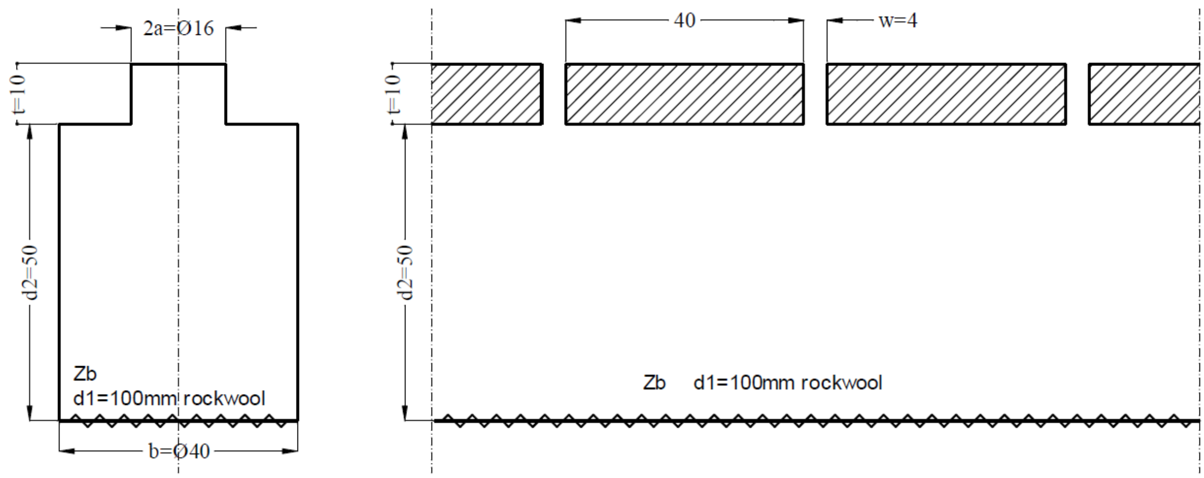

Fig. 5. Geometry and dimensions of analyzed axisymmetric (left) and slotted (right) Helmholtz resonators

Rys. 5. Geometria i wymiary osiowosymetrycznego (po lewej) i dwuwymiarowego szczelinowego (po prawej) rezonatorów Helmholtza

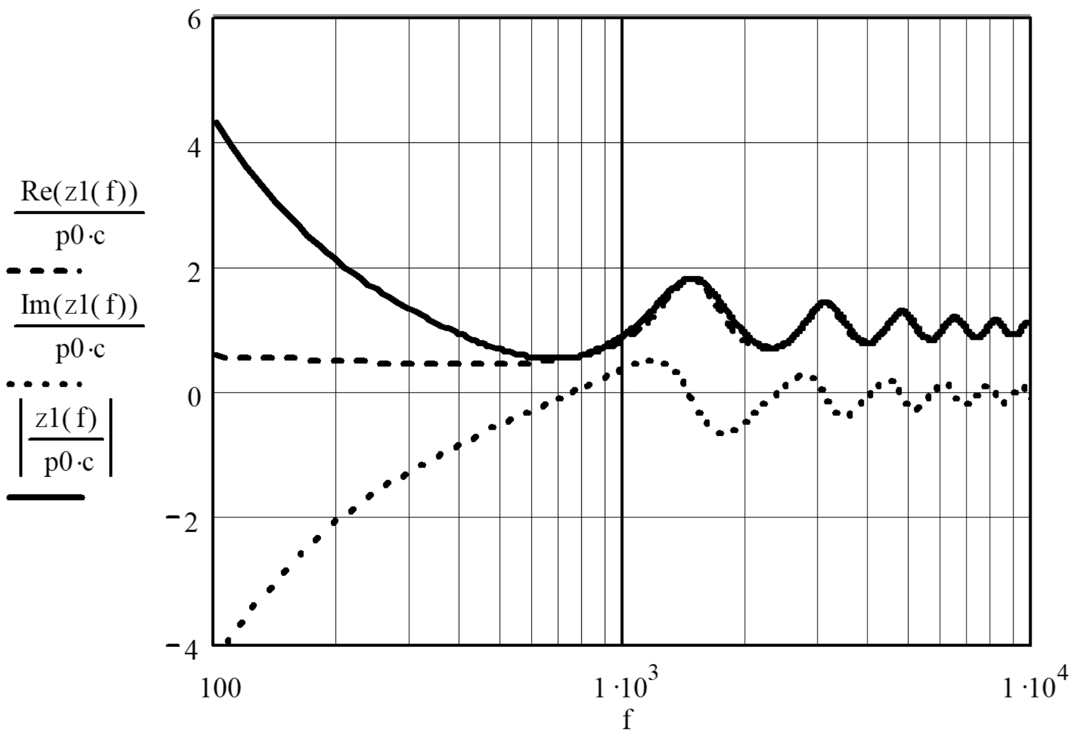

Fig. 6. Acoustic impedance of 100 mm glass wool backed by a rigid wall calculated by Delany-Bazley analytic model
Rys. 6. Znormalizowana jednostkowa impedancja akustyczna 100 mm maty z wełny szklanej umieszczonej na twardej ścianie wyznaczona z analitycznego modelu Delany-Bazley'a

