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## **19. RECONSTRUCTION OF THE IMAGE OF STRUCTURE WELL REFLECTING THE ULTRASOUND WAVE BY USING DOPPLER TOMOGRAPHY METHOD**

### **19.1. Introduction**

The paper will be mainly devoted to the method of image reconstruction called Doppler Tomography (DT). This is an innovative and relatively unknown method of imaging stationary sections of the examined object. A two-transducer ultrasonic transmitter-receiver head is used for measurements. It is set in motion to obtain the Doppler effect. It should be emphasized that Doppler Tomography differs significantly from the well-known and described measurement of blood flow velocity in blood vessels. Both methods use an ultrasonic head and the Doppler effect, but the first method reconstructs the image of a stationary cross-section of an object (e.g. tissue), while the second method shows the speed of moving blood.

The first part of the paper will be devoted to the description of the principle of Doppler Tomography. It will describe in detail how to reconstruct the image using selected algorithms. Basic terms such as Doppler signal, sinogram, Doppler band and others will be explained. The next part will contain results of imaging simulations based on the calculated measurement signal. A special program has been created for the simulation in LabVIEW environment. It allowed to investigate the influence of DT parameters such as for example number of Doppler bands on the imaging quality. Necessary modifications of the method will also be presented to enable more efficient image reconstruction. Additionally, two algorithms created for Doppler Tomography will be presented, allowing to significantly improve the quality of obtained images. The last part will be devoted to the reconstruction of real images of a 2.8 mm diameter brass rod.

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## 19.2. Doppler Tomography method

### 19.2.1. The main concept of image reconstruction using Doppler Tomography

The Doppler effect has been used in medicine since the fifties of the last century, when a Japanese scientist made the first analysis of heart valve motion [4]. One of the most popular methods based on this effect is to measure the velocity of blood flow in blood vessels using a continuous ultrasonic wave [3]. Figure 1 shows the general principle of measurement for this method. It shows a two-transducer ultrasonic transmitter-receiver probe generating a continuous wave of  $f_T$  frequency. This wave enters the blood vessel and bounces off the moving erythrocytes. At this point, due to the Doppler effect, the ultrasonic signal frequency changes to  $f_R$ . The reflected wave returns to the receiving transducer of the probe. The difference between  $f_T$  and  $f_R$  is the Doppler frequency  $f_d$ . It is related to the blood flow velocity by the formula (1).

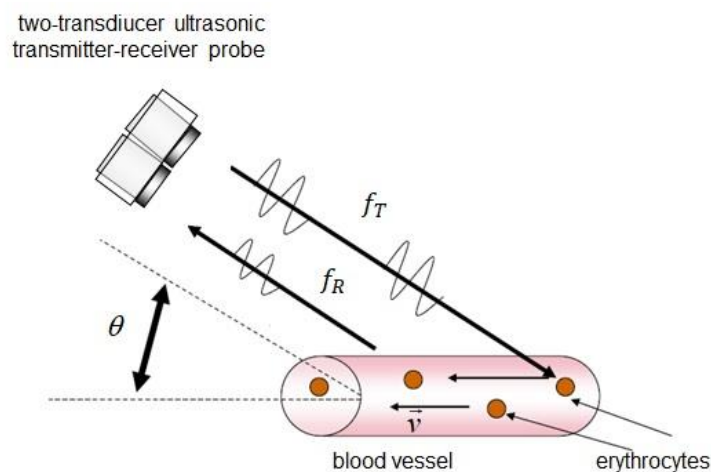


Fig. 1. General principle of measuring blood velocity flow using Doppler effect and ultrasound wave  
Rys. 1. Ogólna zasada pomiaru przepływu prędkości krwi z wykorzystaniem efektu Dopplera i fali ultradźwiękowej

In formula (1)  $\theta$  is the angle of inclination of the probe to the blood vessel,  $c$  is the speed of ultrasound wave propagation in the blood.

$$f_T - f_R = f_d = \frac{2 \cdot f_T \cdot v \cdot \cos(\theta)}{c} \quad (1)$$

Doppler Tomography (DT), otherwise known as continuous wave ultrasound tomography (CWUT), uses the Doppler effect in an unusual way. This method also uses a two-transducer ultrasonic probe, which generates a quasi-plane wave. However, unlike the classical methods,

this probe moves around or along the object being examined. Due to the probe's movement, in accordance with the Doppler effect, the Doppler frequency will be added or subtracted from the frequency of signals reflected from the inclusions in the object. This makes it possible to visualize ultrasound-wave dispersive inclusions located inside the tissue. As shown in Fig. 2, there are two ways of data acquisition in the DT method. In the first way the probe moves along the examined object (Fig. 2a). It is called linear geometry. Another way of DT data acquisition is circular geometry (Fig. 2b), in which transducers move around the imaged tissue section [2]. Currently, the literature lacks comprehensive research on the Doppler CT method, especially simulations, which would allow to analyze the possibility of using this method.

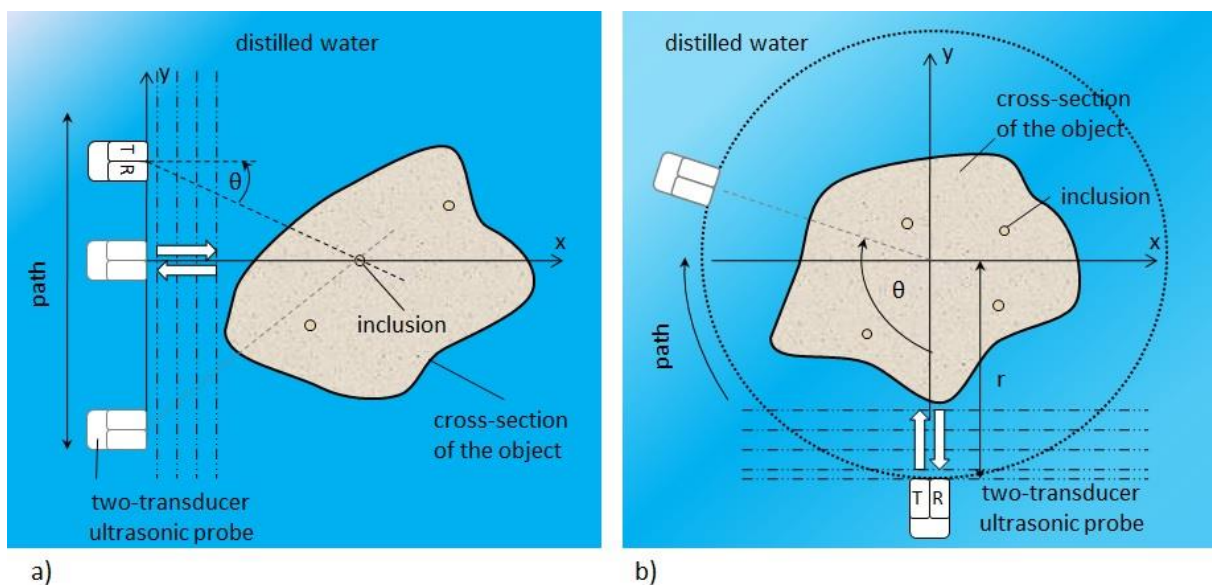


Fig. 2. The principle of data acquisition necessary for image reconstruction for linear geometry DT (a) and circular geometry DT (b)

Rys. 2. Zasada akwizycji danych niezbędnych do rekonstrukcji obrazu dla geometrii liniowej DT (a) i geometrii okrężnej DT (b)

The article focuses on circular geometry. Additionally, it should be noted that the Doppler effect shows that both in the case when the head rotates around the examined inclusion and in the case when the inclusion rotates around the center of rotation, the Doppler frequency remains the same. Assuming the situation from the second case greatly simplifies the explanation of the DT operation. For this reason, in the further part of the article it is the object being tested with the inclusions that will move and the probe will remain stationary. In the actual use of this method it would of course be a significant obstacle. You can imagine a situation when you need to get a picture of the inside of a metal sample with cracks in it. It can be placed on a rotating platform immersed in water and the measurement is done with a stationary probe. Another potential application of this method is imaging the inside of a woman's breast in search of cancerous changes. In this case it is required that the probe rotates around the breast. One can

also imagine such a device to scan the bone structures of human limbs, for example, to visualize fractures.

