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SOME COMPARISON OF DIFFERENT OPTIMIZATION TECHNIQUES ON THE BASE OF RESULTS FOR ARCHETYPE "TRAGEDY OF COMMONS"

Summary. On the base of some chosen system archetype the comparison of different optimization techniques, like "hill climbing" by Coyle (in COSMOS language), in packet Mathematica and in Vensim is presented. Some conclusions from these comparisons are undertaken.

PEWNE PORÓWNANIE RÓŻNYCH TECHNIK OPTYMALIZACYJNYCH NA PRZYKŁADZIE WYNIKÓW OTRZYMANYCH DLA ARCHETYPU „TRAGEDIA WSPÓŁUŻYTKOWANIA"

Streszczenie. Na przykładzie wybranego archetypu systemowego zaprezentowano porównanie różnych technik optymalizacyjnych, takich jak „wspinanie się w górę" w sensie Coyle'a (w języku COSMOS), pakietu Mathematica i języka symulacyjnego Vensim. Sformułowano pewne wnioski na bazie tego porównania.

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1. Introduction

The problem of system archetypes (see [1, 13, 15, 16]) is rather new for field modelers and specially there is lack of attempts to optimize the behaviour of these archetypes. The mean of such optimization can't be overestimated, because knowledge about dynamics of elementary structures can help in better understanding about the sources of dynamics behaviour of complex, nonlinear and multilevels systems.

For simulation and optimization experiments the archetypes named: "tragedy of the commons" was chosen (see [13, pp. 387]). On figure 1 the main structure of such archetype is presented.

The idea of such archetype is: "individuals use commonly available but limited resource solely on the basis of individual need. At the first they are rewarded for using it; eventually, they get diminishing returns, which causes them to intensify their efforts. Eventually, the resource is either significantly depleted, eroded, or entirely used up" (see [13, pp. 387]).

The mathematical equations for "tragedy of the commons" archetype are as follows:

$$\dot{x_1}(t) = \left(\frac{x_3(t)}{a} - b\right) x_1(t),$$

$$\dot{x_2}(t) = \left(\frac{x_3(t)}{c} - d\right) x_2(t),$$

$$\dot{x_3}(t) = -\dot{x_1}(t-e) - \dot{x_2}(t-e),$$

with initial conditions $x_1(0) = x_{10}$, $x_2(0) = x_{20}$, $x_3(0) = x_{30}$. For the aim of illustration of the behaviour of this archetype, the following parameters were chosen:

$$x_{10} = 20, \quad x_{20} = 15, \quad x_{30} = 100.$$

The obtained dynamics for the system is presented on figure 2.

2. Optimization results

The comparison of three different optimization techniques:

- "hill climbing" in COSMOS [3],
- "hill climbing" in Mathematica [7, 17],
- Powell algorithm in Vensim [14]

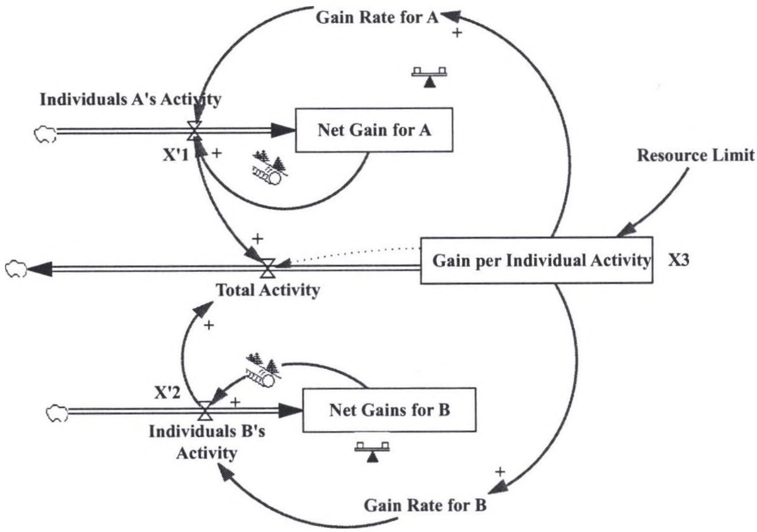


Fig. 1. Block diagram for "tragedy of the commons" archetype (source [1])
Rys. 1. Schemat blokowy dla archetypu „tragedia wspólnoty" (źródło [1])

