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ON SOME BINARY PROGRAMMING FOR MINIMALIZATION OF POLLUTANTS EMISSION

Summary. The article presents a method leading to optimal allocation of financial resources in order to obtain the best possible ecological effect. The issue is related to binary programming with only one condition (i.e. limits of financial resources). The issue has been solved using the Bellman's principle of optimality. The algorithm which at given financial resources defines optimal incidental matrix has been developed. The matrix reveals emission sources and technologies in which the investment is done in optimal way.

O PEWNYM MODELU PROGRAMOWANIA BINARNEGO DLA MINIMALIZACJI EMISJI ZANIECZYSZCZEŃ

Streszczenie. W pracy przedstawiono rozwiązanie problemu optymalnej alokacji zasobów finansowych ze względu na maksymalizację odpowiednio określonego efektu ekologicznego. Problem sprowadza się do pewnego zadania programowania binarnego z jednym ograniczeniem (ograniczenie na zasoby finansowe). Zagadnienie rozwiązano korzystając z zasady optymalności Bellmana. Opracowano algorytm, który przy zadanych środkach finansowych wyznacza optymalną macierz incydencji wskazującą źródła emisji i technologie, w które należy zainwestować środki finansowe, którymi dysponujemy.

1. Introduction

The problem of the most effective making use of available financial resources is often considered in order to take some economical decisions.

In case of a matter connected with air protection there is a problem which often occurs in a natural way. There are limited financial resources which are supposed to be used to modernize emitters of fumes in definite area in order to decrease pollutants emission. The point is to gain a maximal ecological effect using given financial resources.

There are n – emitters (chimneys) in the studied area and it is possible to adapt one of m – technologies,for each of the emitters, which limits the emission of pollutions from definite emitter or does not modernize this emitter at all. The main difficulty is to choose a suitable technology, for particular emitter, which will restrict emission so that the ecological effect will be maximal and financial resources will not be exceeded. The abovementioned problem brings to the following mathematical problem.

2. Mathematical model of the problem

Two matrix of $m \times n$ dimensions are formed by the input data of the considered problem:

$$A, C \in M(m, n)$$

with real positive elements:

$$A = (a_{ik}), \qquad a_{ik} > 0$$

$$C = (c_{ik}), \qquad c_{ik} > 0$$

the elements denote:

a_{ik} – the ecological profit formed by using i – technology to k – emitte is colculated (according to weighted average of varicus pollutants),

c_{ik} – cost of application of i – technology to k – emitter.

The plan of modernization of emitters is definite by binary incidental matrix:

$$X = (x_{ik}) \in M\,(m, n)\,,$$

elements satisfy following conditions:

$$x_{ik} \in \{0,1\} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ k = 1, 2, \ldots, n \end{matrix}\,, \tag{1}$$

$$\sum_{i=1}^{m} x_{ik} \leqslant 1 \qquad k = 1, 2, \ldots, m. \tag{2}$$

X matrix definite this way is a binary matrix with maximum one not zero element (equal to 1) in each column.
If $x_{ik} = 0$ then i – technology is not adapted to modernize k – emitter,
if $x_{ik} = 1$ then i – technology can be adapted for k – emitter.

The condition (2) signifies that maximum one modernizing technology can be adapted for given emitter. The considered problem is brought to maximization of objective function:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_{ij} \longrightarrow \max \tag{3}$$

if conditions (1), (2) are satisfied and:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \leqslant z. \tag{4}$$

Function (3) assigns total ecological profit if the technology definite by X matrix is put into use. The left side of (4) inequality defines a global cost of emitters modernization according to the plan definite by X matrix. Number z on the right side of the (4) inequality defines total input of financial resources earmarked for emitters modernization, $z \geqslant 0$. It is easily seen that the problem is finite i.e. the amount of binary incidental matrix which satisfies (4) inequality is finite, then a maximum of (3) function exists. It is not possible to solve the problem of (3) function maximization with (4) restriction for binary incidental matrixes by checking all of permissible possibilities. When m, n quantities have practical meaning number of permissible binary incidental matrixes is so large that it excludes checking all possibilities.