One of the advantages of a device constructed on the basis of the DT method is the use of an ultrasonic wave, which is very safe in comparison to X-ray radiation. This is especially important when examining female breasts where mammography is the leading method. An additional advantage could be the price of such a device due to the possibility of using a head that contains only two transducers.

The Doppler Tomography method is not a typical method which is based on the measurement of ultrasound wave passage time. This method uses frequency changes to reconstruct the image. Therefore, it is important to understand two key facts.

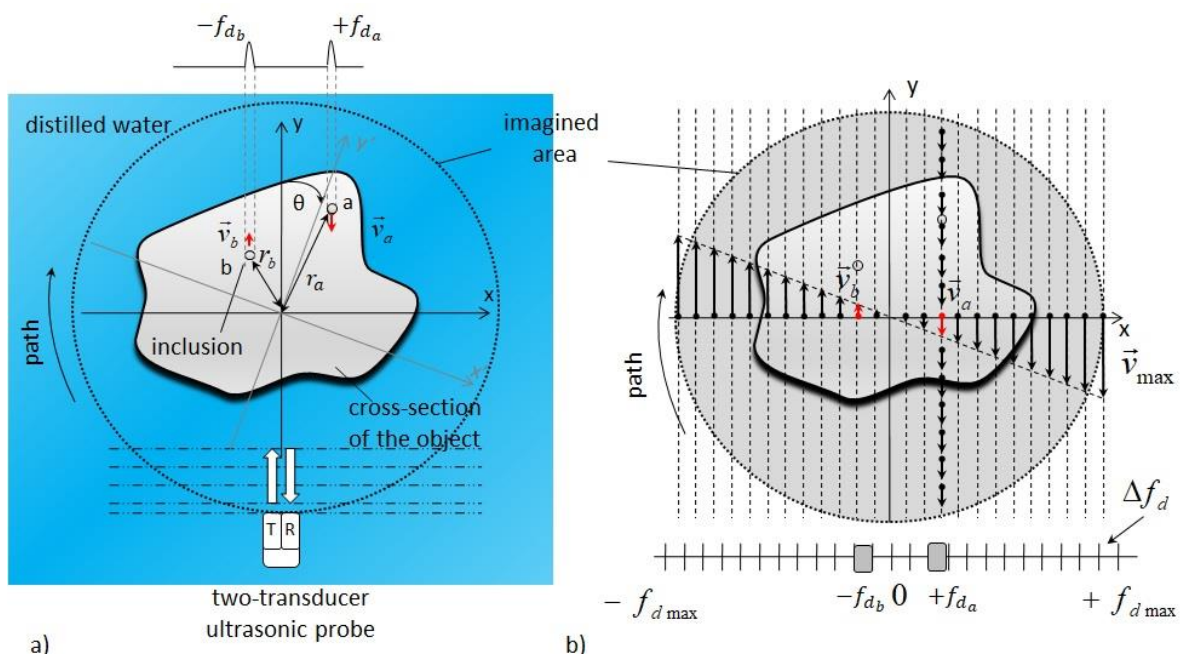


Fig. 3. Example of a cross-section of an object with two inclusions at a given angle of rotation (a), and illustration of the distribution of velocity  $v$  in the direction of wave propagation and Doppler frequency  $f_d$  in the imaging area (b)

Rys. 3. Przykładowy przekrój obiektu z dwoma wtrąceniami przy danym kącie obrotu (a), oraz ilustracja rozkładu prędkości  $v$  w kierunku propagacji fali i częstotliwości dopplerowskiej  $f_d$  w obszarze obrazowania (b)

Let's assume that we are imaging a cross-section of an object that contains inside two  $a$  and  $b$  inclusions well reflecting the ultrasonic wave, as shown in Fig. 3 (a). The DT method implies that at every angle of rotation of the examined object (or ultrasonic head) we register the reflected signals. As mentioned earlier, due to the Doppler effect, the frequency of the reflected signal changes by the Doppler frequency. Let's assume that if, for a given angle of rotation, the inclusion moves in the direction of the probe, this frequency is taken as a positive value, otherwise it is taken with a minus sign (Fig. 3a).

In Fig. 3a the components of velocity in the direction of ultrasound wave propagation of moving inclusions  $v_a$  and  $v_b$  are marked. They are linked to the Doppler frequency by formula (2).

$$f_d = 2 \cdot f_T \cdot \omega_{turn} \cdot r \cdot \frac{\cos(\theta)}{c}, \quad (2)$$

where:  $\omega_{turn}$  – angular rotation speed of the object,  $r$  – the inclusion distance from the center of rotation,  $c$  – speed of the ultrasonic wave in the examined object,  $\theta$  – the angle of rotation of the object under examination,  $f_T$  – frequency of the signal sent from the ultrasonic probe.

By transforming the formula (2) we get a direct relation to speed  $v$  shown below.

$$v = \frac{f_d \cdot c}{2 \cdot f_T \cdot \cos(\theta)} \quad (3)$$

From formula (3) it is clear that the Doppler frequency is directly proportional to the component of the speed of insertion in the direction of wave propagation  $v$ .

Fig. 3b illustrates two facts about speed  $v$ . The first one says that for a given rotation angle, as we move away from the center of rotation, speed  $v$  increases. At the border of the imaging area it reaches its maximum value  $v_{max}$ . On the basis of formula (3) it can be stated that the same dependence is valid for the frequency  $f_d$  and at the mentioned border we have the maximum Doppler frequency  $f_{dmax}$ . The second fact is that on the lines along the wave propagation we have the same velocity values  $v$  and thus the same frequency values  $f_d$ . This allows to determine the range from  $-f_{dmax}$  to  $+f_{dmax}$  and its division into bands of equal width  $\Delta f_d$  (Fig. 3b). These are the so-called Doppler bands. At a given angle of rotation of the tested object to the appropriate bands we record the sums of amplitudes of Doppler frequencies coming from the inclusions. If we perform this operation for all rotation angles, we get a matrix in which the rotation angles will change in the rows and the columns will have the sums of Doppler frequency amplitudes for the individual Doppler bands. This matrix is called a sinogram [3]. On its basis we reconstruct the cross-sectional image of the examined object. For this purpose, it is possible to use one of the algorithms used in computed tomography. In this work, the filtered back projection (FBP) algorithm is used. It was chosen, among others, because it is a fast algorithm and because the image reconstruction can be done from half of the object rotation data.

The algorithm of image reconstruction with the use of Doppler tomography method is shown in Fig. 4. First, the so-called Doppler signal should be recorded for each angle of rotation of the examined object. This signal contains only Doppler frequencies. Then we determine the spectrum of this signal for a given angle. Next, we divide them into Doppler bands. In the next step we create a sinogram based on the data collected in this way. In the last step we reconstruct the image using one of the algorithms used in computed tomography.

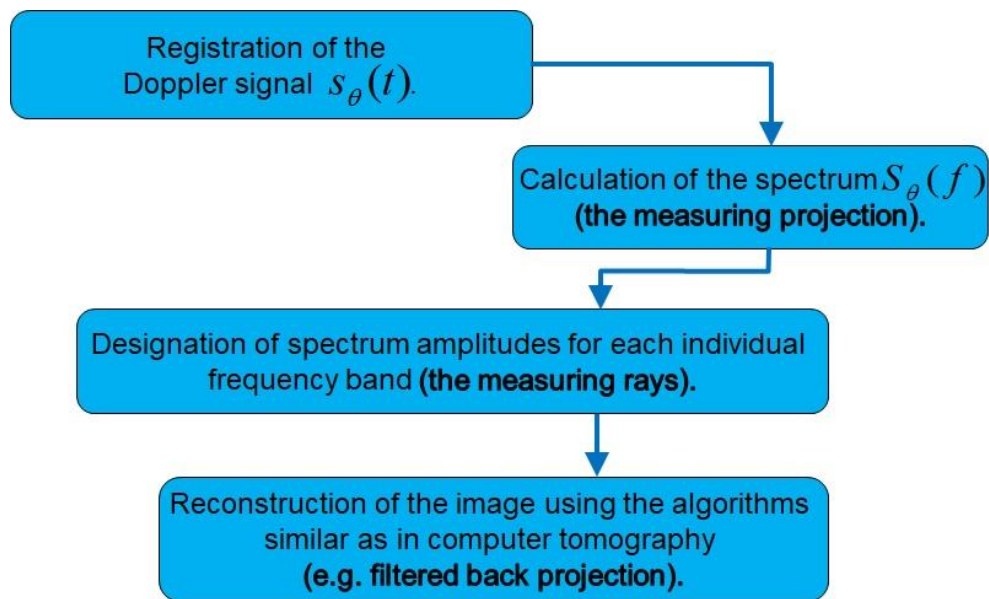


Fig. 4. Image reconstruction algorithm using Doppler Tomography method  
Rys. 4. Algorytm rekonstrukcji obrazu za pomocą metody Tomografii Dopplerowskiej