To correctly capture the radiation impedance of the resonator neck in the FEM model, it is necessary to ensure that acoustic energy can be radiated into the free space without reflections. This can be achieved in two ways: by discretizing a small part of free space and end it by using so-called "infinite" elements (appropriately formulated finite elements ensuring no sound reflections) or by discretizing a small fragment of free space and on its outer edge define acoustic impedance equal to the impedance of medium $\rho_0 c$. The second solution is less accurate and generally requires discretization of a much larger part of the free space than in the first case. However, not all commercial FEM programs allow the use of "infinite" elements, this possibility is only available in selected calculation modules developed exclusively for modeling acoustic phenomena.

For this reason, the latter option of ending the free radiation space is used in this work. No reflections were obtained by imposing acoustic impedance $\rho_0 c$ on its outer edge (Fig. 7). Therefore, the effect of the size of the discretized part of the free space and the way how it ends on the value of the absorption coefficient of the modeled sound-absorbing systems was first investigated. The calculation uses four axisymmetric models, which are shown in Fig. 7. In the first model, the finite element mesh is terminated at the neck of the resonator. Acoustic load in the form of Neumann boundary condition (specific acoustic velocity) is determined at the outlet of the resonator neck, and acoustic impedance Z_b (100 mm thick wool impedance) is determined on the lower wall of the resonator chamber. The acoustic impedance of the resonator is determined by calculating the sound pressure distribution in the resonator neck. With such a definition of the problem, the impedance of radiation of the neck of the resonator is not taken into account. In a further three cases, the free radiation space takes the form of a semi-sphere (in the axisymmetric model of a quarter of a circle) with three different radii ending by impedance boundary condition of $\rho_0 c$. The radius of the free space shall be equal to the double, quadruple and eight-times the radius of the resonator neck. In these three models, the acoustic load is formulated in the form of an acoustic velocity v = 1 m/s at the outer edge of the free space, together with an impedance condition of $\rho_0 c$. These three models are designed to determine the effect of reflections on the outer edge of the free space with a condition of $\rho_0 c$ in the absence of an "infinite" element.

Fig. 8-Fig. 12 provide the results of the acoustic impedance calculations of the resonator neck and the sound absorption coefficient for the tested Helmholtz resonator. In particular, the following patterns can be observed in the example of characteristics of α.

For models with a free radiation space with a radius of 4a and 8a, identical results are obtained, but different than for an area with a radius of 2a. In analytical and numerical calculations, there is a clear effect of the way of taking into account the impedance of the radiated neck on the values and frequency characteristics of α of the resonator. Since only the outer opening of the neck radiates into the free space, while the inner opening ends with a closed chamber, it seems reasonable to take into account the radiation impedance of only one of the two sides of the resonator neck. In this case, taking into account in the analytical model the impedance of radiation only the outer opening of the resonator neck, a good consistency is achieved between the numerical results and the analytical solution. The characteristics obtained by numerical and analytical methods are almost identical in shape, while the differences in values occur practically only near the resonances of the absorber. Such a discrepancy is likely due to the limited and, above all, much smaller, resolution of the frequency for numerical methods.