3. Iterative solution of the problem

To assign optimal solution of the considered problem the Bellman's principle of optimality should be applied because (3) objective function is lenear with regard to columns of X binary incidental matrix. The Bellman's principle of optimality is presented as follows (cf. [1]). Optimal strategy has this propriety that whatever is the initial state and initial decision, next decision must create optimal strategy with regard to the state which is a resuct of the first decision. Partial maximization tasks are studied:

$$\sum_{i=1}^{m} \sum_{j=1}^{k} a_{ij} x_{ij} \longrightarrow \max \tag{5}$$

if following conditions are satisfied:

$$\sum_{i=1}^{m} \sum_{j=1}^{k} c_{ij} x_{ij} \leqslant z, \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad \text{for} \qquad \begin{matrix} i = 1,2,\ldots,m \\ j = 1,2,\ldots,k \end{matrix}, \tag{7}$$

$$\sum_{i=1}^{m} x_{ij} \leqslant 1 \qquad \text{for} \qquad j = 1, 2, \ldots, k. \tag{8}$$

Denote by $f_k(z)$ the function (5) under conditions (6), (7), (8) i.e.:

$$f_k(z) = \sum_{i=1}^{m} \sum_{j=1}^{k} a_{ij} \tilde{x}_{ij}, \tag{9}$$

where:

$$\tilde{X}_k = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \ldots & \tilde{x}_{1k} \\ \tilde{x}_{21} & \tilde{x}_{22} & \ldots & \tilde{x}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \ldots & \tilde{x}_{mk} \end{bmatrix}$$

is a binary incidental matrix which reaches maximum of the function (5), under conditions (6), (7), (8) for estimated value $z \geqslant 0$. It is obvious that for $\tilde{X}_k$ matrix following inequality is satisfied:

$$\sum_{i=1}^{m} \sum_{j=1}^{k} c_{ij} \tilde{x}_{ij} \leqslant z. \tag{10}$$

After transformations (9) and (10), for $k \geqslant 2$ one can write down:

$$f_k(z) = \sum_{i=1}^{m} \sum_{j=1}^{k-1} a_{ij} \tilde{x}_{i,j} + \sum_{i=1}^{m} a_{ik} \tilde{x}_{i,k}, \tag{11}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{k-1} c_{ij} \tilde{x}_{ij} \leqslant z - \sum_{i=1}^{m} c_{ik} \tilde{x}_{ik}. \tag{12}$$

According to the Bellman's principle of optimality we have:

$$f_{k-1}(z - \sum_{i=1}^{m} c_{ik} \tilde{x}_{ik}) = \sum_{i=1}^{m} \sum_{j=1}^{k-1} a_{ik} \tilde{x}_{ik} \quad \text{for} \quad \sum_{i=1}^{m} c_{ik} \tilde{x}_{ik} \leqslant z,$$

therefore on the basis of (11) there is:

$$f_k(z) = f_{k-1}(z - \sum_{i=1}^{m} c_{ik} \tilde{x}_{ik}) + \sum_{i=1}^{m} a_{ik} \tilde{x}_{ik} \quad \text{for} \quad \sum_{i=1}^{m} c_{ik} \tilde{x}_{ik} \leqslant z.$$

k – column of $\tilde{X}_k$ binary incidenta matrix includes maximum one not zero element, thus:

$$f_k(z) = f_{k-1}(z), \quad \text{when} \quad \tilde{x}_{ik} = 0 \quad \text{for} \quad i = 1, 2, \ldots, m \tag{13}$$

$$f_k(z) = f_{k-1}(z - c_{ik}) + a_{ik}, \quad \text{when} \quad \tilde{x}_{ik} = 1 \quad \wedge \quad c_{ik} \leqslant z \tag{14}$$

Therefore one can get an equation on the basis of (13) and (14):

$$f_k(z) = \max\Big(\{f_{k-1}(z)\} \cup$$

$$\cup \{f_{k-1}(z - c_{ik}) + a_{ik}; c_{ik} \leqslant z \wedge i = 1, \ldots, m\}\Big) \tag{15}$$

for $k = 2, 3, \ldots, n$ and $z \geqslant 0$, where function $f_1(z)$ is assigned below. There is:

$$f_1(z) = \max\{a_{11}x_{11} + a_{21}x_{21} + \ldots + a_{m1}x_{m1}\}$$

if following conditions are fulfiled:

$$\begin{aligned} c_{11}x_{11} + c_{21}x_{21} + \ldots + c_{m1}x_{m1} &\leqslant z \\ x_{i1} \in \{0, 1\} & \qquad i = 1, 2, \ldots, m \\ x_{11} + x_{21} + \ldots + x_{m1} &\leqslant 1 \end{aligned}$$