will be presented. "Hill climbing" in COSMOS is the method of embedding simulation in optimization (see: [4, 10, 11]). Like the objective function in so called Direct Optimization, the sums of: individuals A's activity, individuals B's activity and gain per individual Activity, were chosen. The obtained results are compared with these in Mathematica and Vensim (the window of so called "Optimization setup" is presented on figure 3).

The results of optimization in simulation horizon 12 units, are presented in table 1, and some chosen characteristics on figure 4, 5, 6.

The main six types of experiments were undertaken. Let's concentrate on first, for packet COSMOS. Like the objective function the sumx1 was undertaken (remember that sumx1 is the sum of values of variable x_1 during whole horizon of simulation). Like lower and upper limits for parametr b were taken appropriately

$$\min b = 0.00, \qquad \max b = 0.40$$
$$\min d = 0.00, \qquad \max d = 0.40.$$

Length of simulation was 12 and number of iteration was 30. The improvement of objective function was from value: 293.62 to value: 10835.0, so rather large.

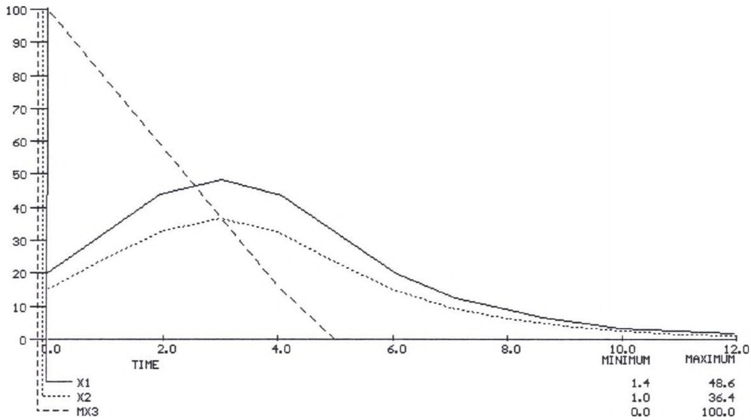


Fig. 2. The dynamics of net gains for A (x_1) and B (x_2), and per individual activity (x_3)

Rys. 2. Dynamika korzyści dla grupy A (x_1) i korzyści dla grupy B (x_2) oraz korzyści z działań indywidualnych (x_3) (sumaryczna)

Now, let's pay attention on first experiment in Mathematica. Similary, the scope of parameters b and d was:

$$\min b = 0.00, \qquad \max b = 0.40$$
$$\min d = 0.00, \qquad \max d = 0.40.$$

The number of iteration was 10. The dynamics of variable "sum of x_1" in whole horizon of simulation, of Figure 5b) is presented (let's notice the final value of "$sumx1$" which is: 6000). Now concentrate on experiment 1 in Vensim. Like so colled "pay off" function the sum of x_1 was taken. The scopes of parameters b and d was the same like in COSMOS and Mathematica. The final value of "pay off" function on the end of simulation (during 22 iteration) was 1180, which is similaty to that obtainted in COSMOS. This is not suprise, the both algorithms chose final value of optimized parameters the same: $b = 0$, $d = 0.4$. Only in Mathematica the values $b = 0$, $d = 0$ was obtained, giving the value of objective function: 6000. Approprietely, we can analyzed the results of remaining experiments, comparing the values in Table 1. Some of them are visualized on Figures 4, 5 and 6.

