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THE RELIABILITY OF TREE AND STAR NETWORKS

Summary. This paper investigated the reliability of tree and star networks. Following measures of network reliability are assumed: the expected number of nodes, that can communicate with the central node; the expected number of node pairs, that are connected by a path through the central node; the expected number of node pairs communicating.

Keywords: star networks, tree networks, reliability

NIEZAWODNOŚĆ SIECI KOMPUTEROWYCH O TOPOLOGIACH GWIAŹDZISTYCH ORAZ DRZEWIASTYCH

Streszczenie. Rozważono sieci o topologiach gwiaździstych oraz drzewiastych. Jako ocenę niezawodności tych sieci przyjęto: średnią liczbę sprawnych węzłów, które mogą komunikować się z węzłem centralnym; średnią liczbę sprawnych par węzłów, które mogą komunikować się przez węzeł centralny; średnią liczbę sprawnych par węzłów, które mogą komunikować się wzajemnie.

Słowa kluczowe: sieci gwiaździste, sieci drzewiaste, niezawodność

1. Introduction

In addition to the other problems such as network cost, average packet delay and network throughput an important consideration in the design of computer network is the network reliability with respect to failures of communication links and nodes. The reliability of network depends strongly on their topology and on the reliability of the communication links and nodes. Work in this area has been focused on the formulation of reliability criteria, development of methods of effective evaluation of reliability criteria and search for networks, which are most reliable in the sense of chosen criteria. There are many methods of the reliability analysis, as well as the algorithms for computing the reliability measures [1, 5]. Almost all these methods used a set of branch or node-disjoint routes, or cuts. For any network, calculation all disjoint routes or cut set can be difficult. In the case when topology of network is tree or star the problem is simpler. In this paper, following measures of the reliability are assumed: the expected number of nodes, that can communicate with the central node; the expected number of node pairs, that are connected by a path through the central node; the expected number of node pairs communicating. Any two nodes can communicate mutually, if links and nodes forming rout among these nodes are no damage states. So, for star a network expected number of nodes, which can communicate, with the central node is equal to the sum of probabilities of reliability routes connecting nodes with central node. Similarly, the expected number of node pairs which are connected by a path through the central node, and the expected number of node pairs communicating are equal to sum probabilities of reliability routes connecting node pairs. For large networks with many hops the recurrence method [2] can be used instead of determining the values of the reliability measures. The calculations of the network reliability were done as a function of the probability of the link being operative. In the paper [3, 4] the numerical calculation for assumed measures of reliability were done as a function of the probability failures of the nodes. Papers [3, 4] investigated the reliability of star, ring and hybrid networks.

2. Reliability analysis

The topology of the computer communication network will be modeled as a graph $G = (\mathrm{N}, \mathrm{E})$, where $\mathrm{N} = \{n_1, n_2, ..., n_N\}$ is a set of nodes, E is a set of links. The network has only duplex links, thus the network can be modeled as the graph with no oriented branches. The link between nodes n_i and n_j is denoted by the unordered pair (i, j). Considered networks have tree or star topologies. Example of two-hop star topology is shown in Fig. 1.

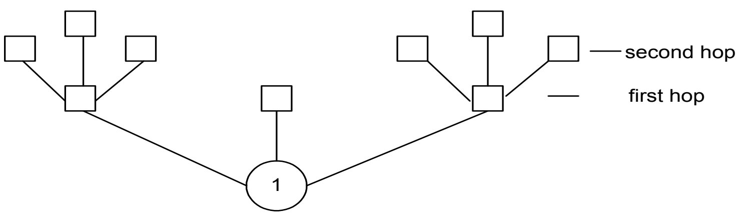

Fig. 1. Two-hop star topology
Rys. 1. Dwustopniowa topologia gwiaździsta

All of the nodes and links fail independently. The probability, that the link (i, j) is in the no damaged state is equal to $p(i, j)$, the probability that the node n_i is in the no damaged state is equal to $p(i)$. The following measures of the reliability are considered: the expected number of nodes that can communicate with the central node (*S*); the expected number of node pairs that are connected by a path through the central node (*R*); the expected number of node pairs communicating (*T*). For the star and tree network, the expected number of node pairs, which are connected by a path through the central node, the expected number of node pairs communicating and the expected number of nodes which can communicate with the central node are equal to the sum of probabilities of reliability routs connecting node pairs. For example, for two-hop star network the formula for expected number of nodes, which can communicate with the central node is following.

