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## NUMERICAL MODELING OF STRESS FIELD IN ROCK STRATA FROM SURFACE DEFORMATION SURVEYS

**Summary.** The surface deformation measurements provide information that may be used in determination of the redistribution of stresses within the rock strata. A methodology has been developed in which displacements at discrete points obtained from geodetic monitoring surveys can serve as an input to deterministic modeling of stress distribution. The knowledge of the stress distribution provides a better understanding of the deformation mechanism. The method has been applied to a complex problem of explaining irregular ground subsidence caused by mining activity and hydrological changes in a large potash mine in eastern Canada.

## NUMERYCZNE MODELOWANIE ROZKŁADU NAPRĘŻEŃ W GÓROTWORZE NA PODSTAWIE POMIARÓW ODKSZTAŁCEŃ NA POWIERZCHNI

**Streszczenie.** Wyniki pomiarów odkształceń na powierzchni dostarczają informacji, które mogą służyć do wyznaczenia rozkładu naprężeń wewnątrz górotworu. W opracowanej metodzie przesunięcia punktów otrzymane z geodezyjnych pomiarów są użyte do opracowania deterministycznego modelu rozkładu naprężeń, z którego można otrzymać lepsze zrozumienie mechanizmu odkształceń. Metoda ta została zastosowana do wytłumaczenia nieregularnego osiadania powierzchni na terenie górniczym dużej kopalni potasu we wschodniej Kanadzie. Nieregularne osiadanie zostało wywołane robotami górniczymi i zmianami hydrologicznymi.

### 1. Introduction

Integrated analysis of deformations of a rock mass includes geometrical analysis and physical interpretation. The geometrical analysis may be based on geodetic monitoring results

from which one can derive displacement and strain fields in the space and time domains (Chrzanowski et al., 1982).

Physical analysis may be based on deterministic modeling, which utilizes the knowledge of the geometry of the object and its environment, causative factors (loads), properties of the material, and physical laws governing the stress-strain relationship. Since complex differential equations must be solved in the process of deterministic modeling, numerical methods, (e.g. the finite element method (FEM)) are generally employed.

In case of rock and soil materials, the in-situ geomechanical properties may significantly differ from the laboratory values. This must be taken under consideration when performing deterministic modeling of deformation.

By comparing the geometrical model of deformations, derived from the observed deformation quantities, with the designed deformations obtained from FEM, one can verify the designed geomechanical parameters (e.g., Szostak - Chrzanowski et al, 2000). In addition, with properly designed monitoring surveys one may also determine the actual deformation mechanism (Chrzanowski and Szostak - Chrzanowski, 1993; Chrzanowski and Szostak - Chrzanowski, 1995) and explain the causes of deformation in case of an abnormal behavior of the investigated object. Thus, the role of deformation monitoring surveys becomes much broader than just the conventional determination of the geometrical status of the deformable object. In this presentation an example is given on the use of geodetic monitoring surveys in determination of effects of mining and resulting hydrological changes on ground subsidence in a potash mine in Canada.

## **2. Basic Concepts and Methodology used in Deterministic Modeling of Rock Deformation**

### **2.1. In-Situ Characteristics of Rock Mass**

In order to form a deterministic problem the following information must be defined: geometry of the problem, material mechanical properties, model of the behavior of the material, acting loads, and boundary conditions. The most critical problem in modeling and predicting rock deformations is to obtain real characteristics of in-situ rock mass. Collection of in-situ characteristics of rock is very difficult and very costly and the data is often incomplete. In laboratory testing, the selected samples may differ from one location to

another, they may be disturbed during the collection, or the laboratory loading conditions may differ from natural conditions. The physical values obtained from laboratory testing require scaling in order to represent a rock mass. The process involves a degree of uncertainty. Generally, four types of scale are distinguished: (1) sample as an intact rock; (2) rock in vicinity of opening with some joints; (3) rock with many joints; and (4) rock as a rock mass (large scale problem). The problem of scale-dependent properties is a main problem in modeling rock behavior (Glaser and Doolin, 2000).

Canadian Centre for Geodetic Engineering (CCGE) has developed a methodology for deterministic modeling of rock deformation where the behavior of the rock mass is treated as a large-scale problem. A concept of the averaged equivalent medium or a concept of the equivalent medium divided into a few interacting blocks is used in the forward analysis for parameter identification. The material model for equivalent medium has been developed on a basis of the determination of the Young modulus as a function of time. Young modulus and strength of the rock material vary through the in-situ rock mass. Generally, Young modulus of in-situ rock masses is smaller than the values obtained in laboratory. (Bieniawski, 1984; Sakurai, 1997). The material strength decreases with the size of the model where large fractures are predominant in the rock considered as a rock mass. Determination of the scaling function is based on the stress distribution (compressive and tensional).