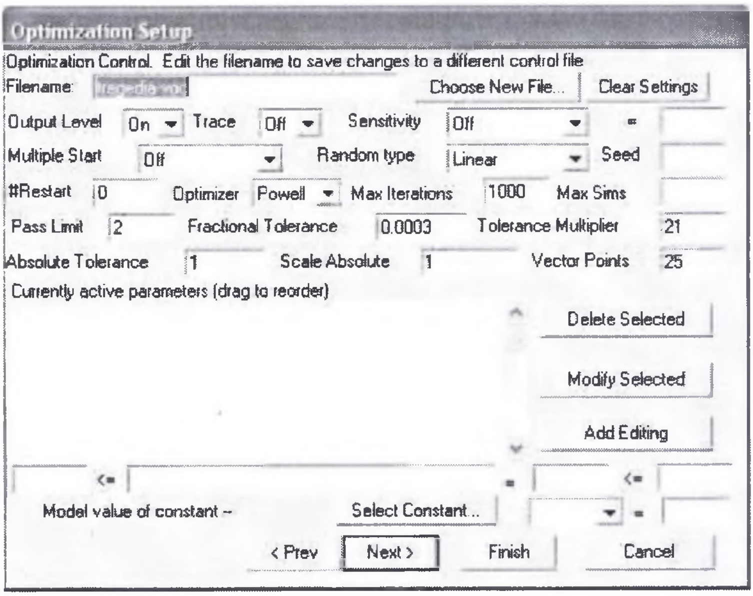


Fig. 3. Window of "optimization setup" in Vensim
Rys. 3. Okno optymalizacji w pakiecie Vensim

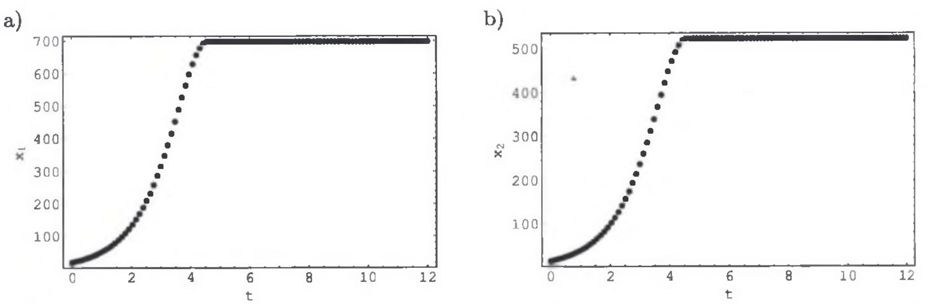


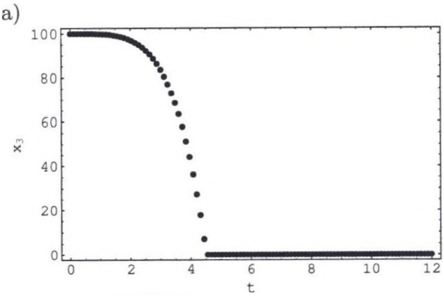
Fig. 4. The dynamics of net gains for A (x_1) (a) and B (x_2) (b) (in optimization experiment with rx_3 in Mathematica)
Rys. 4. Dynamika korzyści dla grupy A (x_1) (a) i korzyści dla grupy B (x_2) (b) (w eksperymencie optymalizacyjnym rx_3 w programie Mathematica)

Table 1

Summarizing of optimization results by use: COSMOS (C), Mathematica (M) and VENSIM (V)

Type of exp.	iteration			optimization parameters			objective function		
	C	M	V	C	M	V	C	M	V
1	30	10	22	$b = 0$ $d = 0.4$	$b = 0$ $d = 0$	$b = 0$ $d = 0.4$	rx_1 : 1083.5	rx_1 : 6000	rx_1 : 1180
2	30	10	110	$b = 0$ $d = 0.399$ $a = 1$ $c = 99.05$	$b = 0$ $d = 0$ $a = 1$ $c = 1$	$b = 0$ $d = 0.4$ $a = 10.98$ $c = 100$	rx_1 : 10^9	rx_1 : 6000	rx_1 : 990.108
3	30	10	22	$b = 0.4$ $d = 0.0$	$b = 0$ $d = 0$	$b = 0.4$ $d = 0.0$	rx_2 : 911.11	rx_2 : 5000	rx_2 : 990.108
4	30	10	97	$b = 0.39$ $d = 0$ $a = 99.703$ $c = 1$	$b = 0$ $d = 0$ $a = 1$ $c = 1$	$b = 0.4$ $d = 0.4$ $a = 100$ $c = 12.12$	rx_2 : 39019	rx_2 : 10^9	rx_2 : 3503.08
5	30	10	13	$b = 0.4$ $d = 0.4$	$b = 0$ $d = 0$	$b = 0.4$ $d = 0.4$	rx_3 : 351	rx_3 : 450	rx_3 : 240.23
6	30	10	25	$b = 0.4$ $d = 0.4$ $a = 100$ $c = 100$	$b = 0$ $d = 0$ $a = 1$ $c = 1$	$b = 0.4$ $d = 0.4$ $a = 100$ $c = 100$	rx_3 : 351	rx_3 : 110	rx_3 : 240.23