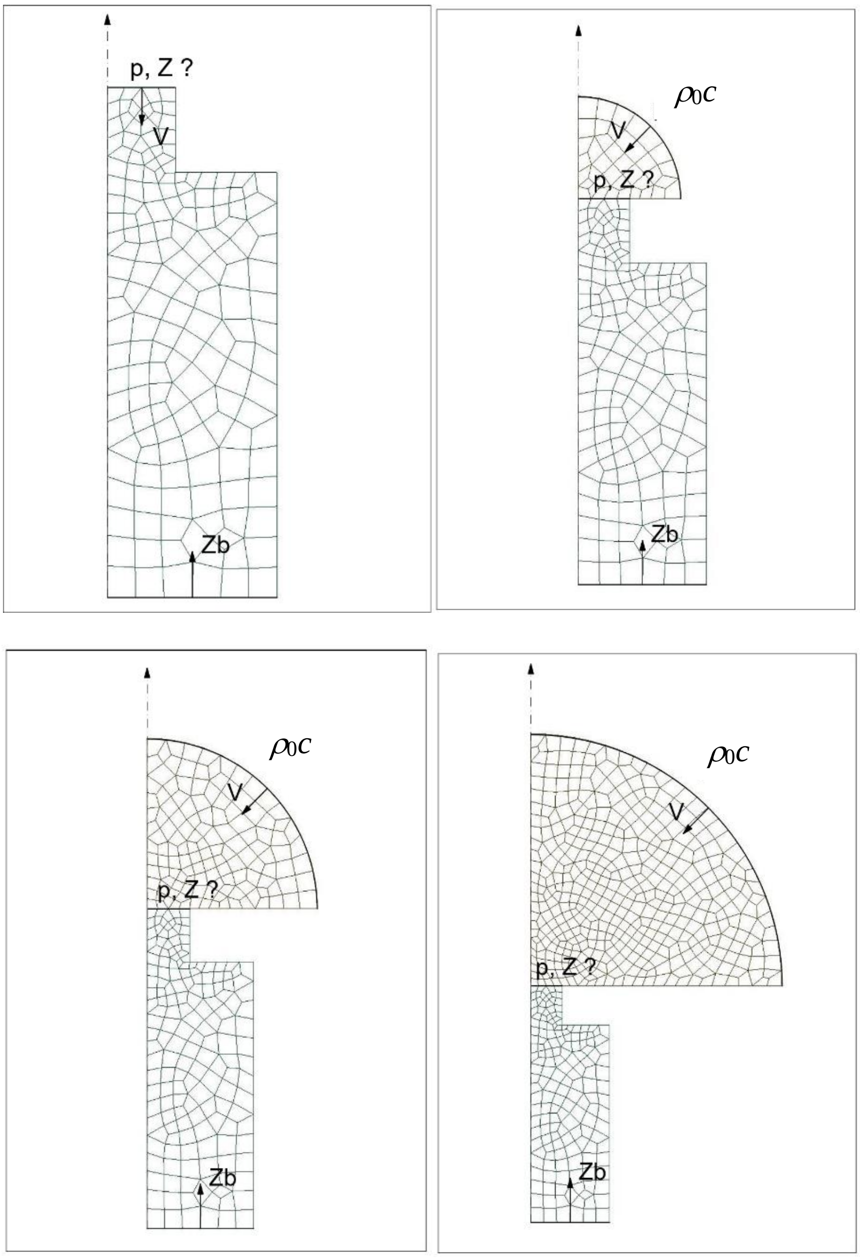

Fig. 7. Axisymmetric finite meshes of Helmholtz resonator for a different approach of termination of free radiation condition

Rys. 7. Osiowosymetryczna siatka elementów skończonych dla czterech różnych sposobów uzyskania warunku swobodnego promieniowania na zewnętrznej granicy obszaru modelu

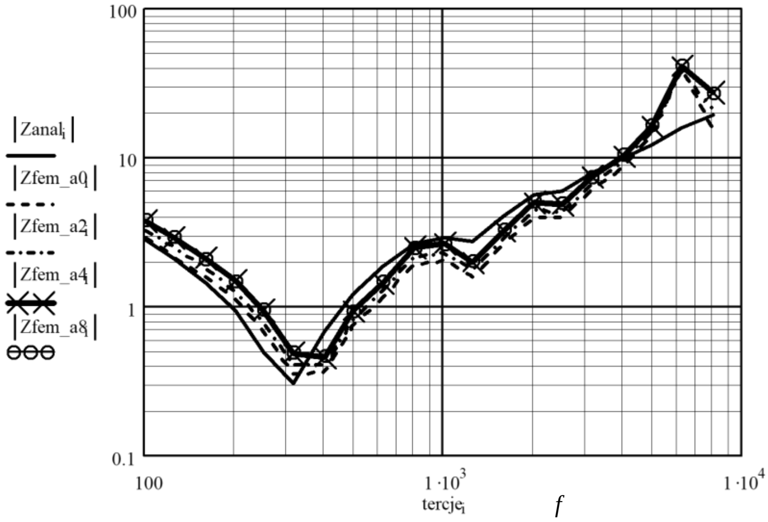

Fig. 8. Magnitude of acoustic impedance of axisymmetric Helmholtz resonator calculated by: Zanal – a alytic method using neck's outer end correction factor, Z_fem_a0 – FEM without radiation impedance, Z_fem_a2 – FEM with a free field of 2*a* radius, Z_fem_a4 – FEM with a free field of 4*a* radius, Z_fem_a8 – FEM with a free field of 8*a* radius

Rys. 8. Moduł znormalizowanej jednostkowej impedancji akustycznej dla osiowosymetrycznego rezonatora Helmholtza uzyskany różnymi metodami: Zanal – metodą analityczną z uwzględnieniem impedancji promieniowania otworu szyjki rezonatora, Z_fem_a0 – metodą FEM bez uwzględnienia impedancji promieniowania, Z_fem_a2 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 2*a*, Z_fem_a4 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 4*a*, Z_fem_a8 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 8*a*

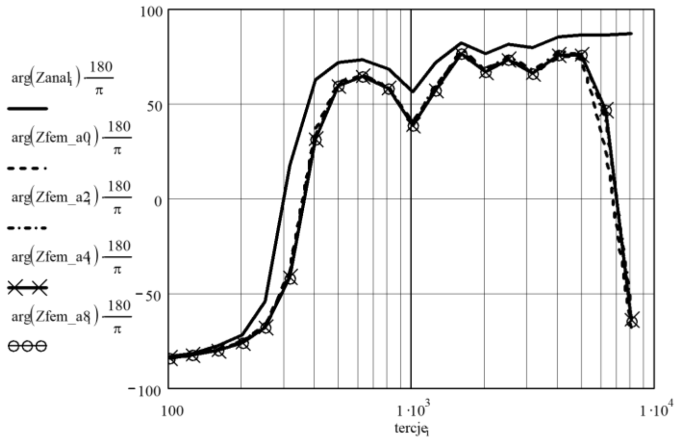

Fig. 9. Phase of acoustic impedance of axisymmetric Helmholtz resonator calculated by: Zanal – analytic method using neck's outer end correction factor, Z_fem_a0 – FEM without radiation impedance, Z_fem_a2 – FEM with a free field of 2*a* radius, Z_fem_a4 – FEM with a free field of 4*a* radius, Z_fem_a8 – FEM with a free field of 8*a* radius