## **2.2. Model of Rock Mass Behavior**

The selection of the material behavior model is another important problem in deterministic modeling. The model of linear elasticity is still most widely used in modeling behavior of rocks, especially hard rocks (Jing, 2003). More sophisticated constitutive models used in rock mechanics are, for example, non-tensional model (Zienkiewicz et al., 1968), anisotropic elasticity, plasticity, elasto-plasticity, and visco-elasticity. Plasticity and elasto-plasticity models used in rock mechanics are typically based on Mohr-Coulomb and Hoek-Brown failure criteria (Hoek and Brown, 1982). Visco-elastic models are used in salt rock or other weak rocks and may be based on a rheological model (Owen and Hinton, 1986). The salt rock is also modeled as a non-Newtonian liquid (Dusseault et al., 1987). Use of more sophisticated constitutive models in rock mechanics may be limited by a difficulty in obtaining necessary parameters.

The CCGE method (known as S-C method) of modeling deformations in mining areas is based on the assumption that the brittle rock has parameters of non-tensional

material(Szostak-Chrzanowski and Chrzanowski 1991b). The salt and potash rock material is considered as a non-Newtonian liquid with high and not constant viscosity (Dusseault et al., 1987; Szostak-Chrzanowski and Chrzanowski, 1991a).

### **2.3. Modeling Hydrological Changes in a Rock Mass**

Change of underground fluid (water or oil) condition due to pumping to the surface or due to water relocation, e.g. an inflow into the mining openings, may cause surface subsidence. Vertical flow of ground water, horizontal flow, and inelastic compaction of the material of aquifer systems are important parameters in modeling changes in an aquifer. The aquifer may consolidate or compact due to increased stress. For example, a confined aquifer with an initial thickness of 45 m consolidates 0.20 m when the head is lowered by 25 m (Fetter, 1942). The compaction of material in the aquifer produces redistribution of stress in the surrounding rock. Thus the process of the ground subsidence calculation above the aquifer follows the same model as the calculation of subsidence due to mining openings.

## **3. Stress Redistribution in PCS Potash and Salt Mine**

Mining of a large deposit of high grade sylvinitic in New Brunswick, Canada, has been carried out since the mid 1980s. Potash and salt mining takes place at depths between 400 m to 700 m within a 25 km long dome-shaped salt pillow in which the potash is preserved in steeply dipping flanks (Figure 1). A strong, arch shaped, cap rock provides an excellent natural support for the overlain brittle rocks. Potash is mined by using a mechanized cut-and-fill method with up to 100% extraction in the 1000 m long and about 150 m high stopes. Unsupported openings are up to 25 m wide. The potash deposit is structurally complex with a variable dip and width. Salt mining is by multi-level room-and-pillar method.

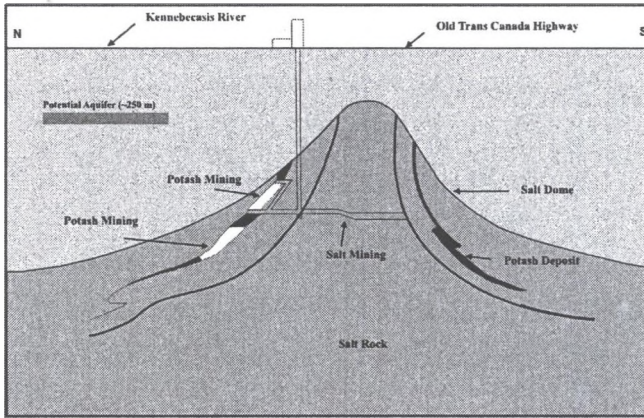


Fig. 1. Geological cross-section  
 Rys. 1. Przekrój geologiczny

Annual monitoring of ground subsidence over the potash and salt mining operation has been carried out by the Canadian Center for Geodetic Engineering since 1989. Fig. 2 shows the layout of the mining workings and the location of points of monitoring surveys consisting of precision levelling, traversing with robotic total stations, and GPS surveys. In 1995, a finite element analysis was performed to model the maximum expected subsidence along a selected cross-section (line crossing the observed maximum subsidence in Fig. 2). A summary of the results was presented in (Chrzanowski et al. 1998). The expected subsidence profile was to follow a regular shape with its maximum subsidence located above the room-and-pillar salt extraction (approximately above the center of the salt dome).