a)



b)

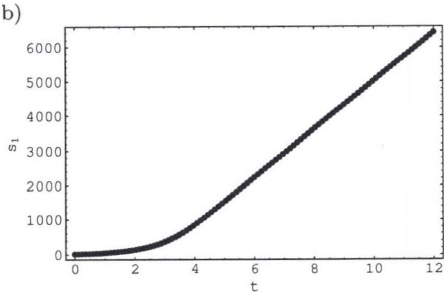


Fig. 5. The dynamics of net gains per individual activity x_3 (a) and the dynamics of sum of x_1 (b) (in optimization experiment with rx_3 in Mathematica)

Rys. 5. Dynamika korzyści indywidualnych x_3 (a) i dynamika sumy korzyści x_1 (b) (w eksperymencie optymalizacyjnym rx_3 w programie Mathematica)

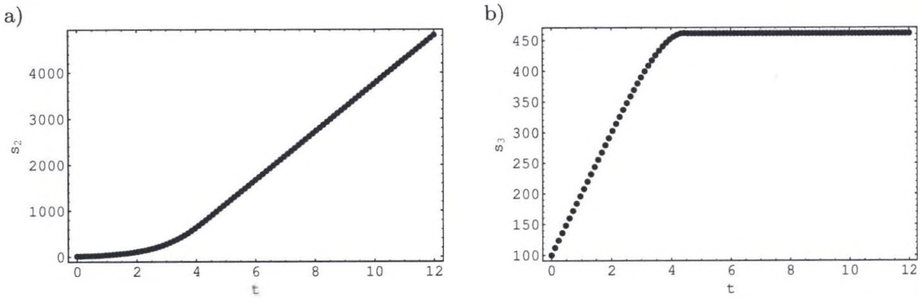


Fig. 6. The dynamics of sum of x_2 (a) and x_3 (b) (in optimization experiment with rx_3 in Mathematica)

Rys. 6. Dynamika sumy korzyści x_2 (a) i korzyści x_3 (b) (w eksperymencie optymalizacyjnym rx_3 w programie Mathematica)

3. Conclusions

After optimization using three different techniques, in chosen horizon of simulation, the conclusion are as follows:

- In some experiments the local maximum founded in COSMOS is smaller then this in Mathematica or Vensim, but for some experiments are better. (This is not surprised. Prof. Coyle in his book [4] has warned that in "hill climbing" sometimes it is required careful choosing of the initial value of parameters, step multiplier and number of iterations).
- The simulation and optimization experiments on systems archetypes, like for example "tragedy of commons", seems to gain knowledge about not only dynamics of elementary structures, but about complex, nonlinear, multilevels systems too.
- It will be interesting in future to compare describing results with that in GA (Genetic Algorithm). Some tests authors have undertaken already. They are still in progress.

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Streszczenie

W artykule przedstawiono wyniki sześciu typów eksperymentów dokonywanych przy użyciu pakietów symulacyjnych: COSMOS, Mathematica i Vensim (w sumie 18 eksperymentów). Wymagały one odmiennych liczb iteracji, otrzymane wyniki wskazują na odmienność końcowych wartości badanych parametrów, a tym samym też funkcji celu. Wyniki pokazują, iż celowe mogą być dalsze prace w tym zakresie, z wykorzystaniem np. algorytmów genetycznych.