Rys. 9. Faza znormalizowanej jednostkowej impedancji akustycznej dla osiowosymetrycznego rezonatora Helmholtza uzyskany różnymi metodami: Zanal – metodą analityczną z uwzględnieniem impedancji promieniowania otworu szyjki rezonatora, Z_fem_a0 – metodą FEM bez uwzględnienia impedancji promieniowania, Z_fem_a2 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 2*a*, Z_fem_a4 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 4*a*, Z_fem_a8 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 8*a*

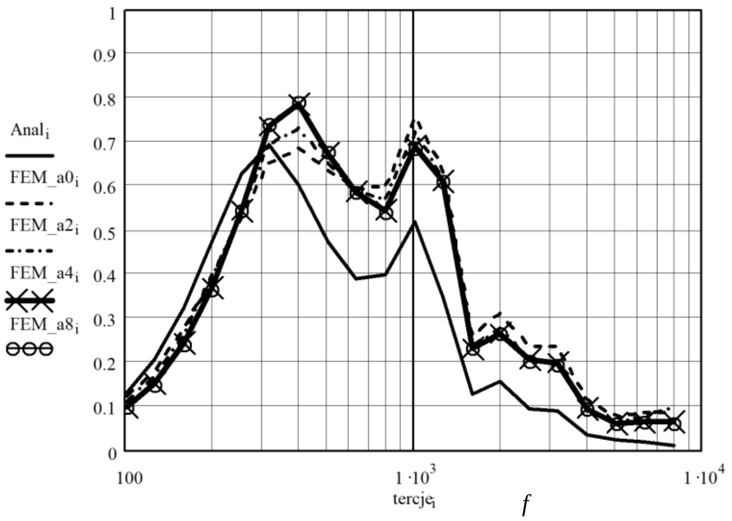

Fig. 10. Sound absorption coefficient of axisymmetric Helmholtz resonator calculated by: Anal − analytic method and both side radiation correction factor, FEM_a0 – FEM without radiation impedance, FEM_a2 – FEM with a free field of 2*a* radius, FEM_a4 – FEM with a free field of 4*a* radius, FEM_a8 – FEM with a free field of 8*a* radius

Rys. 10. Fizyczny współczynnik pochłaniania dźwięku osiowosymetrycznego rezonatora Helmholtza wyznaczony: Anal − metodą analityczną z uwzględnieniem impedancji promieniowania wlotu i wylotu szyjki rezonatora, FEM_a0 – metodą FEM bez uwzględnienia impedancji promieniowania, FEM_a2 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 2*a*, FEM_a4 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 4*a*, FEM_a8 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 8*a*

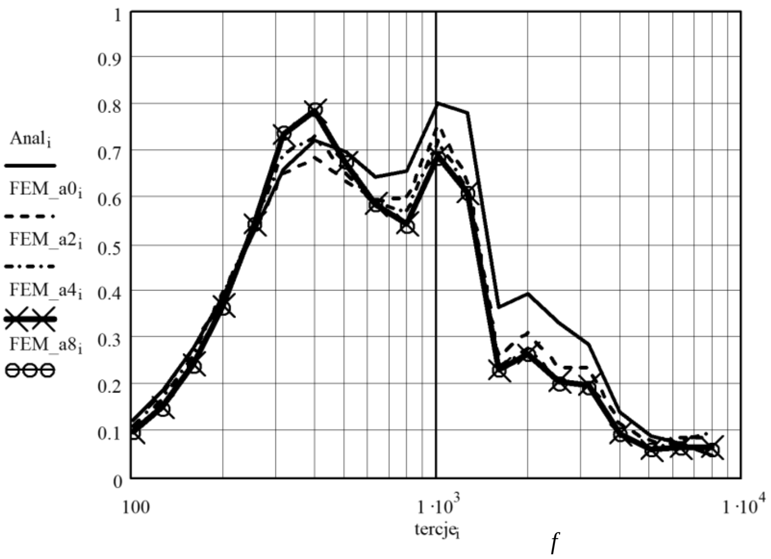

Fig. 11. Sound absorption coefficient of axisymmetric Helmholtz resonator calculated by: Anal − analytic method without radiation impedance, FEM_a0 – FEM without radiation impedance, FEM_a2 – FEM with a free field of 2*a* radius, FEM_a4 – FEM with a free field of 4*a* radius, FEM_a8 – FEM with a free field of 8*a* radius