### 19.2.2. Filtered back projection algorithm

The measurement data for the filtered back projection algorithm for the general case are in the form of a continuous function  $p_\theta(s)$ , where  $s \in (-1, 1)$ . The parameter  $s$  is the standardized distance of the ultrasonic probe to the center of rotation of the tested object. It is worth noting that the center of rotation corresponds to the center of the imaging zone. First, the Fourier transform of  $p_\theta(s)$  function should be determined. This procedure allows to determine the function in the spatial frequency domain. Formula (4) describes this operation.

$$P_\theta(\xi) = \int_{-1}^1 p_\theta(s) \cdot e^{-j\xi s} d\xi, \quad (4)$$

where:  $\xi = -\xi_m, \dots, \xi_m$  and  $\xi_m = 1/\Delta\lambda$  is the maximum value of the spatial frequency that occurs in measurements in the geometry of parallel radiant projections. The parameter  $\Delta\lambda$  indicates the offset step for this geometry.

It should be noted that the FBP algorithm assumes that measurements carried out in a single wave can be treated as a two-dimensional filtration operation in the spatial frequency domain ( $\xi = 1/\lambda$ ) [2].

For the purpose of the Doppler Tomography method, it is necessary to determine the operations in the field of discrete variables, and not for continuous variables. Additionally it is necessary to define parameters of FBP algorithm for DT method. Therefore, the maximum frequency of Doppler frequency changes is indicated as  $W$ . The variable  $N_p$  will indicate the number of projections and  $M_p$  the number of rays. For the DT method the number of angles of

rotation (of the object or probe) is the number of projections. The number of rays equals the number of Doppler bands. Based on Nyquist's theorem about sampling, it can be concluded that  $p_\theta(mT)$  projection values should be sampled with a period of  $T = 1/(2W)$ , where  $m = -N/2, \dots, 0, \dots, N/2-1$ , for a sufficiently high value of  $N$ . For the DT method the  $T$  period is equal to the width of the  $\Delta f_d$  Doppler bands.

In the first stage of image reconstruction using the FBP algorithm, for each projection one calculates the approximation of Fourier transform  $S_\theta(w)$ . For this purpose the formula (5) should be used.

$$S_\theta(w) \approx S_\theta\left(m \frac{2W}{N}\right) = \frac{1}{2W} \sum_{k=-N/2}^{N/2-1} p_\theta\left(\frac{k}{2W}\right) \cdot e^{j2\pi(mk/N)} \quad (5)$$

where  $N$  – number of projection samples equal to the number of  $M_p$  measurement rays (equal to the number of Doppler frequency bands), supplemented by zeroes so that the number  $N$  is the power of number 2 (due to the use of FFT).

In this way  $S_\theta(w)$  is a discrete function of the amplitude change frequency of Doppler frequencies at a given angle of rotation  $\theta$ .

Then each projection is filtered in the frequency domain and multiplied by the window function (e.g. Hamming) to eliminate noise in the reconstructed image [1]. The formula (6) is used for this purpose.

$$Q_\theta\left(\frac{k}{2W}\right) \approx \left(\frac{2W}{N}\right) \sum_{m=-N/2}^{N/2} S_\theta\left(m \frac{2W}{N}\right) \cdot \left|m \frac{2W}{N}\right| \cdot H\left(m \frac{2W}{N}\right) \cdot e^{j2\pi(mk/N)}, \quad (6)$$

where  $k = -N/2, \dots, 0, \dots, N/2$ , and  $H(m(2W/N))$  is the window function used.

In the third and last part one reconstructs the image using the formula (7) of back projection.

$$f(x, y) = \frac{\pi}{K} \sum_{i=1}^K Q_{\theta_i}(x \cdot \cos\theta_i + y \cdot \sin\theta_i), \quad (7)$$

where  $K = N_p/2$ , which means that it is sufficient to use the range  $\pi$  for the angle  $\theta_i$ .

## 19.3. Simulation of image reconstruction with DT

### 19.3.1. Doppler signal simulation

In order to simulate the image reconstruction using the Doppler Tomography method it is necessary to calculate the Doppler signal. As mentioned before it is a signal that contains only Doppler frequencies.

In the simulations carried out it was assumed that we reconstruct the image of a limited number of inclusions. They are infinitely small and disperse the ultrasonic wave in each direction the same way.

It should be noted that the Doppler signal is a chirp signal. Suppose that we calculate this signal for a single inclusion placed at a distance  $r_0$  from the center of rotation and at an angle of  $\alpha_0$  to the ultrasonic probe. The change of Doppler frequency for this inclusion during rotation can be calculated from formula (8). It was derived from formula (2).

$$f_d(t) = \frac{2 \cdot f_T \cdot f_{turn} \cdot 2 \cdot \pi \cdot r_0 \cdot \cos(2 \cdot \pi \cdot f_{turn} \cdot t + \alpha_0)}{c}, \quad (8)$$

where  $f_{turn}$  is the frequency of object rotation.

It should be noted that due to the nature of Doppler signal changes it can be presented as  $s(t) = A \cdot \sin(\varphi(t))$ , where  $\varphi(t)$  is the modulation function searched for and  $A$  is the signal amplitude. Additionally  $\varphi(t)$  and  $f_d(t)$  can be linked with each other using formula (9).

$$f_d(t) = \frac{1}{2\pi} \frac{d\varphi(t)}{dt} \quad (9)$$

Based on formula (8) and (9) it is possible to derive formula (10) defining the Doppler signal.

$$s(t) = A \sin \left( \frac{4 \cdot \pi \cdot f_T \cdot r_0}{c} \cdot (\sin(2\pi \cdot f_{turn} \cdot t + \alpha_0) - \sin \alpha_0) \right) \quad (10)$$

In case of simulation of multiple interjections, it is necessary to calculate the Doppler signal for each of them and then to add the results.

### 19.3.2. Overlay algorithm

In its original form, the Doppler Tomography method has several flaws that need to be corrected. The first one is the necessity of recording the reflected signal for particular angles of rotation. This causes that during the measurement for a certain number of rotation angles, the probe should be switched on and off in due time. It is much easier to register the signal for a full rotation (or several rotations) of the probe and divide it into an appropriate number of angles during data processing.

The second flaw is related to the resolution of the reconstructed image, which depends on the number of Doppler bands and resolution of the Doppler signal spectrum. Let's consider a case in which we simulate an image from a single inclusion, which is located 3 cm from the center of rotation and at an angle of  $0^\circ$  to the probe. Additionally, let's assume that  $\omega_{turn} = 2$  turns per second;  $c = 1482$  m/s;  $f_T = 4.7$  MHz. Basing on the formula for Doppler signal (10) we calculate samples of this signal for the full rotation of the inclusion. The number of samples will be equal to 10000. Then we divide these samples into 398 rotation angles  $\theta$ . The number results from the fact that the FBP algorithm reconstructs the image from half a turn. The number of angles to be reconstructed is 200. This number guarantees good accuracy of the calculated



image. With equal distribution of samples, the number for one angle  $\theta$  is 50. Since it is necessary to specify the middle sample for the image reconstruction, the number of samples should be an odd number and therefore we increase this value to 51. For the calculated Doppler signal the sampling period is 25  $\mu$ s. This means that the sampling rate is  $f_s = 40$  kHz. Formula (11) shows the definition of spectrum resolution, which is equal to the quotient of the sampling rate  $f_s$  and the number of samples of  $N$  signal.

$$\Delta f = \frac{f_s}{N} \quad (11)$$

So, in our case it is equal to  $\Delta f = 784$  Hz. Let us assume that the zone we want to image has a diameter of 10 cm. Based on formula (2) one can calculate the maximum Doppler frequency, which in this case is equal to  $f_{dmax} = 3984$  Hz. Looking at the resolution of the spectrum and the maximum Doppler frequency one comes to the conclusion that it is possible to determine maximum 11 Doppler bands only. This means that the sinogram created for image reconstruction will have very little data. As a result, the image will have a very low resolution of 11x11 pixels. Taking into account that we reconstruct an area of 10 cm in diameter, the area of one pixel will be about 1 cm<sup>2</sup>. A very important conclusion is the fact that increasing the sampling rate in such a situation does not increase the spectrum resolution. This is because when we increase the frequency  $f_s$  the number of  $N$  samples of the signal increases simultaneously. This causes that the resolution  $\Delta f$  remains at a similar level.