According to this:

$$f_1(z) = \begin{cases} 0 & \text{for} \quad 0 \leqslant z < p_1, \\ max\,\{a_{i1}; c_{i1} \leqslant z, i = 1, \ldots, m\} & \text{for} \quad z \geqslant p_1, \end{cases} \tag{16}$$

where:

$$p_1 = \min\{c_{11}, c_{21}, \ldots, c_{m1}\}$$

It is possible to assign function $f_1(z)$ using (15) equation pressuming $f_0(z) \equiv 0$ for $z \geqslant 0$. The abovementioned considerations lead to the inference that maximum of the function (3), under conditions (6), (7), (8), can be determined using a sequence of iterations:

$$f_0(z) = 0$$

$$f_k(z) = \max\Big(\{f_{k-1}(z)\}\cup$$

$$\cup\,\{f_{k-1}(z-c_{ik})+a_{ik};\, c_{ik}\leqslant z\wedge i=1,\dots,m,k=1,\dots,n\}\Big) \tag{17}$$

for $z \geqslant 0$. Function $f_n(z)$ defines maximum of the function (5) under conditions (6), (7), (8).

4. Properties of the function $f_k(z)$

Function $f_1(z)$ (formula (16)) is a right-handed continuous, non-decreasing, staircase function, then also functions:

$$f_k(z), \quad k = 2,3,\dots,n$$

have the same properties (cf. (15)).

Maximal binary incidental matrix X_m will be constructed now. Following denotations are accepted:

$$\begin{aligned}
d_k &= \max\{\, a_{1k};\ a_{2k},\dots,a_{mk}\},\\
z_k &= \min\{\, c_{ik};\ a_{ik}=d_k \ \wedge\ i=1,2,\dots,m\},\\
v_k &= \min\{\, i;\ c_{ik}=z_k \ \wedge\ i=1,2,\dots,m\},\\
q_k &= \sum_{i=1}^{k} z_i,\\
D_k &= \sum_{i=1}^{k} d_i,
\end{aligned} \tag{18}$$

for $k = 1,2,\dots,n$. Maximal binary incidental matrix is definite as fallows:

$$X_m = (x_{ik})$$

where:

$$x_{ik} = \begin{cases} 0 & \text{for} \quad i \neq v_k \\ 1 & \text{for} \quad i = v_k \end{cases}, \qquad \begin{matrix} i = 1,2,\dots,m \\ k = 1,2,\dots,n \end{matrix}.$$

Taking into consideration binary incidental matrix in (4) we obtain:

$$f_k(z) = D_k \quad \text{for} \quad z \geqslant q_k. \tag{19}$$

Using (17) define a set U_k of all points of discontinuity of the function $f_k(z)$, $k = 1, 2, \dots, n$:

$$U_k = \left\{ \sum_{i=1}^{m} \sum_{j=1}^{k} c_{ij} x_{ij}, \quad x_{ij} \in \{0,1\} \quad \wedge \quad \begin{array}{c} i = 1, 2, \dots, m \\ k = 1, 2, \dots, m \end{array} \right\} \quad \cap$$

$$\cap \quad [0, q_k]. \tag{20}$$

For those sets the following conditions are fulfiled:

$$U_1 \subset U_2 \subset \dots \subset U_n.$$

The staircase function $f_k(z)$ can be written by a pair of vectors:

$$(Z_k, \ F_k), \tag{21}$$

where vector Z_k is formed by elements of the set U_k in ascending order i.e.:

$$\begin{aligned} Z_k &= [z_0, z_1, \dots, z_{s_k}], \\ 0 &= z_0 < z_1 < \dots < z_{s_k} = p_k, \\ U_k &= \{ \ z_k; \ \ k = 1, 2, \dots, s_k \}. \end{aligned} \tag{22}$$

Vector F_k is a value of the function $f_k(z)$ i.e. $f_k = f_k(z_i)$ $i = 0, 1, \dots, s_k$,

$$F_k = [f_0, f_1, \dots f_{s_k}];$$

in this case $f_0 = 0$, $f_{s_k} = D_k$. The following function is presented by the pair of vectors (21):

$$f_k(z) = \begin{cases} f_i & \text{for} \quad z_i \leqslant z < z_{i+1} \quad \wedge \quad i = 0, 1, \dots, s_k \\ f_{s_k} & \text{for} \quad z \geqslant q_k. \end{cases} \tag{23}$$

Let:

$$U_k' = \{\ z_{i+1};\ \ f_i = f_{i+1} \quad \wedge \quad i = 0, 1, \ldots, s_k - 1\}$$