In 1997, a significant increase in water inflow to the mine was noticed at lower levels of potash extraction near the investigated cross-section and a secondary subsidence basin started occurring on the surface at the north end of the investigated cross-section. An underground aquifer located above the potash mining in the area of the secondary subsidence was suspected to be a source of inflowing water. At the same time, exploratory and mitigative drillings from underground workings revealed that the caprock and rock strata above potash mining are much weaker than previously expected, showing multiple cracks and a significant void of about 20 m width and 180 m height. Fig. 2 shows the current subsidence isolines (contours) developed between 1996 and 2006. Fig. 3 shows the accumulated subsidence profile along C line between 1996 and 2006 with pt. C<sub>4</sub> being at the centre of the primary subsidence profile and pt. C<sub>0</sub> being approximately at the centre of the secondary subsidence.

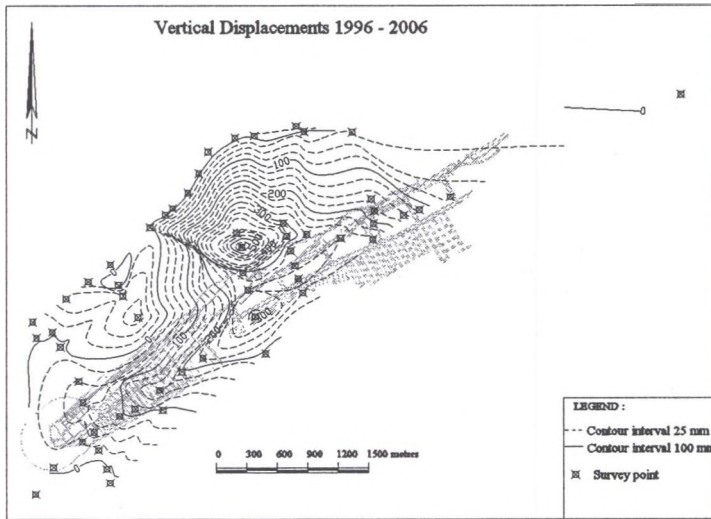


Fig. 2. Subsidence isolines (contours) developed between 1996 and 2006  
 Rys. 2. Kontury osiadania pomiędzy 1996 i 2006

The FEM analysis of ground subsidence was performed to explain whether the water inflow from an unknown aquifer and its compaction, or the weaker rock strata and the creation of the void could cause the development of the secondary subsidence basin. The described methodology, based on the large-scale problem modeling, was used in deterministic modeling of the stress distribution and rock mass deformations caused by salt and potash mining and by hydrological changes between 1996 and 2006. The selection of the best model was based on the correlation between the measured and calculated subsidence (Fig. 3).

The effect of the hypothetical aquifer was modeled assuming that the centre of the aquifer was located below the centre of the secondary subsidence. The FEM modeling indicated that the most probable depth of the aquifer was at 150 m (Szostak - Chrzanowski and Chrzanowski, 2001) having width of 330 m and height of 40 m. The FEM modeling of the subsidence caused by salt and potash mining and by hydrological changes was repeated using some additional information on the discovered voids above potash stopes and water inflow into the upper stopes.

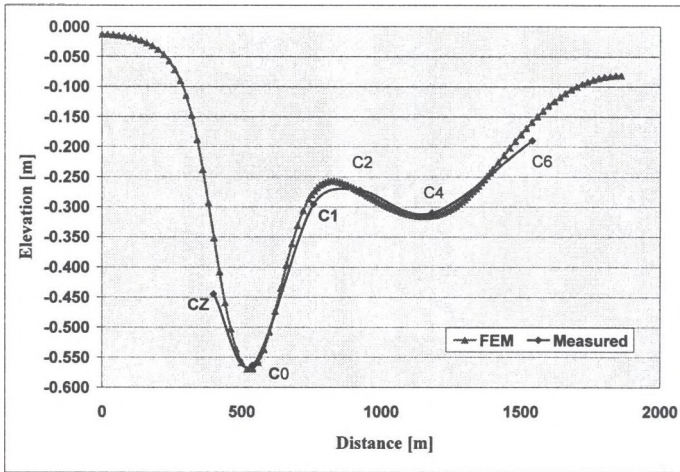


Fig. 3. Measured and FEM Surface Subsidence

Rys. 3. Pomierzone i MES wyznaczone osiadanie powierzchni

FEM analysis of the rock mass response to the mining activity and to the hydrological changes addressed:

1. determination of the redistribution of stresses in the rock mass and determination of zones with the maximum stress values in the areas of:
  - salt dome over potash and salt mining,
  - potential aquifer,
  - washout zones in the upper parts of potash mining,
  - cap rock;
2. determination of changes of mechanical parameters of the rock mass and determination of their time function.