The solution to this problem can be to increase the number of  $N$  samples while leaving the sampling frequency at the same level. This will increase the interval for each rotation angle. As a result, as shown in Fig. 5, the Doppler signal segments for each  $\theta$  angle will overlap. That is why this solution is called the overlap algorithm.

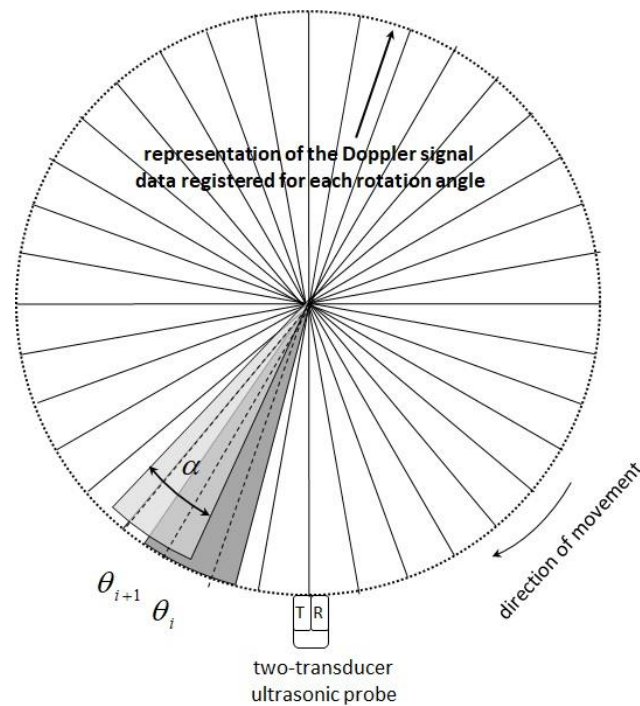


Fig. 5. The idea of overlay algorithm presented for two adjacent angles  $\theta_i$  and  $\theta_{i+1}$

Rys. 5. Idea algorytmu nakładkowania przedstawiona dla dwóch sąsiadujących kątów  $\theta_i$  oraz  $\theta_{i+1}$

The parameter  $\alpha$  is the length of the Doppler signal for the rotation angles. Depending on the value of this parameter, the resolution and blur of the inclusions will change. Both these parameters are determined by the pixel values of the image. The resolution at 3 dB drop and the blur at 10% of the maximum pixel value of the inclusion image. It should be noted that above a certain value of  $\alpha$  an abnormal elongation of the inclusion image occurs and both these parameters deteriorate [5]. For the simulation conditions described at the beginning of this chapter, the optimal value is  $\alpha = 90^\circ$ , where  $360^\circ$  is the length of the Doppler signal from the full revolution [5].

### 19.3.3. Additional zeroes method

Another way to increase the resolution of the inclusion image is to add additional zero samples to the Doppler signal for all  $\theta$  angles of rotation. Fig. 6a shows a normalized Doppler signal for the angle  $\theta_i$  (blue diagram) coming from a simulated inclusion set at a distance of 4 cm from the center of rotation and at an angle  $0^\circ$  to the ultrasonic probe. In this figure 32 additional zero samples are also marked in orange. The frequency of rotation of the inclusion was 1 turn per second, frequency of the probe  $f_T = 4.7$  MHz, diameter of the reconstructed imaging zone is 10 cm. The number of samples of the Doppler signal for the full rotation of the inclusion was 10000 and the number of rotation angles (as in chapter 3.1) was 398. On the basis

of these data it can be concluded that the number of samples for one angle is equal to 25, and that the sampling period is 0.1ms.

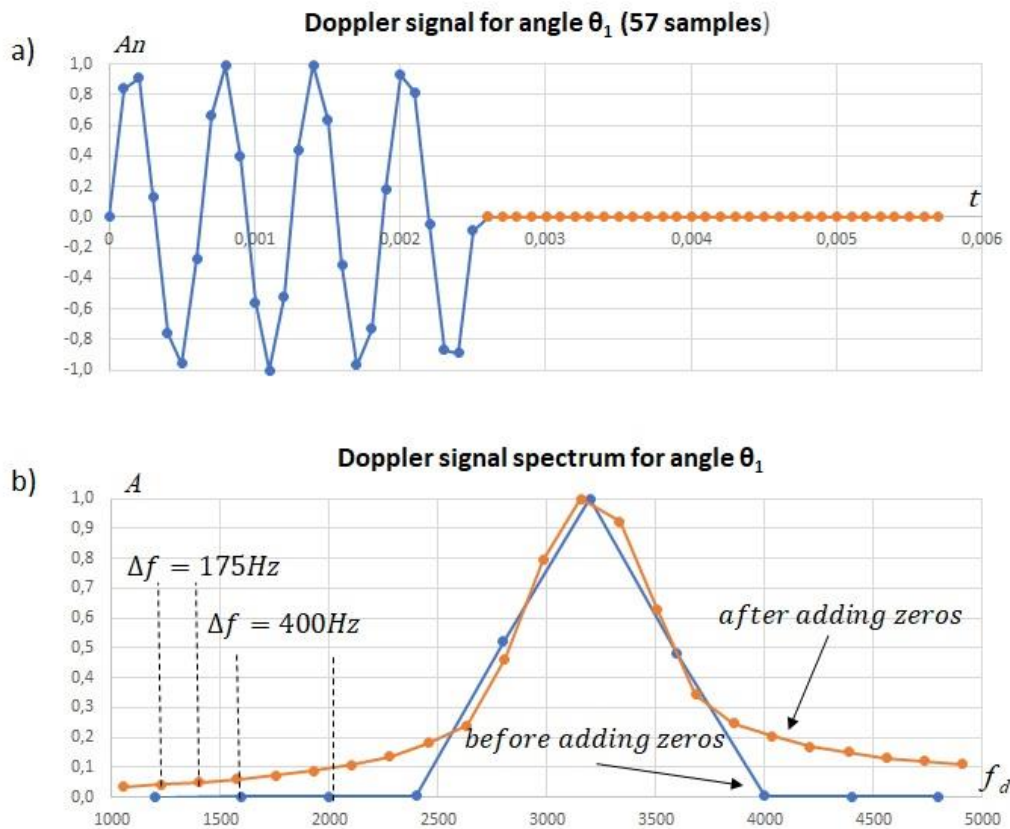


Fig. 6 Doppler signal for angle  $\theta_1$  before addition (blue chart) and with additional 32 zero samples (a), Doppler signal spectrum for angle  $\theta_1$  before adding (blue chart) and after adding zero samples (orange chart) (b)

Rys. 6. Sygnał dopplerowski dla kąta  $\theta_1$  przed dodaniem (wykres niebieski) oraz z dodatkowymi 32 zerowymi próbkami (a), widma sygnału dopplerowskiego dla kąta  $\theta_1$  przed dodaniem (wykres niebieski) i po dodaniu zerowych próbek (wykres pomarańczowy) (b)

Fig. 6b shows the spectrum of Doppler signals before and after adding zero samples. Using formula (11) it is possible to calculate the resolution of the spectrum for both cases. For the signal without additional samples the resolution is 400 Hz, while for the case with zero samples the result is much better and equals 175 Hz. This results in more Doppler bands and better resolution image reconstruction.

