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MODELING SURFACES BY SUBDIVISION METHODS

Summary. This paper presents general survey of subdivision schemes using for modeling 3D object of arbitrary topological type. The way of analyzing main properties of subdivision surfaces is also presented. This mathematical set of tools allow to use subdivision as well defined predict block in multiresolution analysis of surface meshes by second generation wavelet.

Keywords: subdivision surfaces, lifting scheme, second generation wavelets, multiresolution

MODELOWANIE POWIERZCHNI PRZY UŻYCIU METOD PODPODZIAŁU

Streszczenie. Artykuł prezentuje przegląd metod modelowania obiektów 3D za pomocą podpodziału powierzchni. Został przedstawiony sposób analizy głównych właściwości powierzchni, które powstały w wyniku zastosowania schematów podpodziału. Ten zestaw matematycznych narzędzi pozwala na użycie schematów podpodziału jako bloku predykcji w wielorozdzielczej analizie siatek powierzchni za pomocą falek drugiej generacji.

Słowa kluczowe: podpodział powierzchni, schemat liftingu, falki drugiej generacji, wielorozdzielczość

1. Introduction

Historically subdivision schemes were developed as generalizations of uniform B-spline knot insertion algorithms to settings with topologically irregular control meshes. Generally subdivision defines a smooth curve or surface as the limit of a sequence of successive refinements [1]. The main advantage of subdivision surfaces on the other representations is that they can be defined by control meshes with arbitrary connectivity. Other properties of

subdivision surfaces are: scalability, numerical stability, code simplicity, efficiency, local definition and affine invariance. Recently subdivision arouse lot of interest in computer graphic research in modeling, compression, progressive transmission, LOD (level of detail) rendering and multiresolution processing of surfaces. We can see application of this method in animation films (Geri's Game, A Bug's Life and Toy Story 2) and games (Quake 3). Subdivision surfaces were also integrated into a number of commercial modeling and rendering systems.

2. Subdivision surfaces

This article presented subdivision schemes for surface meshes composed of triangles as the most popular primitive in computer graphics.

Triangle mesh is a pair M=(P, T) where P is a set of n points $p_i=(x_i, y_i, z_i)$ with $1 \le i \le n$ and T is a simplical complex which contains information about topology of mesh. The complex is a set of three types subsets called simplies: vertices $v = \{i\} \in V$, edges $e = \{i, j\} \in E$, faces $f = \{i, j, k\} \in F$.

Two vertices $\{i\}$ and $\{j\}$ are neighbours if $\{a, b\} \in E$. The 1-ring neighbourhood of vertex $\{i\}$ is the set $N(i) = \{b \mid \{a, b\} \in E\}$. The valence of a vertex is the number of edges meeting at this vertex. There are two types of vertex in triangle mesh: ordinary (regular vertex with valence equals 6) and extraordinary. For each simplex *s*, its topological realization |s| is the strictly convex hull. The topological realization |T| is defined as $\bigcup_{s \in M} |s|$. The star of a vertex $\{i\}$ is the set of simplices $star(i) = \bigcup_{i \in s, s \in M} s$.

Given these definitions, a topological mesh |T| defines 2-manifold if for each $\{i\}$, |star(i)| is homeomorphic to a disk in \mathbb{R}^2 .

The extraordinary vertex count is linked to the genus of the manifold by Euler formula:

$$v - e + f = 2 - 2g \tag{1}$$

where g is the genus of mesh and v, e, and f are the number of vertices, edges and faces in closed mesh, respectively.

The domain of subdivision surface is initial control mesh M^0 (corresponding polygonal complex). The subdivision process produces a hierarchy sequence of polyhedra with increasing numbers of faces and vertices $(M^1, M^2,...)$. This hierarchy is connected with multiresolution techniques as wavelets analysis of surface mesh [19, 18, 16].

Each refinement consist of two steps (fig. 1): splitting $(M^{j} - >M^{j'})$ and averaging $(M^{j'} - >M^{j+1})$.



Rys. 1. Pojedynczy krok podpodziału: podział i uśrednianie

The first step, topological subdivision, splits each face or edge. The second step, geometric positioning, computes positions for new vertices as a local affine combination of old vertices via subdivision relation:

$$P^{j} = S^{j-1} P^{j-1} \tag{2}$$

where *S* is a subdivision matrix and *j* indicates the level of subdivision process (number of subdivision steps). This produces the nested sets of vertices $V^j \subset V^{j+1}$. In descriptions of subdivision schemes the subdivision masks are given for odd vertices (new vertices added to mesh) and even (vertices from previous level V^j).

