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DEFLECTION ROUTING WITH HAMILTONIAN CYCLES

Summary. For the mixed routing algorithm running on the networks with non buffering nodes this article presents an improvement in which an Eulerian cycle is replaced with Hamiltonian cycles. The new upper bound on a data packet's end to end number of hops is equal to or lower than the original upper bound.

Keywords: deflection routing, Hamiltonian cycle, Eulerian cycle

TRASOWANIE Z DEFLEKSJĄ I CYKLAMI HAMILTONA

Streszczenie. W artykule proponuje się ulepszenie mieszanego algorytmu trasującego dla sieci z węzłami, które nie przechowują pakietów (ang. non-buffering nodes). Ulepszenie polega na wykorzystaniu cykli Hamiltona w zamian cyklu Eulera, co sprawia, że górna granica na liczbę skoków pakietu jest mniejsza od (albo w najbardziej niekorzystnym przypadku równa) górnej granicy przed wprowadzeniem ulepszenia.

Słowa kluczowe: trasowanie z defleksją, cykl Hamiltona, cykl Eulera

1. Introduction

The growth of the Internet and the demand for high quality multimedia network services drive the research of high speed networking. The concept of all-optical networks [1] is being intensely researched on, as the prospective technology of the future Internet. In all-optical networks the optical signal traverses the network without undergoing the bottlenecking electro-optical conversion. There are many difficulties to be resolved, most notably the lack of optical memories.

Xu et al report in [2] that currently the only optical element capable of storing an optical signal is the optical delay line. As the optical memories required to implement store-and-forward routing are not available, the optical switches cannot buffer packets[1] but only delay them, and then relay using deflection routing (DR). Therefore DR is applicable and nowadays essential to all-optical switches.

This article is concerned with the part of convergence routing (CR) used in the mixed deflection and convergence routing (MR) algorithm discussed in Section 3. It is not the purpose here to report research on DR.

The paper is organized as follows. Section 2 presents the model of the network. Section 3 reviews an existing MR algorithm, while Section 4 introduces an improvement in the MR algorithm. In Section 5 some examples are given, and Section 6 concludes this paper.

2. Network Model

The network is homogeneous, i.e. each node is either a source node, an intermediate node or a destination node for a packet. A packet is injected into the network in a source node, traverses through intermediate nodes, and finally reaches the destination node, where it is removed from the network.

The network operates synchronously: time is the same for every node and is divided into time slots. During a time slot a link transmits only one packet, while a node receives packets from the adjacent nodes, admits new packets into the network, relays packets to neighboring nodes and removes these packets for which this node is the destination.

The article pertains to directed networks, but an undirected network can be converted into a directed one by replacing every undirected link by two directed links. Similarly, an undirected cycle (i.e. a closed path) can be replaced by two directed cycles running in opposite directions.

Among various measures of a network and a routing algorithm, the end-to-end number of hops is of particular interest in this article. The end-to-end number of hops is the number of links a packet traverses in the network from the moment it is admitted into the network (injected by the source node) to the moment the packet is removed from the network (absorbed by the destination node). The end-to-end number of hops is equal to the number of time slots a packet resides in the network, since every time slot a packet traverses one link. The ending guarantee defined in [3] is an integer that specifies the upper bound on the end-to-end number of hops. The stretch of a cycle (defined in [3]) is an integer equal to the length of a longest path of all shortest paths (between pairs of nodes) which go along the cycle.

[1] From now on the term "packet" refers to a data packet.

3. Mixed Deflection

An acute problem of DR is that a packet may travel in a network for a long time, and potentially never reach its destination due to the probabilistic nature of DR. Since this routing strategy offers no ending guarantee, some packets are considered lost after a long delay.

To cure the problem of indefinite end-to-end number of hops, and to provide an ending guarantee, Barth et al in [3] propose a mixed algorithm. They incorporate into deflection routing Eulerian routing which is convergence routing [5] on an Eulerian cycle. Convergence routing, like deflection routing, does not violate the non-buffering constraint so prominent of the contemporary all-optical network technology. The DR and CR algorithms are mixed together to achieve two qualities: efficiency and an ending guarantee. First, DR provides a good mean end-to-end number of hops (consequently also a good network throughput), which is desired in MR. Second, the virtue of CR is an ending guarantee, and thus the MR algorithm with a skillfully incorporated CR would also ensure an ending guarantee.