Fig. 7 presents the results of the simulation of image reconstruction of the previously mentioned inclusion for the overlay algorithm parameter  $\alpha = 90^\circ$  and the number of additional zero samples equal to 0, 8, 16, 32, ..., 1024.

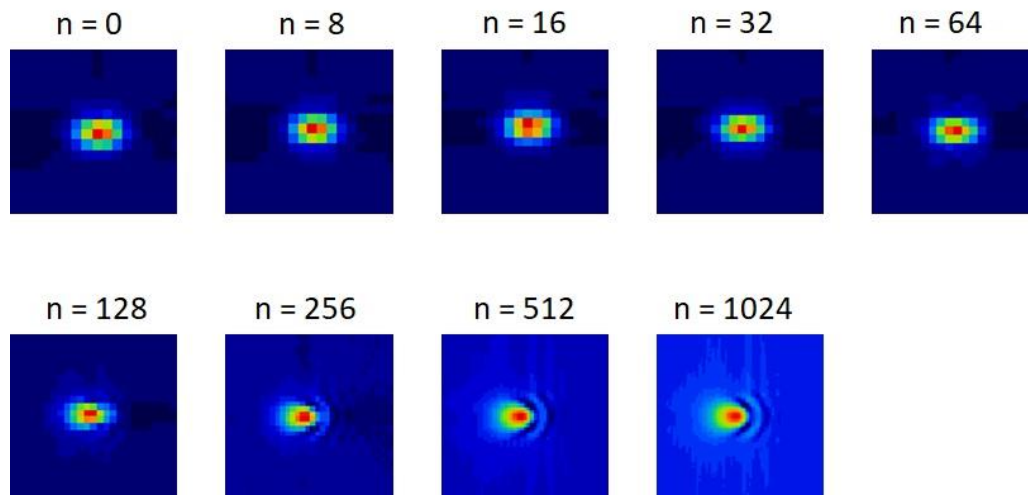


Fig. 7. Simulation of reconstruction of an inclusion image set at a distance of 4cm from the center of rotation when 0, 8, 16, 32, ..., 1024 additional zero samples are added to the Doppler signal (for rotation angles)

Rys. 7. Symulacja rekonstrukcja obrazu wtrącenia ustawionego w odległości 4cm od środka obrotu w przypadku dodania do sygnału dopplerowskiego (dla kątów obrotu) 0, 8, 16, 32, ..., 1024 dodatkowych zerowych próbek

It shows a significant improvement in resolution when the number of additional  $n$ -zero samples is increased. For  $n = 0$  the resolution is  $99 \times 99$  pixels and for  $n = 1024$  it is  $507 \times 507$  pixels. This shows that the resolution of the image has improved about five times. The resolution and blur of the inclusions image was also examined. The pixel values of the image were examined in two planes: vertical and horizontal. The results are shown in Fig. 8. In this figure we can see that both the resolution and the blur improve for the increasing value of parameter  $n$ . However, above the value of 256 there is no increase in parameter quality. This means that the  $n = 256$  value is optimal. It should be noted that for higher values of  $n$ , the amplitudes of the Doppler signal spectrum decrease. This may cause loss of detail in the image.

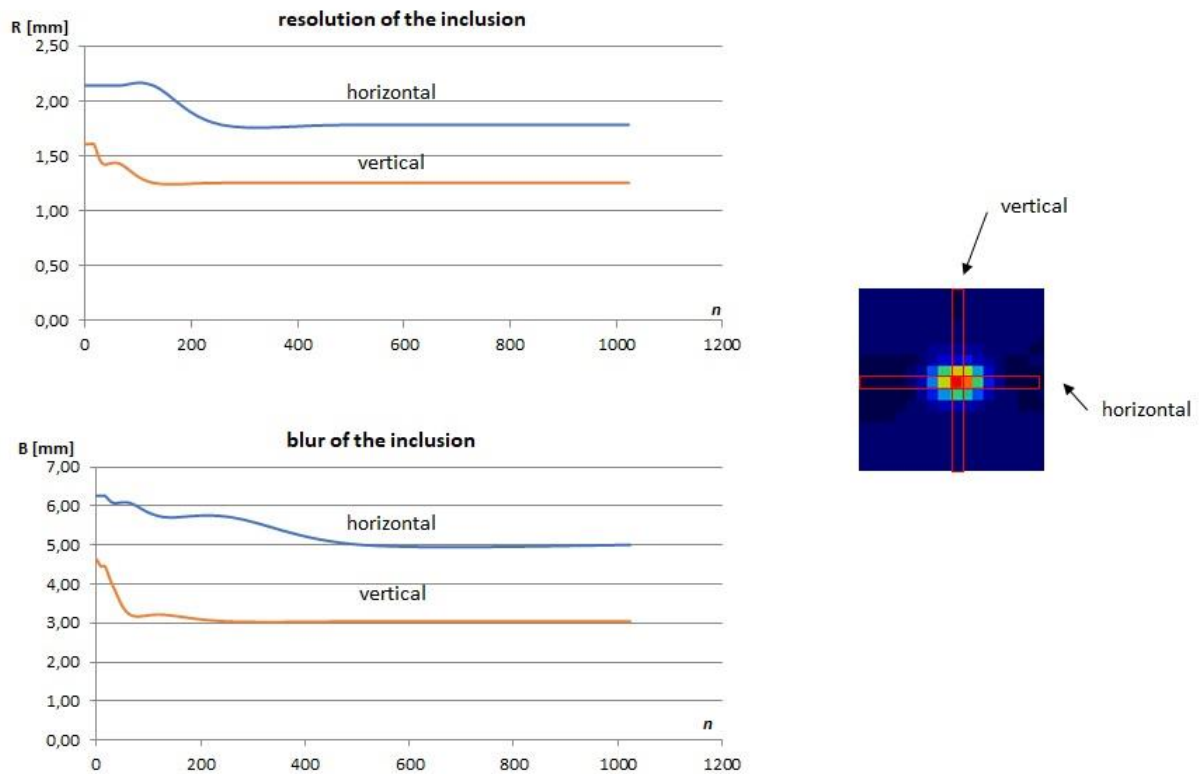


Fig. 8. Measurement results of resolution and blur of the inclusions in the vertical and horizontal plane for the number of additional zero samples in the Doppler signal (for angles of rotation) equal to  $n = 0, 8, 16, 32, \dots, 1024$

Rys. 8. Wyniki pomiaru rozdzielczości i rozmycia obrazu wtrącenia w płaszczyźnie pionowej i poziomej dla liczby dodatkowych próbek zerowych w sygnale dopplerowskim (dla kątów obrotu) równych  $n=0, 8, 16, 32, \dots, 1024$

## 19.4. Measurement and reconstruction of the image by DT method for real inclusion

### 19.4.1. Measuring system

The measurement system is shown in Fig. 9. The elements of this system are numbered. The most important device marked with number 1 is the Doppler blood flow meter UD48 by Sonomed. The device has been modified accordingly. It has, among other things, the possibility of connecting the oscilloscope to the Doppler signal outputs. This flowmeter has two channels A and B in which motion of an object moving in the direction from and to the probe is detected. It also has a possibility to connect two ultrasonic probes for continuous wave of 4 MHz and 8 MHz. The probe used in this experiment and marked on Fig. 9 with number 2 works on 4 MHz frequency. However, it was designed and made with the Doppler Tomography method in mind.