Subdivision schemes, for which the subdivision matrix is independent of the level of subdivision, are known as stationary schemes. When subdivision matrix depend on the level j this is a non-stationary scheme. Limit of this process is a subdivision surface:

$$\sigma = \lim_{i \to \infty} S^{j} P^{0} \tag{3}$$

We can also define subdivision as linear map Z for simplical complex T:

$$Z: T^{j} \to T^{j+1} \tag{4}$$

Subdivision basic function ϕ_v (scaling function) is the result of subdivision process on initial control mesh represent by Kronecker delta δ_v (value one at vertex *v* and zero at all other vertices):

$$\phi_{\nu} = Z^{\infty} \delta_{\nu}$$
⁽⁵⁾

3. Analysis of subdivision schemes

3.1. Support

To study region of limit surface of given subdivision scheme first it must be determined vertices (vertex and its neighbourhood) in control mesh which influence on this region. Each vertex influences finite part of surface. This size of neighborhood depends on the number of non-zero entries in each row of the subdivision matrix. This matrix depends on the valence of the original vertex in the centre. Coefficients in this matrix rows sum up to one because scheme should be affine invariance [1, 3, 4].

3.2. Spectral analysis

The eigenvalues and eigenvectors of the subdivision matrix allow to analyze how the control points in the invariant neighbourhoods change from level to level. The local subdivision matrix *S* has size *n x n* and has real eigenvectors x_0 , x_1 , ..., x_{n-1} , with corresponding real eigenvalues λ_0 , λ_1 , ..., λ_{n-1} . For affine invariance schemes with respect to translations and rotations $\lambda_0 = I$. Stable and converging subdivision schemes will have all the remaining λ_i less than 1 [7].

Main equations of eigenanalysis for subdivision matrix S are:

$$Sx_{Ri} = \lambda_i x_{Ri}$$

$$x_{Li}S = \lambda_i x_{Li}$$

$$X_L = X_R^{-1}$$
(6)

Where index *R* stand for right and *L* for left eigenvectors.

Diagonal matrix of all eigenvalues $\Lambda = diag(\lambda_i)$ is called spectre of matrix *S*. Multiplying a set of neighbours vertices \overline{p} by local subdivision matrix *S*(7) is the way to determine what happens in the limit of an infinite number of steps of stationary subdivision [3, 4, 6].

$$P^{j} = S^{j}P^{0}$$

$$\overline{p}^{\infty} = \lim_{j \to \infty} S^{j} \overline{p}^{0} = \lim_{j \to \infty} \sum_{i=0}^{n-1} (x_{Li} \lambda_{i} x_{Ri})^{j} \overline{p}^{0} = \lim_{j \to \infty} \sum_{i=0}^{n-1} x_{Li} (\lambda_{i})^{j} x_{Ri} \overline{p}^{0} = a_{0}$$

$$a_{0} = x_{L0} \overline{p}^{0}$$

$$(7)$$

Coefficient a_0 is computed using dominant left eigenvector, that is, the left eigenvector associated with largest eignevalue.

Technique of eigenanalysis is mostly used for characterize the limit surface behave near an extraordinary point [6, 28]. Spectral analysis of local subdivision matrix make also possible to derive necessary and sufficient conditions for stationary subdivision scheme to produce smooth limit surface (chapter 3.3).

3.3. Smoothness and continuity

Analysis of continuity consists of two parts, determining degree of continuity in the regular regions (composed of valence 6 vertices for triangles meshes and valence 4 vertices for quad meshes) and at extraordinary points. If the scheme is derived from tensor product or box-spline the continuity is references to the underlying basis function in the regular part. If it is not derived from any known surface representation, the continuity of the limit surface should be analyzed by using sufficient conditions based on z-Transforms or difference scheme and degree of reproduction polynomial [8]. In this case it can be also analyzed by spectral analysis (chapter 3.2) which allow characterize the limit surface round the vertex.

Spectral decomposition of eigenvetors x_i and eigenvalues λ_i may be presented as a Taylor expansion:

$$P^{j} = S^{j} P^{0} = \sum_{i=0}^{n} \lambda_{i}^{j} a_{i} x_{Ri}$$
(8)

where a_i are the Taylor coefficients. This has the same geometrical represents as Taylor expansion: components for i=0 are responsible for central position of vertex at limit surface, components for i=1, 2 define tangent plane, elements for i=3, 4, 5 are responsible for the curvature (cup and two saddles). This give conditions for a subdivision scheme to coverage at an extraordinary vertex $\lambda_0 = 1 > \lambda_i$. The second largest (subdominant) eigenvalues λ_1 and λ_2 determine tangent plane. Useful schemes are rotationally symmetric and this give additional conditions $\lambda_1 = \lambda_2$ and $\lambda_4 = \lambda_5$.