Then the set:

$$U_k \setminus U_k'$$

is a set of points of discontinuity of the function $f_k(z)$. Omitting in (21) points of discontinuity of the function $f_k(z)$ which belong to the set U_k' and omitting dependent variables in those points one can get a representation of staircase function $f_k(z)$ in the simplest form. A sequence of vectors:

$$(Z_k, F_k) \qquad k = 1, 2, \ldots, n$$

which defining functions:

$$f_k(z) \qquad k = 1, 2, \ldots, n,$$

is now get as follows:

1. On the basis of (20) and (22) a sequence of vectors is assigned:

$$Z_1, Z_2, \ldots, Z_n$$

2. As given vector F_{k-1} is known, so F_k vector is:

$$F_k = [f_0, f_1, \ldots, f_{s_k}]$$

According to (17) we have:

$$f_v = f_k(z_v) = max\Big(\{f_{k-1}(z_v)\} \ \ \cup \ \ \{f_{k-1}(z_v - c_{ik}) + a_{ik};$$

$$z_k \leqslant a_{ik} \ \ \wedge \ \ i = 1, 2, \ldots, m\}\Big)$$

where:

$$f_{k-1}(z_v - c_{ik}), \qquad c_{ik} \leqslant z_v, \qquad i = 1, 2, \ldots, m$$

is definite according to the pair of vectors (Z_{k-1}, F_{k-1}) and the formula (23).

5. Assignment of binary incidental matrix which reaches maximum of (3) function

Described algorithm leads to assignment of maximum of objective function (3) at changeable restriction ($f_n(z)$ function). It is necessary to assign incidental matrix for each stabilized value z ($z \geqslant 0$):

$$\tilde{X} = (\tilde{x}_{ij}) \in M\,(m, n) \tag{24}$$

such that:

$$f_n(z) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}\tilde{x}_{ij}. \tag{25}$$

A vector V_k is introduced for this purpose:

$$V_k = [v_1, v_2, \ldots, v_{s_k}],$$

we note, on this vector, which of values from the set occurring in the formula (24) reaches maximum. We introduce the set $B_{v,k}$ of coordinates of the vector V_k as follows:

$$\begin{array}{llll} \text{if} & f_v = f_{v-1}(z_v) & \text{than} & 0 \in B_{v,k}, \\ \text{if} & f_v = f_{v-1}(z_v - c_{ik}) + a_{ik} & \text{than} & i \in B_{v,k}; \quad \text{for} \quad i = 1, 2, \ldots, m \end{array}$$

If $B_{v,k} = \{s\}$, then $v_v = s$, in the case where the set $B_{v,k}$ consists of more than one element we put:

$$v_v = -\max\{B_{v,k}\}. \tag{26}$$

If the set of all binary incidental matrixes fulfiling condition (26) is to be determined, all sets $B_{v,k}$ containing more than one element are to be memorized. Minus sign with V_k vector coordinate results in ambiguity when formula (24) reaches maximum. On the basis of sequence of vectors:

$$(\ Z_k, F_k, V_k\ ) \qquad k = 1, 2, \ldots, n$$

incidental matrix can be determined when objective function (compare (26)) reaches maximum for optional $z \geqslant 0$. Taking into consideration information about ambiguity when reaches maximum (24) (compare (27)) one can identify, if incidental (24) matrix fulfiling condition (26) is assigned without any doubt. With given additionally all of $B_{v,k}$ sets containing more than one element one can assign a set of all (25) binary incidental matrix fulfiling the (26) condition. It should be noticed that for $z \geqslant D_n$ (compare(18)) incidental matrix reaching maximum of objective function is maximum binary incidental matrix definite by (19) formula. Assignment of binary incidental matrix which reaches maximum of $f_n(z)$ objective function for $z \geqslant D_n$. There is a coordinate z_i in z_n vector that:

$$z_i \leqslant z < z_{i+1},$$

v_i value is assigned using V_n vector. If $v_i = 0$ then $x_{in} = 0$ for $i = 1, 2, \ldots, n$, but if $v_i = s > 0$ then:

$$\begin{aligned} &x_{sn} = 1 \\ &x_{in} = 0 \quad \text{for} \quad i \neq s \quad \wedge \quad i = 1, 2, \ldots, n. \end{aligned}$$