Rys. 11. Fizyczny współczynnik pochłaniania dźwięku osiowosymetrycznego rezonatora Helmholtza wyznaczony: Anal – metodą analityczną bez uwzględnieniem impedancji promieniowania wlotu i wylotu szyjki rezonatora, FEM_a0 – metodą FEM bez uwzględnienia impedancji promieniowania, FEM_a2 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 2*a*, FEM_a4 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 4*a*, FEM_a8 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 8*a*

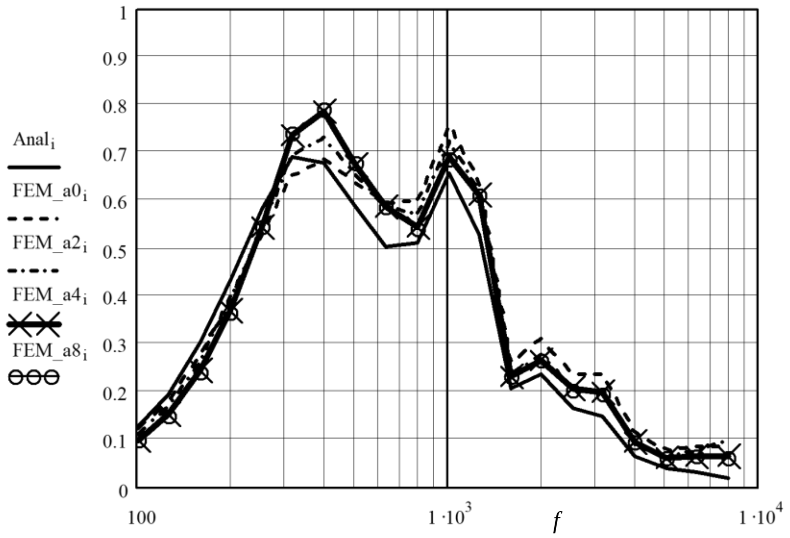

Fig. 12. Sound absorption coefficient of axisymmetric Helmholtz resonator calculated by: Anal − analytic method and external side radiation impedance only, FEM_a0 – FEM without radiation impedance, FEM_a2 – FEM with a free field of 2*a* radius, FEM_a4 – FEM with a free field of 4*a* radius, FEM_a8 – FEM with a free field of 8*a* radius

Rys. 12. Fizyczny współczynnik pochłaniania dźwięku osiowosymetrycznego rezonatora Helmholtza wyznaczony: Anal − metodą analityczną z uwzględnieniem impedancji promieniowania tylko wylotu szyjki rezonatora, FEM_a0 – metodą FEM bez uwzględnienia impedancji promieniowania, FEM_a2 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 2*a*, FEM_a4 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 4*a*, FEM_a8 – metodą FEM dla obszaru swobodnego promieniowania o promieniu 8*a*

For further verification, another Helmholtz absorber, the slotted system, was also analyzed. The selected system consisted of panels 40 mm wide and 10 mm thick (this is also the thickness of the slit), located at a distance of 50 mm from the rear wall of the resonator (Fig. 5). Acoustic impedance corresponding to a layer of mineral wool 100 mm thick has been tiled on the back wall of the chamber. The width of the slit was 4 mm, so the fraction of open space was ε = 9%. Such a system is still Helmholtz absorber, in which the slit is the neck of the resonator. A single-slit system (no mutual radiation impedance) and a system with three slits were considered. The two-dimensional finite element models are shown in Fig. 13. The free radiation space with a radius of 4*w*/2 was used in the model. The acoustic load is defined by the Neumann boundary condition (*v* = 1 m/s) on the outer edge of the free radiation space together with the impedance condition $\rho_0 c$.

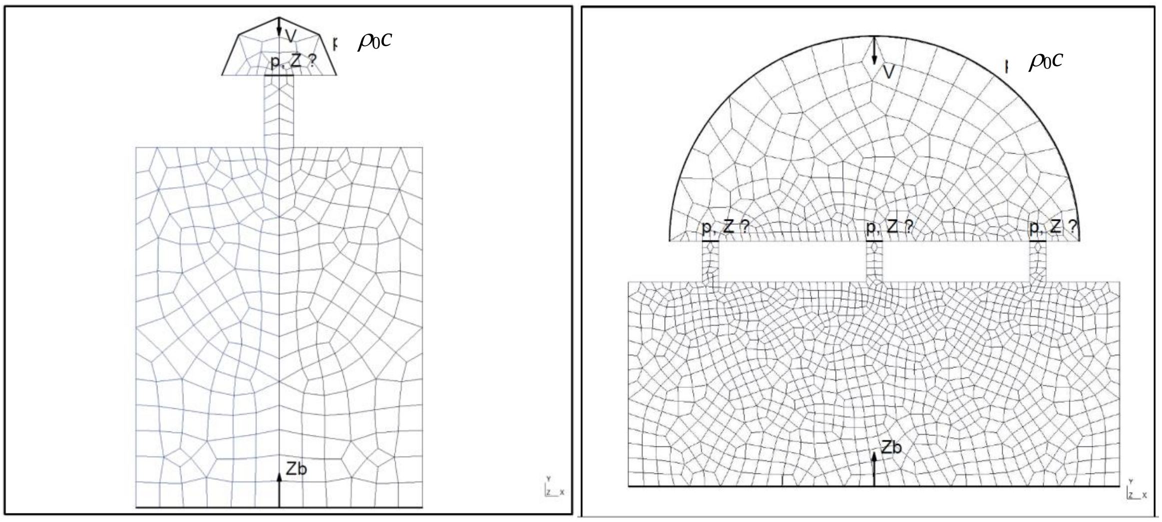

Fig. 13. Two-dimension meshes of slotted Helmholtz resonator with one (left) and 3 slits (right)
Rys. 13. 2D siatka elementów skończonych dla modelu szczelinowego rezonatora Helmholtza z jedną (po lewej) i 3 szczelinami (po prawej)