FEM analysis was performed for 2002, 2003, 2004, 2005, and 2006. The input data for each year was based on the output data of previous year and the Young modulus of the “flow - in” zone of salt was calibrated using monitoring subsidence data. Fig. 6 shows Young modulus values as a function of time. There is a progressing development of the tensional stresses (anisotropic elements) above the aquifer and above the top of the caprock. The analysis has confirmed that the void between the aquifer and the caprock had to be increased in the vicinity of the caprock.

The redistribution of stresses (Fig. 4. and Fig. 5.) explains the mechanism of response of the rock mass to the mining activity and to the hydrological changes.

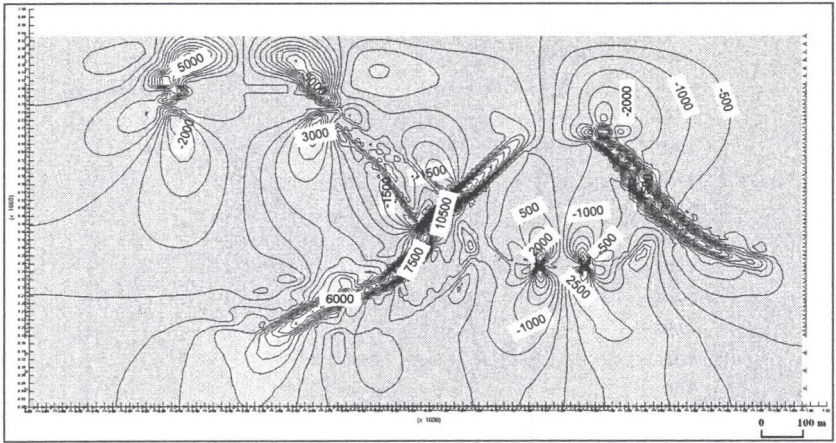


Fig. 4. Distribution of Maximum Shear Stress (MPa)  
 Rys. 4. Rozkład maksymalnych naprężeń ścinających (MPa)

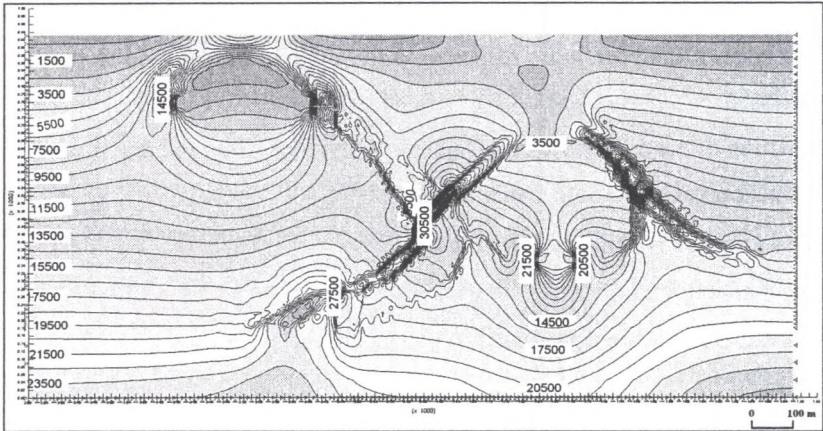


Fig. 5. Distribution of Maximum Stress (MPa)  
 Rys. 5. Rozkład maksymalnych naprężeń

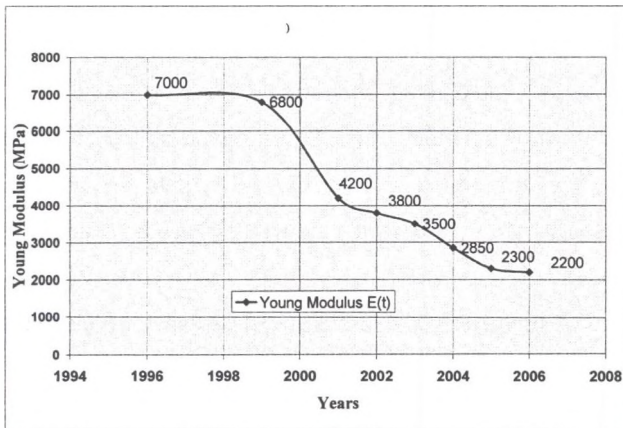


Fig. 6. Young modulus values as a function of time  
 Rys. 6. Moduł Younga jako funkcja czasu



## 4. Conclusions

The monitoring surveys alone describe only the geometrical changes of the surface. In order to explain the physical meaning of the changes, the results of surveys must be combined with deterministic modeling of the phenomena.

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