The mixed algorithm works as follows. Each packet can be in either of two modes on its journey: the deflection mode or the convergence mode. In these two modes a packet is routed according to different rules. A packet at its source node always starts in the deflection mode, and follows the rules of DR. Each packet keeps a record of the number of suffered deflections by storing this number within its header. The preferable scenario is the packet's arrival at the destination node while it is still in the deflection mode. This, however, might take an indefinitely long time and therefore must be restrained. Conversely (the worse scenario), the packet arrives to its destination in the convergence mode, i.e. after the packet's conversion to the convergence mode. The conversion takes place when the number of experienced deflections reaches the threshold number of deflections, S.

The reason for using an Eulerian cycle in CR is the property that a packet can instantly enter the convergence mode at any node whenever necessary, i.e. once it reaches S deflections. A packet may begin its Eulerian journey on any available outbound link, since every link belongs to an Eulerian cycle.

Note that a stretch of an Eulerian cycle can be painfully long, even equal to the number of network's links. Once a packet gets to such a cycle, it stays there for a considerably long time. Not only does a packet travel longer, but it also blocks the network, and prevents the network from admitting more packets, thus deteriorating the network throughput.

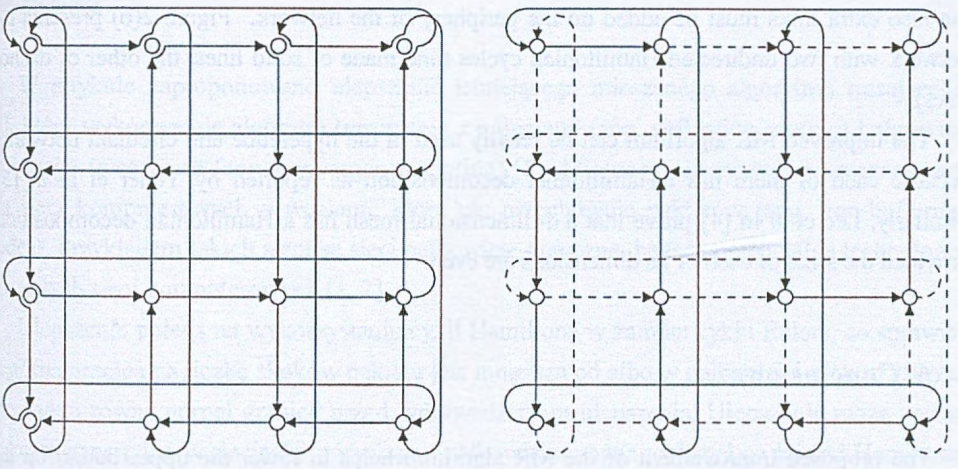
4. Improvement

The central notion of the article is an improvement, where an Eulerian cycle is replaced by several Hamiltonian cycles. As for CR, a packet reaches its destination in a possibly smaller number of hops when routed on a Hamiltonian cycle rather than on an Eulerian cycle. Thus for the mixed algorithm the improvement offers an ending guarantee which is equal to or lower than the ending guarantee of the original MR algorithm.

The Hamiltonian cycles are obtained by a Hamiltonian decomposition, and therefore the modified algorithm is applicable only to those networks that yield a Hamiltonian decomposition. A Hamiltonian decomposition is a set of Hamiltonian cycles, such that every pair of cycles is link-disjoint, and every link of the network belongs to one cycle. A Hamiltonian decomposition offers the property of an instant packet entrance to the convergence mode which is ensured by each link of the network belonging to a Hamiltonian cycle. Note that there is an important difference between the convergence routing on an Eulerian cycle and on several Hamiltonian cycles. The difference is in the stretch of a Hamiltonian cycle and an Eulerian cycle. Since a Hamiltonian cycle is a closed path which visits every network's node, its length and stretch are equal to the number of network's nodes. An Eulerian cycle is always of length equal to the number of links in the network, but its stretch depends on the exact form of the Eulerian cycle. The stretch of an Eulerian cycle, however, is never smaller than the stretch of a Hamiltonian cycle. Therefore if the network has a Hamiltonian decomposition then in CR it is more advantageous to employ Hamiltonian cycles instead of an Eulerian cycle.