The analytical calculations used two different formulations of the impedance of the slit: given by Beranek [4] and by Smiths and Kosten [5].

Fig. 14 - Fig. 16 provide the results of acoustic impedance calculations and sound absorption coefficient α in the analytical method for a single-slot system and a FEM model with one and three slots.

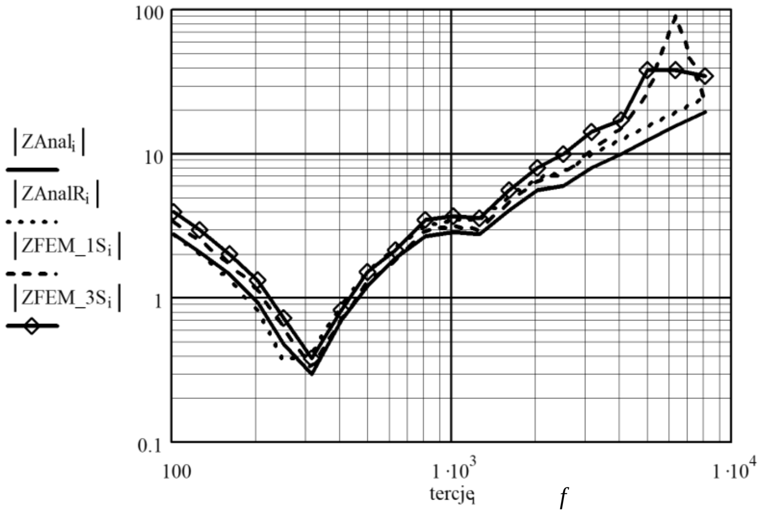

Fig. 14. Magnitude of acoustic impedance of slotted Helmholtz resonator calculated by: Zanal – analytic method by Beranek, ZanalR – the analytic method by Smiths and Kosten, ZFEM_1S – FEM without mutual radiation impedance (1 slit), ZFEM_3S – FEM with mutual radiation impedance (3 slits)
Rys. 14. Moduł znormalizowanej jednostkowej impedancji akustycznej szczelinowego rezonatora Helmholtza uzyskany: Zanal – metodą analityczną z wykorzystaniem wzoru Beranka, ZanalR – metodą analityczną z wykorzystaniem wzorów Smithsa i Kostena, ZFEM_1S – metodą FEM bez uwzględnienia wzajemnej impedancji promieniowania (jedna szczelina), ZFEM_3S – metodą FEM z uwzględnienia wzajemnej impedancji promieniowania (3 szczeliny)

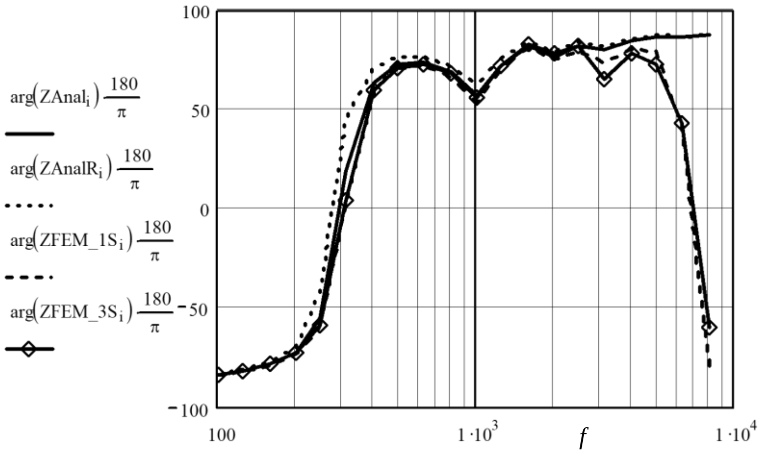

Fig. 15. Phase of acoustic impedance of slotted Helmholtz resonator calculated by: Zanal – the analytic method by Beranek, ZanalR − the analytic method by Kosten, ZFEM_1S – FEM without mutual radiation impedance (1 slit), ZFEM_3S – FEM with mutual radiation impedance (3 slits)

Rys. 15. Faza znormalizowanej jednostkowej impedancji akustycznej szczelinowego rezonatora Helmholtza uzyskany: Zanal − metodą analityczną z wykorzystaniem wzoru Beranka, ZanalR − metodą analityczną z wykorzystaniem wzorów Smithsa i Kostena, ZFEM_1S – metodą FEM bez uwzględnienia wzajemnej impedancji promieniowania (jedna szczelina), ZFEM_3S – metodą FEM z uwzględnienia wzajemnej impedancji promieniowania (3 szczeliny)