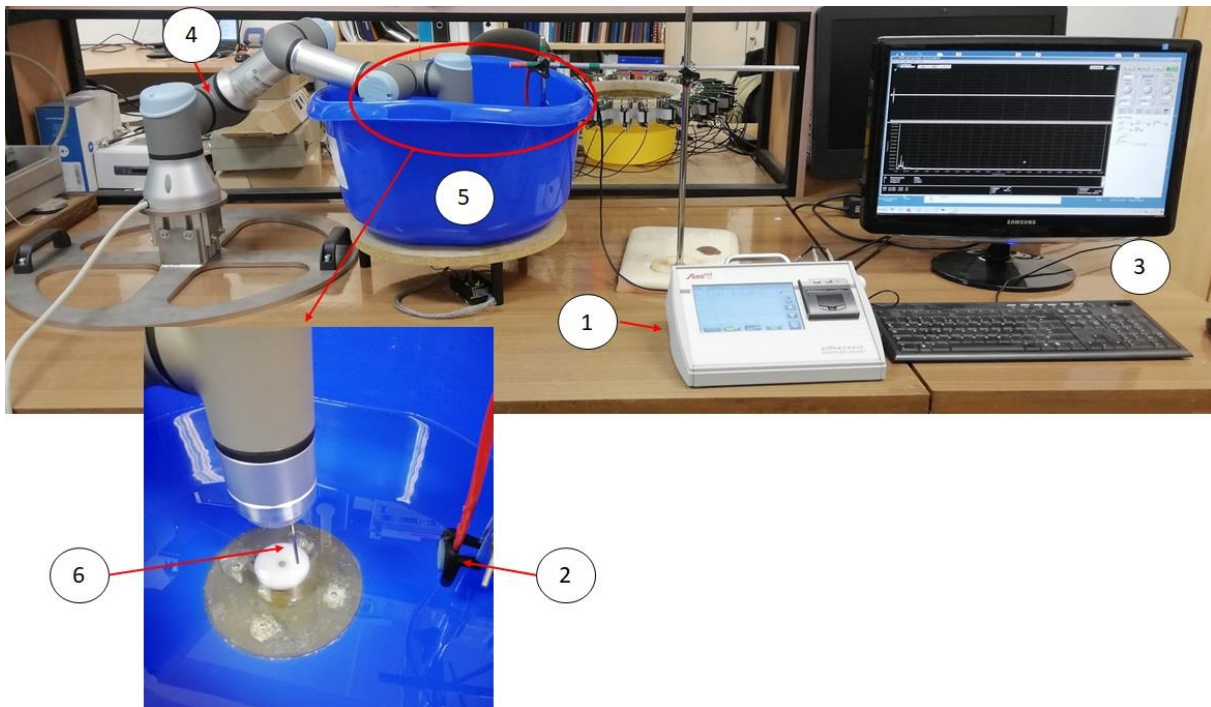


Fig. 9. Elements of the measuring station for data acquisition in the Doppler Tomography method (description of elements in text)

Rys. 9. Elementy stanowiska pomiarowego do akwizycji danych w metodzie Tomografii dopplerowskiej (opis elementów w tekście)

The Doppler signal from the Doppler flow meter is recorded by means of an oscilloscopic card of National Instruments company placed in the computer. In Fig. 9 it is marked with the number 3. A program has been installed on the computer in LabVIEW environment to process, preview and record samples of the Doppler signal. It allows to monitor whether the measurements are running correctly. Another necessary element is a multifunctional UR3 robot from Universal Robots. It has the ability to program and perform complex movements. It allows for precise positioning and movement of the examined object. It has the ability, among other things, the task of a specific speed of movement of the object mounted on its third joint. In the DT method it is one of the most important parameters for correct reconstruction of the image. In Fig. 9 it has been marked with number 4. In order to create proper measurement conditions a vessel filled with distilled water (marked with number 5) was used, in which the ultrasonic probe and the imaged element were immersed. In this case, it is a narrow brass cylinder (rod) with a diameter of 2.8 mm. In Fig. 9 it is marked with number 6.

#### 19.4.2. Data measurement and reconstruction of a 2.8 mm diameter brass rod image

The rod was attached to the robot arm shown in Fig. 9 at a distance of 10 mm from the center of rotation and at an angle  $0^\circ$  to the ultrasonic probe. The speed of rotation of the imaged element was set to 1 rotation per second. First, using a Doppler flow meter and oscilloscope



card, the Doppler signal was recorded at ten full rotations of the rod. The whole signal contained 200 000 samples. It is worth reminding that the UD48 flow meter has the possibility of recording the signal in two channels A and B. As already mentioned, the first one (channel A) will record the signal when the bar moves in the direction away from the probe, while the second one (channel B) will perform data acquisition when the element moves to the probe. In order to limit interference, the measurements were averaged for 10 revolutions. The result is shown in Fig. 10.

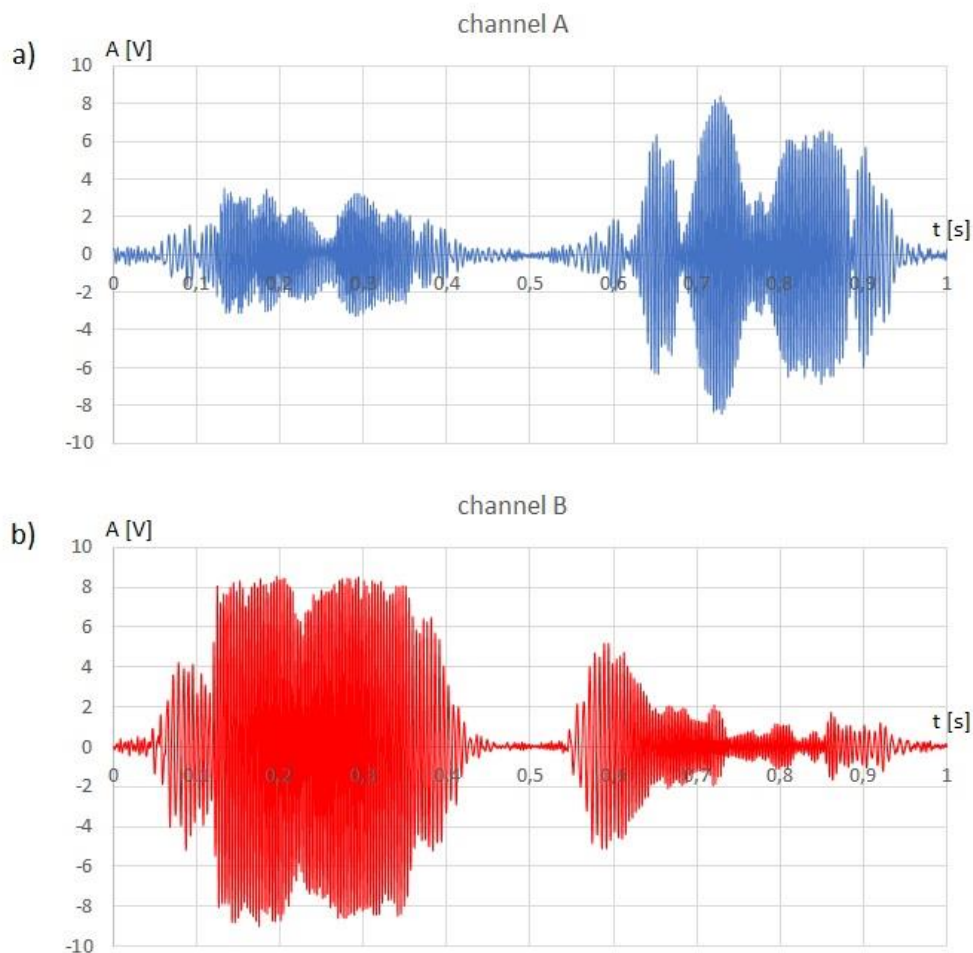


Fig. 10. Doppler signal registered for full rotation of the brass rod for channel A (a) and channel B (b)  
 Rys. 10. Sygnał dopplerowski zarejestrowany dla pełnego obrotu pręta mosiężnego dla kanał A (a) i kanału B (b)

In Fig. 10a it is clearly visible that a low amplitude signal was recorded for the first half second for channel A. In theory, the signal should be zero when the bar moves away. This is the crosstalk from channel B. A similar situation is observed in Fig. 10b when crosstalk from channel A appears between 0.5 s and 1 s. This effect will cause an additional shadow of the rod to appear in the picture. It will be located symmetrically to the center of rotation.

The next step is to determine the Doppler signals for each rotation angle  $\theta$ . Since the FBP algorithm reconstructs the image on the basis of data from the half-turn, the number of angles is assumed to be 998 for the whole rotation. This gives the number of 500 angles per half turn and allows to calculate the image with good accuracy. At this stage it is necessary to use the overlay algorithm to ensure proper resolution of the image. In chapter 3.2 it has been established that  $\alpha = 9^\circ$  [5] should be used to obtain optimal imaging results. For the Doppler signals calculated in this way we calculate the spectra, which are then divided into Doppler bands. After completing the necessary operations, a sinogram is obtained. Its graphical interpretation is shown in Fig. 11. In the figure the biggest values of the Doppler spectrum amplitudes are marked in red and the smallest values are marked in blue. It should also be noted that the diameter of the reconstructed image is assumed to be 50 mm when calculating the sinogram.