In case analyze smoothness it must be shown that a limit surface is C^m manifold in the neighborhood of vertex v, which means that is locally graph of a C^m function. This can be checked by examine of characteristic map [7, 9, 11, 10] which is map of limit surface to the plane defined be subdominant eigenvectors:

$$\Phi(x_1, x_2) \colon U \to R^2 \tag{9}$$

where U is matrix of the regular neighborhood of the extraordinary vertex. If the limit surface is tangent plane continuity we have $\lambda_0 = 1 > \lambda_1 = \lambda_2 > \lambda_i$. This was widen [7] to more stronger condition, if additionally the characteristic map is regular and injectivity then subdivision surfaces are C^l surface for almost every control net. This can be proven by showing that the Jacobian determinant of the resulting characteristic map Φ has the same sign. In similar way a sufficient conditions for C^m continuity may be written:

$$\lambda_0 = 1 > \lambda_1 = \lambda_2^m > \lambda_i \tag{10}$$

The necessary conditions for C^m continuous limit surface was formulated [10, 5], but building a stationary subdivision matrix that satisfied these conditions for arbitrary valence extraordinary vertices is rather difficult and schemes are complex.

3.4. Artifacts

For stationary subdivision surfaces there are at least three kinds of artifact [2]: longitudinal, lateral and rotational.

Longitudinal artifact are ripples which run in the same direction as the ridges in surface. This artifact relate to the regular regions of control mesh.

Lateral artifacts are ripples which run in the across the ridges in surface. This artifact relate to the regular regions of control mesh. It is possible that, even in the regular case, curvature in one direction can cause periodic lumps and bumps in the other.

Rotational artifacts are related to extraordinary points. These are ripples around the extraordinary point of high valence. For some schemes, as the valence increases, the magnitude of the third largest eigenvalue approaches the magnitude of the subdominant eigenvalues. As an example we consider surfaces generated by the Loop scheme near vertices of high valence (fig. 2).

It is possible to eliminate this problem by prescribing the eigenvalues of the subdivision matrix and deriving suitable subdivision coefficients. This approach was used to derive coefficients of the Modified Butterfly scheme [14].

a)





Fig. 2. Rotational artifacts: a) initial mesh, b) mesh after five steps of Loop subdivision
Rys. 2. Artefakty rotacyjne: a) siatka bazowa, b) siatka po pięciu krokach schematu podpodziału Loopa

4. Subdivision schemes

Stationary subdivision schemes may be classified by four main criteria [1]. First is the method of coefficient determination – uniform (coefficients are the same, used only on regular meshes), semi-uniform (coefficients depend on the connectivity – ordinary and extraordinary vertex), non-uniform (coefficients depend on the connectivity and geometry of the mesh). Second criterion is the type of refinement rule – primal (face split, vertex insertion), dual (vertex split, corner cutting). Third is whether the scheme is approximating or interpolating. The last one is the type of control mesh – triangular, quadrilateral, hexagonal. This article presents only selected schemes for triangular schemes.

4.1. Semi-uniform subdivision schemes

4.1.1. Loop scheme

The Loop scheme (fig. 5b) is a simple approximating primal scheme for triangular meshes [12]. The scheme is based on the three-directional box-spline, which produces C^2 continuous surfaces over regular meshes and C^1 continuous at extraordinary vertices. Mask for odd and even vertices give weights for counting new positions of vertices (fig. 3).

Weights for even vertices depend on valence of vertex *n*, $\beta(n) = \frac{1}{n} (\frac{5}{8} - (\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n})^2)$.



Rys. 3. Maska schematu podpodziału Loopa

	$\left[1-\frac{5}{8}\beta(5)\right]$	$\beta(5)$	$\beta(5)$	$\beta(5)$	$\beta(5)$	$\beta(5)$
<i>S</i> ₅ =	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	0	0	$\frac{1}{8}$
	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	0	0
	$\frac{3}{8}$	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	0
	$\frac{3}{8}$	0	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
	$\frac{3}{8}$	$\frac{1}{8}$	0	0	$\frac{1}{8}$	$\frac{3}{8}$

Example 6×6 local subdivision matrix for one ring of neighbors:

4.1.2. Modified Butterfly scheme

The Butterfly scheme [13, 14] is interpolating (fig. 5c), primal subdivision scheme and produces C^1 continuous surface for arbitrary mesh. For this scheme is two masks for odd vertices – regular and extraordinary (fig. 4).