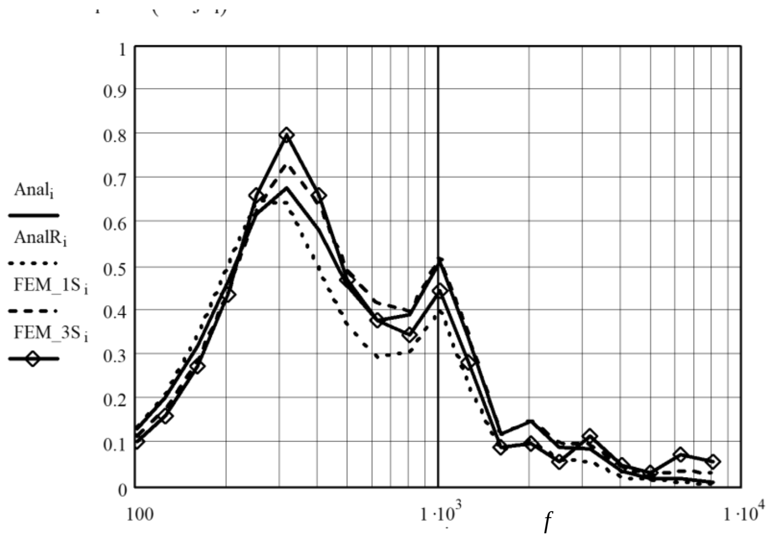

Fig. 16. Sound absorption coefficient of slotted Helmholtz resonator calculated by: Zanal – the analytic method by Beranek, ZanalR − the analytic method by Kosten, ZFEM_1S – FEM without mutual radiation impedance (1 slit), ZFEM_3S – FEM with mutual radiation impedance (3 slits)

Rys. 16. Fizyczny współczynnik pochłaniania dźwięku szczelinowego rezonatora Helmholtza wyznaczony: Zanal – metodą analityczną z wykorzystaniem wzoru Beranka, ZanalR – metodą analityczną z wykorzystaniem wzorów Smithsa i Kostena, ZFEM_1S – metodą FEM bez uwzględnienia wzajemnej impedancji promieniowania (jedna szczelina), ZFEM_3S – metodą FEM z uwzględnienia wzajemnej impedancji promieniowania (3 szczeliny)

Analyzing the presented results, it can be seen that numerical calculations quite well coincide with analytical ones. In FEM calculations, the shape of the characteristics, the frequencies of the two resonances are correctly obtained, but there are discrepancies in the absorption values in the resonances. Besides, numerical calculations show that due to the consideration of mutual radiation impedance, the sound absorption for the basic resonance increases and at the same time decreases for the second resonance. Therefore, omitting the mutual impedance of radiation, which is the case for the most commonly used analytical methods of perforated systems, can significantly reduce the usefulness of the method.

Fig. 17 provides examples of the results of calculating the pressure distribution and acoustic velocity using the FEM for a group of three slotted resonators for frequencies near the resonance of the system. Based on the analysis of the acoustic velocity distribution, it can be seen that for frequencies near the resonance of the system, the greatest speed value is in the neck of the resonator. This is consistent with the physics of the Helmholtz resonator and confirms the correctness of the modeling results obtained. Also based on the sound pressure distribution, it can be concluded that the geometry of the free radiation area used and the set velocity boundary condition causes a flat wave to incidence on the system.

Modeling has shown that the sound absorption coefficient of the Helmholtz resonator can be correctly obtained using proposed methodology. FEM models with a well-formulated "infinite" boundary condition at the limit of the free radiation space allow reliable results to be obtained for the complex geometry of sound-absorbing systems, taking into account any acoustic impedance on the inner walls of the resonator (e.g. acoustically soft walls). Besides it is possible to take into account losses in the resonator in the form of acoustic fabrics or absorbing fibrous materials in a wide configuration of their placement in the neck or inside the resonator chamber. The proposed method also makes it possible to assess the effect of both the impedance of the radiation of the neck outlet and the mutual impedance of radiation for the group of resonators on the value of the Helmholtz resonator sound absorption.