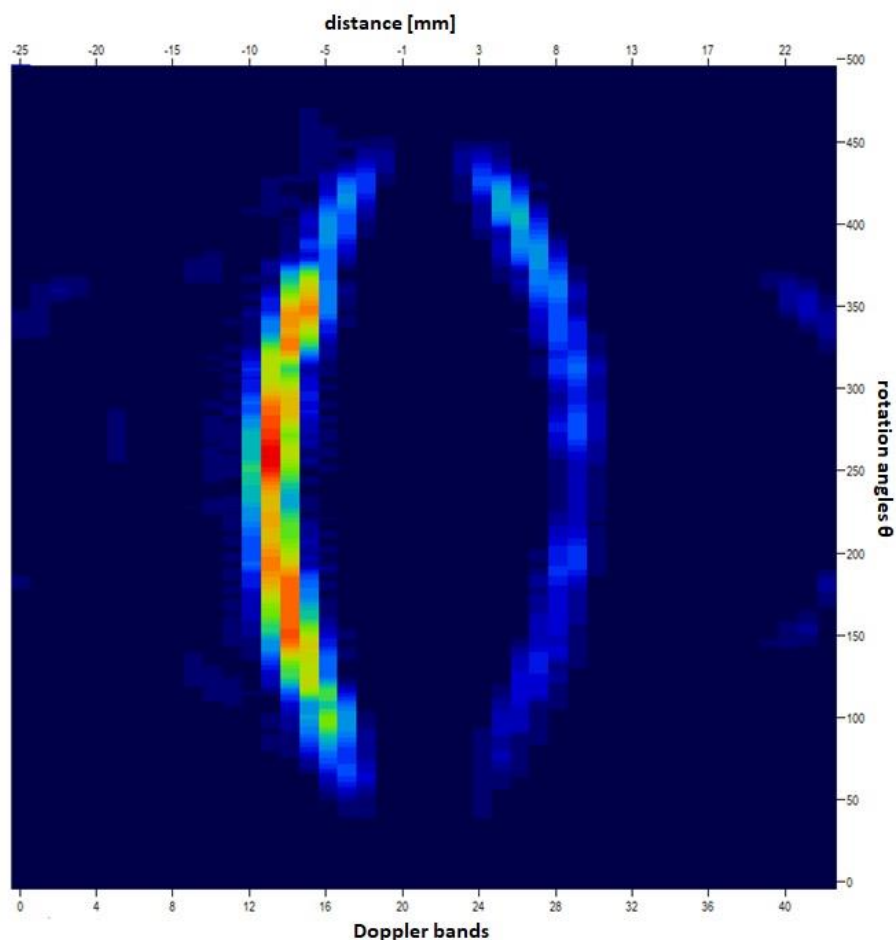


Fig. 11. Graphic interpretation of the sinogram for the reconstruction of the brass bar image  
Rys. 11. Graficzna interpretacja sinogramu dla rekonstrukcji obrazu mosiężnego pręta

Fig. 11 shows a symmetrical image. The left side of this picture is correct, while the right side is the result of the previously mentioned crosstalk between channels A and B.



In the last stage, based on the sinogram, we reconstruct the image using the FBP algorithm. The result of this reconstruction together with the photo of the physical rod is shown in Fig. 12.

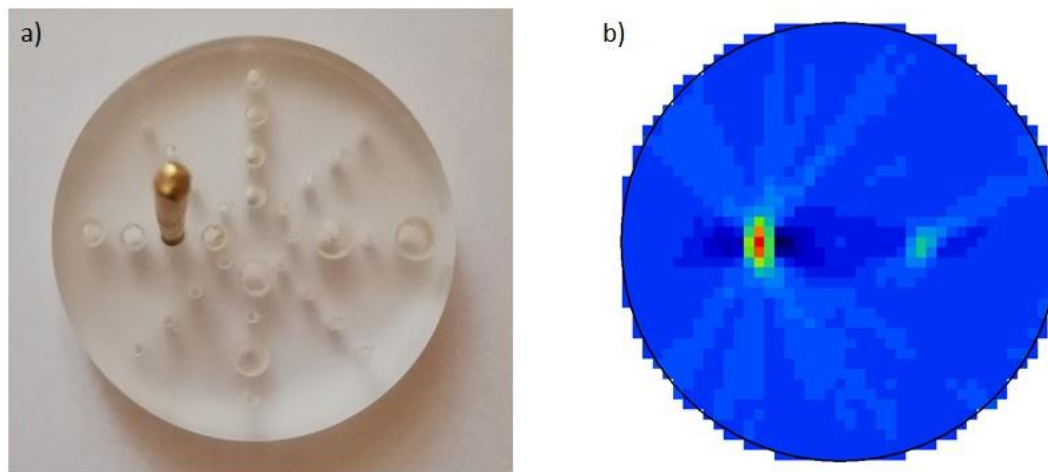


Fig. 12. Picture of a brass rod screwed to the disc, which was attached to the robot (a); reconstruction of the rod image using Doppler Tomography method (b)

Rys. 12. Zdjęcie pręta mosiężnego przykręconego do krążka, który został przymocowany do robota (a); rekonstrukcja obrazu pręta za pomocą metody Tomografii dopplerowskiej (b)

As expected in Fig. 12b we can see its shadow in addition to the image of the rod. Both images are symmetrical to the center of rotation. The resolution of such a reconstructed image is 43x43 pixels. The image quality can be improved by using the additional zeroes method described in chapter 3.3. For this purpose new sinograms have been calculated. The comparison of two results for 0 and 1024 additional zeroes is shown in Fig. 13. It is clear that after applying this method the number of Doppler bands increased from 43 to 129. This fact directly translates into a 3-fold improvement in image resolution. In order to better observe the process of improving imaging quality, Fig. 14 shows the results of reconstruction after applying 8, 16, 32, ..., 512 and 1024 additional zero samples in the Doppler signal for angles of rotation. The images show a slight elongation of the rod shape in the vertical plane. Such an effect on the image has an overlay algorithm if the parameter  $\alpha$  is too high. However, using a smaller value of this parameter would result in a significant deterioration of image resolution.

In order to determine the resolution and blurring of the rod image, its pixel values were examined. In Fig. 15 there are graphs showing how pixel values change depending on the position in the image. As mentioned before, it was assumed that the resolution of the examined object will be determined with a 3 dB decrease from the maximum pixel value. The total blur of the image of an object will be examined at 10% of the maximum value. These measurements were made in two planes: vertical and horizontal.

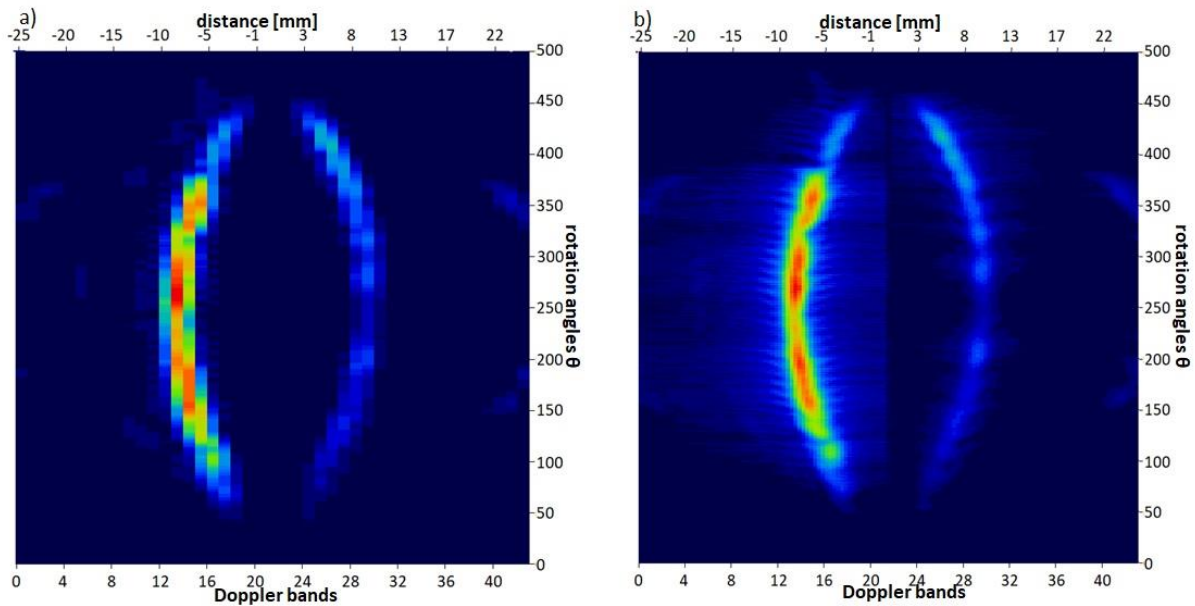


Fig. 13. Graphical interpretation of the sinogram calculated for the brass bar without an algorithm to improve the image resolution (a) and with the applied method of additional 1024 zero samples (b)