Fig. 4. Mask for Modified Butterfly subdivision Rys. 4. Maska zmodyfikowanego schematu podpodziału Butterfly

The coefficients $s_i = \frac{1}{n}(\frac{1}{4} + \cos\frac{2i\pi}{n} + \frac{1}{2}\cos\frac{4i\pi}{n})$ for $n \ge 5$ and $s_0 = \frac{5}{12}$, $s_{1,2} = -\frac{1}{12}$ for n = 3, $s_0 = \frac{3}{8}$, $s_2 = -\frac{1}{8}$, $s_{1,3} = 0$, n = 4.



Fig. 5. Subdivision surfaces: a) initial mesh, b) mesh after five steps of Loop subdivision,c) mesh after five steps of Modified Butterfly subdivision

Rys. 5. Powierzchnie podpodziału: a) siatka bazowa, b) siatka po pięciu krokach schematu Loopa, c) siatka po pięciu krokach zmodyfikowanego schematu Butterfly

4.1.3. $\sqrt{3}$ scheme

The $\sqrt{3}$ scheme [15] is approximating, dual subdivision scheme and produces C^2 continuous surface for arbitrary mesh. First midpoints inserted in the center of each face and then original edges are flipped (fig. 6).



Fig. 6. $\sqrt{3}$ subdivision scheme Rys. 6. Schemat podpodziału $\sqrt{3}$

4.2. Other subdivision schemes

Example subdivision schemes for quadrilateral meshes [1] are: Catmull-Clark, Doo-Sabin, Kobbelt.

The non-uniform schemes (variational subdivision) are derived from digital geometry processing which consist, among other things, set of tools to smoothing meshes (discrete fairing) [22]. This schemes can be used to build predict operator (chapter 5.2) in multiresolution analysis of irregular surface meshes by second generation wavelets [23, 26, 27], to build multiresolution hierarchies [20, 21] and to perform multiresolution editing on irregular meshes [25]. The subdivision operator computes positions of vertex which minimize value of some function for example: minimize of second order differences defined at every edge [20, 21], surface curvature minimization [23]. The disadvantage of these schemes are

slower work, much higher complex and not complete mathematical tool for analysis. The non-uniform subdivision schemes may be used to processing mesh with attributes (e.g. color, texture coordinates, temperature) by defined relaxation operator for this attributes [23, 24].

Another group of subdivision schemes is adaptive subdivision which is used to obtain region of interest (ROI) with more details in chosen area or in place where is more complex surface. Flat regions of the surface are sufficiently well approximated by large triangles. The major difficulties that emerge from adaptive refinement, are caused by the fact that triangles from different refinement levels have to be joined, which may cause gaps if neighboring faces which are not refined. One way to fix this is to use red-green triangulation [17]. This approach was application in $\sqrt{3}$ [15].

5. Subdivision in surface analysis by second generation wavelets

5.1. Second generation wavelets

The second generation wavelets [27, 26, 19] are generalization of biorthogonal classic wavelets (called first generation wavelets). Second generation wavelets are not necessarily translated and dilated of one function (mother function) so they can not be constructed by Fourier transform. Second generation wavelets may use many basic function but must be satisfied refinement relation – basic function of previous level must be linear combination of functions of next level of resolution. This is necessary to define nested space used for describing multiresolution analysis. Lifting scheme (fig. 7) [27, 26, 19] is a simple but powerful tool to construct second generation wavelets. Main advantage of this solution is the possibility to build wavelet analysis on non-standard structures of data (irregular samples, bounded domains, curves, surfaces, manifolds) with keeping all powerful properties of first generation wavelets such as speed and good ability to approximation.



Rys. 7. Schemat liftingu

A general lifting scheme consists of three types of operation: split (S), predict (P) and update (U). The basic idea is to split first a signal $s_{j+1,k}$ into its even $s_{j+1,2k}$ and odd $s_{j+1,2k+1}$

samples. Next predict the odd signal from the even part. What is missed by the prediction is called the detail $d_{j,k}$. Then the even samples are adjusted (update operation) to serve the coarse version of the original signal. The adjustment is needed to maintain the same average for the fine and coarse versions of the same signal.

5.2. Subdivision in predict block

In this section subdivision will be presented as a powerful paradigm to build predictors for the lifting scheme. In this context subdivision is a inverse wavelet transform with no detail coefficients. Subdivision basic function (4) is scaling function of transform.