The problem of maximization of objective function are to be considered:

$$f_{n-1}(z) \qquad \text{if} \quad v_i = 0.$$

or

$$f_{n-1}(z - a_{sn}) \quad \text{if} \quad v_i > 0.$$

Proceeding analogically for vector X_{n-1}, V_{n-1}. Succesion for:

$$k = n, n-1, n-2, \ldots, 1$$

a binary incidental matrix which reaches maximum of objective function is assigned. If during the process of (24) incidental matrix assignment fulfiling (25) conditions $v_i < 0$ has been appeared, it signifies ambiguity in these assignment. In this case adequate $B_{v,k}$ set should be used to assign all (24) incidental matrix, fulfiling (25) conditions. It isillustrated by an example. Abovementioned algorithm has been used to optimalize emission of air pollutions which rise during production of heat and electric energy in the area of Bytom. It is not possible to gain those results by so far using intuitive operations.

6. Presentation of algorithm on numerical example

The above mentioned problem are to be studied for matrixes:

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 4 & 7 & 1 \\ 5 & 4 & 3 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 3 & 4 \\ 5 & 5 & 9 \\ 7 & 9 & 10 \end{bmatrix}$$

For those task maxima binary incidental matrix is:

$$X_m = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

moreover:

$$z_1 = 7 \quad z_2 = 5 \quad z_3 = 4$$
$$q_1 = 7 \quad q_2 = 12 \quad q_3 = 16,$$

$$Z_1 = [0,1,5,7]$$
$$Z_2 = [0,1,3,4,5,6,7,8,9,10,12]$$
$$Z_3 = [0,1,3,4,5,6,7,8,9,10,11,12,13,14,15,16].$$

According to (19):

$$f_1(0) = 0, \quad f_1(1) = 1, \quad f_1(5) = 4 \quad f_1(7) = 5.$$

$f_1(z)$ function is definite by vectors:

$$Z_1 = [0, 1, 5, 7]$$
$$F_1 = [0, 1, 4, 5]$$
$$V_1 = [0, 1, 2, 3],$$

thus:

$$f_1(z) = \begin{cases} 0 & \text{for} \quad 0 \leqslant z < 1 \\ 1 & \text{for} \quad 1 \leqslant z < 5 \\ 4 & \text{for} \quad 5 \leqslant z < 7 \\ 5 & \text{for} \qquad\quad z \geqslant 7 \end{cases}$$

According to the formula (15) we have:

$$f_2(z) = \max\{f_1(z), f_1(z-3)+4, f_1(z-5)+7, f_1(z-9)+4\},$$

thus we obtaine:

$$\begin{aligned}
f_2(0) &= 0 \\
f_2(1) &= \max\{f_1(1)\} = 1 \\
f_2(3) &= \max\{f_1(3), f_1(0)+4\} = \max\{1, 4\} = 4 \\
f_2(5) &= \max\{f_1(5), f_1(2)+4, f_1(0)+7\} = \max\{1, 1+4, 7\} = 7 \\
f_2(6) &= \max\{f_1(6), f_1(3)+4, f_1(1)+7\} = \max\{4, 1+4, 1+7\} = 8 \\
f_2(10) &= \max\{f_1(10), f_1(7)+4, f_1(5)+7, f_1(1)+4\} = \\
&= \max\{5, 5+4, 4+7, 1+4\} = 11 \\
f_2(12) &= \max\{f_1(12), f_1(9)+4, f_1(7)+7, f_1(3)+4\} = \\
&= \max\{5, 5+4, 5+7, 1+4\} = 12
\end{aligned}$$

$$\begin{aligned}
Z_2 &= [0, 1, 3, 5, 6, 10, 12] \\
F_2 &= [0, 1, 4, 7, 8, 11, 12] \\
V_2 &= [0, 1, 1, 2, 2, 2, 2]
\end{aligned}$$

The function $f_3(z)$ is assigned analogically and as the result of it there are three vectors:

$$\begin{aligned}
Z_3 &= [0, 1, 3, 4, 5, 6, 7, 8, 9, 10, 14, 16] \\
F_3 &= [0, 1, 4, 6, 7, 8, 10, 11, 13, 14, 17, 18] \\
V_3 &= [0, 0, 1, 1, -1, 0, 1, 1, 1, 1, 1, 1]
\end{aligned}$$

Some icidental matrix fulfiling (25) condition for various values z are assigned. For $z = 5$ there is ambiguity.