5. Examples

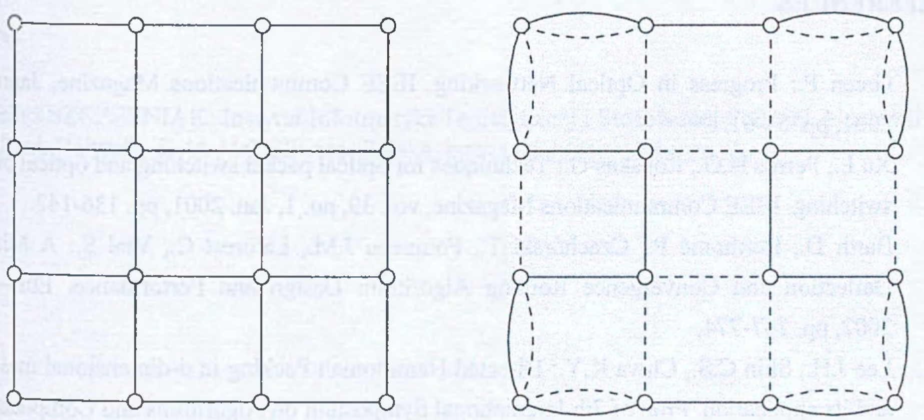
The Manhattan Street Network (MSN) is a torus of a 2D grid, where each of the nodes has two inbound links and two outbound links. The network is Eulerian (i.e. contains an Eulerian cycle) and has a Hamiltonian decomposition under the condition that the number of rows and columns is even. Figure 1(a) shows an Eulerian cycle of a sample MSN, while Figure 1(b) depicts two directed Hamiltonian cycles (one made of solid lines, the other of dashed lines). The network contains 32 links, and consequently this is the length of the Eulerian cycle. The stretch of the Eulerian cycle is 27. Each of the Hamiltonian cycles, however, contains 16 links and is of stretch 16. The difference between the stretch of a Hamiltonian cycle and the stretch of the Eulerian cycle is 11, which motivates the use of the Hamiltonian cycles.



(a) An Eulerian cycle
(a) Cykl Eulera

(b) Two Hamiltonian cycles
(b) Dwa cykle Hamiltona

Fig. 1. Cycles in the Manhattan street network of size 4
Rys. 1. Cykle w sieci „Manhattan street network" rozmiaru 4



(a) Original version
(a) Wersja niezmieniona

(b) Modified version
(b) Wersja zmieniona

Fig. 2. Two versions of an undirected 2D grid
Rys. 2. Dwie wersje nieskierowanej kraty dwuwymiarowej

There are networks which have to be modified so as to benefit from the use of Hamiltonian cycles. An example of such a network is the undirected 2D grid in Figure 2(a). It is Eulerian for the odd network size (the number of rows and columns), but two conditions must be met to make this network have a Hamiltonian decomposition. Not only the network size must be even, but also extra links must be added on the periphery of the network. Figure 2(b) presents the network with two undirected Hamiltonian cycles (one made of solid lines, the other of dashed lines).

The improved MR algorithm can be readily used in the hypercube and circulant networks, because each of them has a Hamiltonian decomposition as reported by Yener et al in [5]. Similarly, Lee et al in [4] prove that a d-dimensional mesh has a Hamiltonian decomposition, provided the sizes of each of its dimensions are even.

6. Conclusion

The proposed improvement of the MR algorithm helps to lower the upper bound on the end-to-end number of hops. This improvement, however, is dependent on a Hamiltonian decomposition of a network, and cannot can be introduced, unless the network has a Hamiltonian decomposition.

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Omówienie

W artykule zaproponowano ulepszenie istniejącego mieszanego algorytmu trasującego [3], który wykorzystuje algorytm trasowania z odbiciami (ang. deflection routing) i algorytm zbieżnego trasowania (ang. convergence routing) [5]. Mieszany algorytm jest przeznaczony dla sieci komputerowych z węzłami, które nie przechowują pakietów (ang. non-buffering nodes). Przykładem takich sieci są sieci całkowicie optyczne, będące potencjalną technologią przyszłych sieci komputerowych [1, 2].

Ulepszenie polega na wykorzystaniu cykli Hamiltona w zamian cyklu Eulera, co sprawia, że górna granica na liczbę skoków pakietu jest mniejsza od albo w najbardziej niekorzystnym przypadku równa górnej granicy przed wprowadzeniem ulepszenia. Ulepszenie może zostać wykorzystane tylko w sieciach, których graf całkowicie można rozłożyć na ścieżki Hamiltona (ang. Hamiltonian decomposition).

Przedstawiono przykłady rodzajów sieci, w których można zastosować udoskonalony mieszany algorytm: „Manhattan street network" przedstawiona na rys. 1, dwuwymiarowa krata przedstawiona na rys. 2, hipersześcian [5] oraz wielowymiarowa krata [4].

Adres

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