Following properties of wavelet transformation are possible to check by analyze subdivision scheme:

- Compact support This is a result of locality of subdivision method (chapter 3.1).
- Polynomial reproduction and smoothness The degree of the reproduction polynomial determines the number of vanishing moments of the analysis.
- Refinability Subdivision generates nested sets of points and the basic function satisfies a refinement relation.
- Stability of transform The condition necessary for the existence of stable scaling basis is convergence (chapter 3.2) of the subdivision scheme. This criterion is possible to check when is one scaling function (regular or semi-regular mesh) but for irregular setting (nonuniform subdivision) there is no intermediate link between convergence of subdivision and stability [26].

Semi-uniform subdivision schemes can be used in multiresolution analysis for meshes with subdivision connectivity [16, 18, 19, 28], for irregular mesh it can be used non-uniform schemes [20, 21, 23, 24] but there is no enough analysis tool to check properties of transform. This is a direction for future research.

5.3. Example predict block

Predict block (P) in the lifting scheme (chapter 5.1) is responsible for computing value of vertex with odd index based on even ones. The difference between the value of odd vertex and its prediction is detailed (wavelet coefficient):

$$d_{j,k} = s_{j+1,2k+1} - P(s_{j+1,2k}) \tag{11}$$

The inverse operation is the recover of odd values by adding details and prediction information:

$$s_{j+1,2k+1} = d_{j,k} + P(s_{j,k})$$
(12)

To use Loop subdivision (chapter 4.1.1) given mask must be redefined, so it become lifting operations which have an inverse of the same style [29]:

$$v_{n} = \frac{3}{8}(v_{0} + v_{1}) + \frac{1}{8}(v_{2} + v_{3})$$

$$v_{o} = (1 - \beta(n))v_{o}' + \beta(n)\sum_{i=1}^{n}v_{n}$$
(13)

where v_n is a new added vertex (with odd indexes) predicted by its neighbours (with even indexes) – v_0 , v_1 , v_2 , v_3 . The second equitation is the approximation step of Loop subdivision and it is part of update block.

In the same way, the predict operator may be defined for Butterfly scheme (chapter 4.1.2) [30]:

$$v_n = \frac{1}{2}(v_0 + v_1) + \frac{1}{8}(v_2 + v_3) - \frac{1}{16}(v_4 + v_5 + v_6 + v_7)$$
(14)

6. Summary

Subdivision surfaces are defined as a limit of repeated refinement of 3D control point mesh. Defining mathematical techniques to analysis smoothness, continuity, convergence and artifacts of surfaces has become the subject of more recent research. This mainly apply to uniform and semi-uniform subdivision schemes. It follows that these schemes can be used in building second generation wavelets to multiresolution analysis of surface meshes with subdivision connectivity. In case of irregular meshes there can be apply non-uniform scheme but it requires defining set of mathematical tools to analyze such transformation.

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Omówienie

Artykuł prezentuje metodę podpodziału powierzchni (*subdivision surfaces*) jako narzędzia modelowania obiektów 3D. Metoda ta definiuje gładką krzywą lub powierzchnię w skończonej liczbie elementarnych kroków. Pojedynczy krok (rys. 1) polega na dodaniu nowych wierzchołków (*splitting*) oraz określeniu współrzędnych pozostałych wierzchołków (*averaging*). Podstawowe równania procesu podpodziału i tworzonej powierzchni zostały przedstawione (2, 3, 4, 5).

Artykuł prezentuje również zestaw narzędzi, umożliwiających analizę powstałej powierzchni, między innymi: określenie ciągłości, gładkości oraz możliwych artefaktów. Przedstawiona analiza głównie opiera się na badaniu macierzy podpodziału (*subdivision matrix*) za pomocą analizy spektralnej (7, 8). Została również przedstawiona główna klasyfikacja schematów podpodziału łącznie z przykładami (rys. 5), takimi jak: schemat Loopa, schemat Butterfly, schemat $\sqrt{3}$, schematy niejednolite i adaptacyjne.

Dzięki dobrze zdefiniowanym zestawie narzędzi matematycznych schematy podpodziału mogą zostać wykorzystane do budowy bloku predykcji w schemacie liftingu (rys. 7). To umożliwia przeprowadzenie wielorozdzielczej analizy siatki powierzchni za pomocą falek drugiej generacji. Równania bloku predykcji dla schematów Loopa (13) i zmodyfikowanego schematu Buttefly (14) zostały przedstawione.

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