The problem for $z = 10$ are to be studied. Using Z_3, F_3, V_3 vectors one can get: $k = 10$

$$v_3(10) = 1 \qquad \tilde{x}_{13} = 1$$
$$c_{13} = 4 \qquad z' = 10 - 4 = 6$$

For $z = z'$ from Z_2, F_2, V_2 vectors one can get: $k = 6$

$$v_2(6) = 2 \qquad \tilde{x}_{22} = 1$$
$$c_{22} = 5 \qquad z' = 6 - 5 = 1$$

Then for $z = 1$ there is a result from Z_1, F_1, V_1 vectors:

$$v_1(1) = 1 \qquad \tilde{x}_{11} = 1$$

The solution of this problem is a matrix:

$$X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The task for $z = 5$ are to be considered: from Z_3, F_3, V_3 vectors one can get: $k = 5$, $B_{v,k} = \{0, 1\}$

There are therefore two solutions of the task $v_3(5) = 1$ or $v_3(5) = 0$.

If:

$$v_3(5) = 1 \quad \text{then} \qquad \tilde{x}_{13} = 1$$
$$c_{13} = 4 \qquad z' = 5 - 4 = 1$$

For: $z = z'$ from Z_2, F_2, V_2 one can get $k = 2$:

$v_2(2) = 0$ the second emitter is not modernized.

For $z = 1$ from Z_1, F_2, V_2:

$$v_1(1) = 1 \qquad \tilde{x}_{11} = 1$$

The solution of the problem is a matrix:

$$X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

If:

$$v_3(5) = 0$$

the third emitter is not modernized. Because of the fact that thera are no expenses still $z = 5$ one can get from Z_2, F_2, V_2: $k = 5$

$$v_2(5) = 2 \qquad \tilde{x}_{22} = 1$$
$$c_{22} = 5 \qquad z' = 5 - 5 = 0.$$

Because $z = 0$ the first emitter cannot be modernized because of lack of financial resources. The solution of the task is such a matrix:

$$X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

There are two equivalent solutions of the task for $z = 5$.

References

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Streszczenie

W pracy przedstawiono rozwiązanie problemu optymalnej alokacji zasobów finansowych ze względu na maksymalizację odpowiednio określonego efektu ekologicznego. Problem sprowadza się do pewnego zadania programowania binarnego z jednym ograniczeniem (ograniczenie na zasoby finansowe). Zagadnienie rozwiązano korzystając z zasady optymalności Bellmana.

Opracowano algorytm, który przy zadanych środkach finansowych wyznacza optymalną macierz incydencji wskazującą źródła emisji i technologie, w które należy zainwestować środki finansowe, którymi dysponujemy.

Rozpatrywane zagadnienie sprowadza się do maksymalizacji funkcji celu postaci:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_{ij} \longrightarrow \max \tag{1}$$

przy ograniczeniach:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \leqslant z, \tag{2}$$

$$x_{ij} \in \{0,1\}, \qquad \begin{array}{l} i = 1,2,\ldots,m \\ j = 1,2,\ldots,n \end{array} \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} \leqslant 1, \tag{4}$$

Do wyznaczenia optymalnego rozwiązania danego zagadnienia zastosowano zasadę optymalności Bellmana, gdyż funkcja celu jest liniowa względem kolumn binarnej macierzy incydencji. Przez $f_k(z)(z \geqslant 0), k = 1, 2, \ldots, n$ oznaczono maksimum funkcji:

$$\sum_{i=1}^{m}\sum_{j=1}^{k} a_{ij}x_{ij} \longrightarrow \max \tag{5}$$

przy ograniczeniach (2), (3) oraz:

$$\sum_{i=1}^{m}\sum_{j=1}^{k} c_{ij}x_{ij} \leqslant z. \tag{6}$$

Otrzymano równanie:

$$f_k(z) = \max\left(\{f_{k-1}(z)\} \cup \{f_{k-1}(z - c_{ik}) + a_{ik}; c_{ik} \leqslant z \wedge i = 1, \ldots, m\}\right)$$

z którego można określić iteracyjnie rozwiązanie rozpatrywanego zagadnienia (1) przy ograniczeniach (2), (3), (4).

Algorytm wyznacza binarną macierz icydencji:

$$\tilde{X} = (\tilde{x}_{ij}) \tag{7}$$

realizującą maksimum:

$$f_n(z) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\tilde{x}_{ij}$$

dla każdej wartości $z \geqslant 0$. Algorytm powyższy daje możliwość rozwiązania zagadnienia o dużym wymiarze.