Rys. 13. Graficzna interpretacja sinogramu wyliczonego dla pręta mosiężnego bez algorytmu poprawy rozdzielczości obrazu (a) i z zastosowaną metoda dodatkowych próbek zerowych w liczbie 1024 (b)

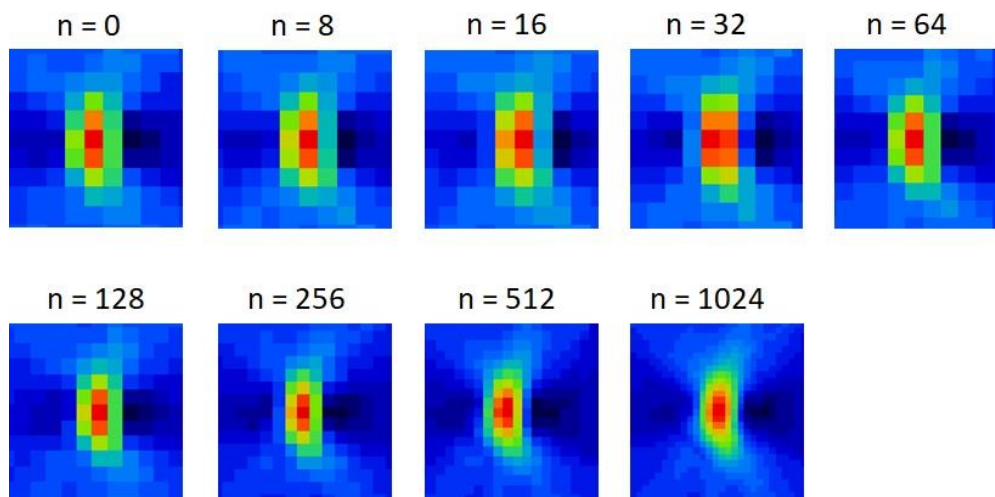


Fig. 14. The result of the reconstruction of the brass bar image using the additional zero-sample method is  $n = 0, 8, 16, 32, \dots, 1024$ , respectively

Rys. 14. Wynik rekonstrukcji obrazu mosiężnego pręta przy zastosowaniu metody dodatkowych zerowych próbek w ilości odpowiednio  $n = 0, 8, 16, 32, \dots, 1024$

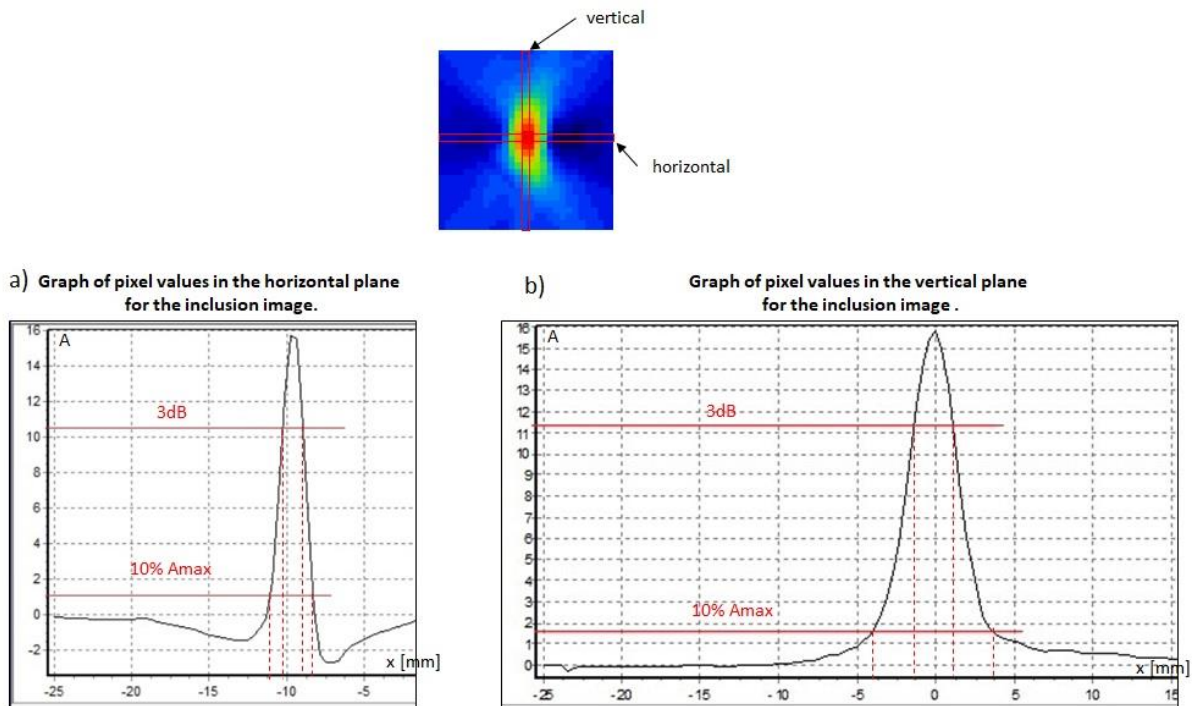


Fig. 15. Graph of pixel values in the horizontal plane for the inclusion image (a); Graph of pixel values in the vertical plane for the inclusion image (b)

Rys. 15. Wykres wartości pikseli obrazu pręta w płaszczyźnie poziomej (a); Wykres wartości pikseli obrazu pręta w płaszczyźnie pionowej (b)

For the horizontal plane, the resolution is 1.44 mm and the total blur is 2.81 mm. This is consistent with the real dimensions of a rod with a diameter of 2.80 mm. However, for the vertical plane, discrepancies appear because the resolution of the point is 2.50 mm and the total blur is 7.14 mm. In addition, the coordinates of the image center have been determined, which are equal (-9.38 mm, 0 mm). This means that the DT method reconstructed the image with a 0.63 mm error for the  $x$  coordinate and a zero error for the  $y$  coordinate.

## 19.5. Conclusions

The Doppler Tomography method is an innovative and little-known method for reconstruction of the stationary image of the examined object. It requires additional solutions and algorithms in order to obtain a good quality image. The overlay algorithm is such a solution. By selecting the appropriate length of the Doppler signal for the angles of rotation (parameter  $\alpha$ ) it allows to significantly improve the image resolution. However, it may introduce unwanted shape distortion of the examined object. If the  $\alpha$  parameter is too high, it is manifested by the extension of the insertion in a certain plane [5].

An additional solution to increase image resolution is to use the method of additional zero samples for the Doppler signal at given angles of rotation. As shown in Fig. 14 the resolution

can be increased up to three times. However, it is necessary to choose optimal parameters taking into account the introduced distortion and resolution of the image.

When measuring real components, one of the problems in the measuring system may be crosstalk. In Fig. 10 this was manifested by a signal from channel A in channel B and vice versa. The crosstalk is most likely formed in the ultrasonic probe between the transducers. In the next part of the investigation of this method we will certainly start work on eliminating this error.

It can be stated at this stage that the Doppler Tomography method is able to reconstruct the image of small inclusions that reflect the ultrasonic wave well, with good image accuracy and resolution. However, it requires further research.

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