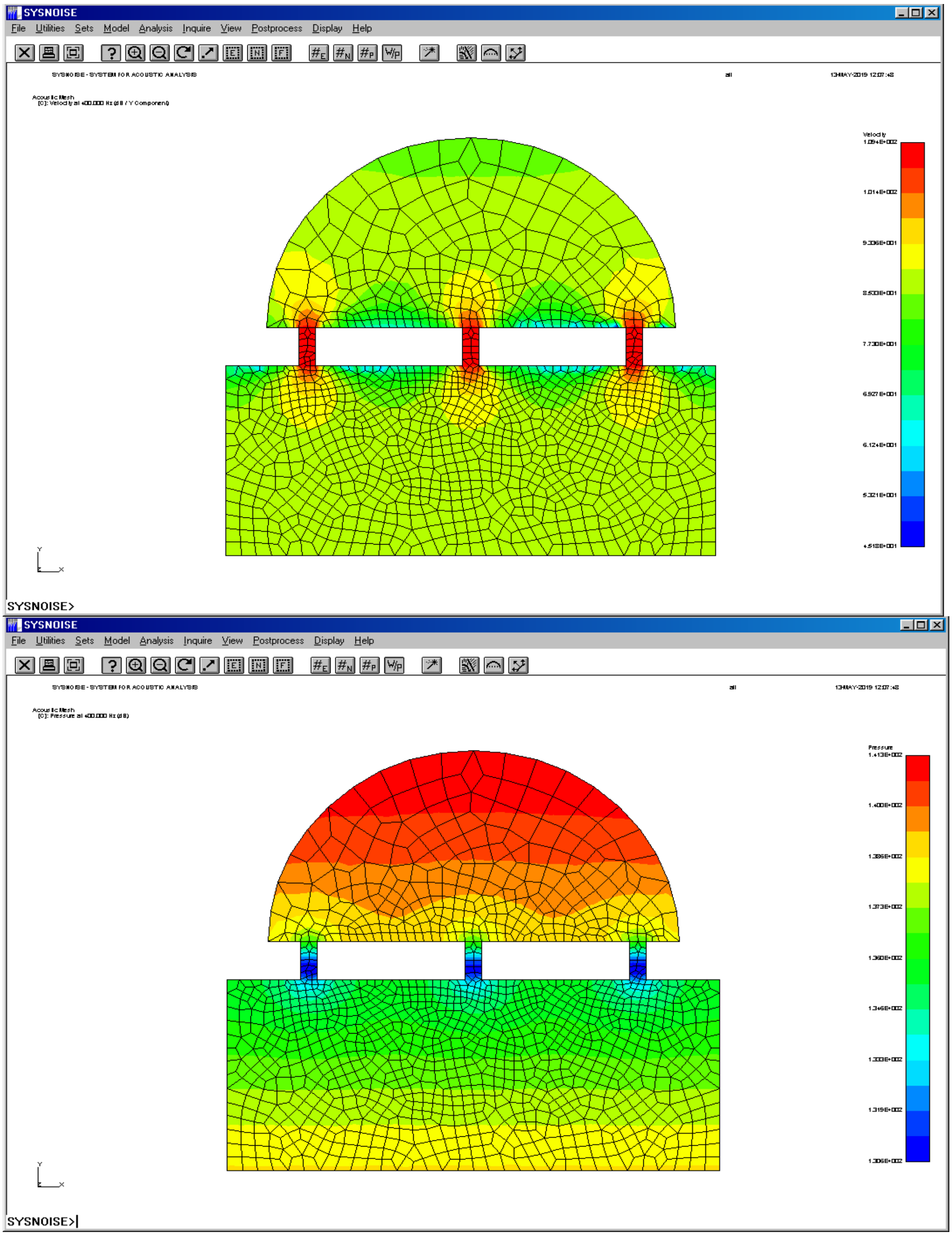

Fig. 17. Acoustic velocity (top) and pressure (bottom) in slotted Helmholtz resonator at a frequency near its basic resonance

Rys. 17. Rozkład prędkości akustycznej (góra) i ciśnienia akustycznego (dół) dla szczelinowego rezonatora Helmholtza w pobliżu jego podstawowego rezonansu

3.4. Conclusions

The sound absorption coefficient of the Helmholtz absorbers can be determined on the base of acoustic pressure and velocity distribution in the neck of a resonator. The proposed methodology used for determining that distribution using FEM produces reliable results. Moreover, the proposed method makes it possible to take into account the radiation of the resonator’s neck, including the mutual radiation impedance. It was also shown that the mutual radiation impedance has a significant impact on the sound-absorbing properties of the resonant absorber.

The proposed methodology is validated for classical circular hole shaped and slotted systems. A good agreement between analytical methods and the proposed approach is achieved. However, this finding requires further verification by comparing the modeling results with the results of measurements of the actual Helmholtz resonators.

References

1. Vigran T.E., Pettersen O.K.: The absorption of slotted panels revisited. Proc. of the Forum Acusticum, Budapest, Hungary 2005, 2037-2040.
2. Tang S.K.: On Helmholtz resonators with tapered necks. Journal of Sound and Vibration. vol. 279, 2005, 1085-1096.
3. Delany M.A., Bazley, M.A.: Acoustic properties of fibrous absorbent materials. Appl. Acoust. 3,1970, 105-116.
4. Beranek L.: Acoustics, Acoustical Society of America. New York 1996.
5. Smits J.M.H., Kosten C.W.: Sound absorption by slit resonators. Acustica 1, 1951, 114-122.
6. Cox T.J., D’Antonio P.: Acoustic Absorbers and Diffusers. Theory, design and application. Taylor&Francis, New York 2009.

Acknowledgments

This work is supported by The National Centre for Research and Development in the framework of project No. 11IR/0008/18 conducted in the Department of Acoustics, Multimedia and Signal Processing at Wrocław University of Science and Technology.