$$S = \sum_{i=2}^{M} (p(1,i)p(i)(1+\sum_{j=1}^{N_i} p(i,i_j)p(i_j))) \tag{1}$$

For two-hop star network the nodes are numbered in this way. The node 1 is the central node. The nodes directly connected with the central node are numbered from 2 to *M*. Next, the non-numbered nodes directly connected to node n_i ($i = 1,2,...,M$) are numbered from i_1 to i_{N_i}. The formula for the expected number of node pairs, which are connected by a path through the central node, and for the expected number of node pairs communicating is similar. For large networks with many hops can be difficult write these formulas. In this case the recurrence method [2] can be used instead. The idea of the recurrence method is the following.

Suppose, that for each node *i* except 1 is designed a node $F(i)$, such that $F(i) < i$ and $(i, F(i))$ is a link in the network. In Fig. 2 the network contains two subtrees, one with the central node *i* and the other with the central node $j = F(i)$.

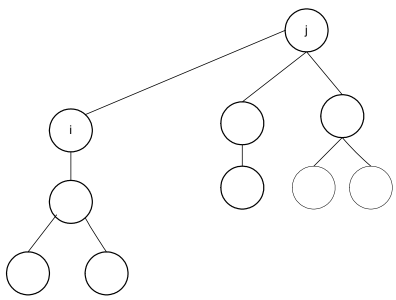

Fig. 2. Illustration for recurrence relations
Rys. 2. Ilustracja metody rekurencyjnej

The values of the measures of reliability for subtrees are known. The values of reliability measures for the tree obtained by joining *i* and *j* by link (i, j) determine following recurrence relations (2) [2], where $S(i)(S(j))$ is the expected number of nodes in the subtree which communicate with the central node *i* (*j*), including *i* (*j*), $R(i)(R(j))$ the expected number of node pairs in the subtree communicating through central node, $T(i)(T(j))$ the expected number of node pairs communicating in the subtree, $S'J), R'(j), T'(j)$ are the values of the measures of the reliability measures for the tree obtained by joining node *i* and node *j* by link (i, j).

$$\begin{aligned} R(j)' &\leftarrow R(j) + (S(i)S(j) + R(i)p(j))p(i, j) \\ T(j)' &\leftarrow T(i) + T(j) + S(i)S(j)p(i, j) \\ S(j)' &\leftarrow S(j) + S(i)p(j)p(i, j) \end{aligned} \tag{2}$$

At first, the network consists with N subtrees having one node. For this network $R(i) = 0$, $S(i) = p(i)$, $T(i) = 0$, $i = 1,2,...,N$. Set *i=N,* let *j=F(i),* determine the values of reliability measures for the tree obtained by joining *i* and *j* by link (i, j). Set *i* to *i-1* procedure joining subtrees is continuing. This procedure is continuing to obtain the initial network. When the algorithm stops, *R(*1*)* is the expected number of node pairs, which are connected by a path through the central node, *S*(1) is the expected number of nodes, which can communicate with the central node, and *T*(1) is the expected number of node pairs communicating.

3. Numerical results

The values of reliability measures were counted for one, two, three, four and hundred-hop star networks. Example of two-hop star topology is shown in Fig. 1. The links fail independently with the same probability $1 - p$ and the nodes are failure-free. The values of reliability measures were counted for the networks containing hundred and one nodes. The distribution of the nodes is following. The two-hop star network contains ten nodes in the first hop, and each node of the first hop is connected with nine nodes of second hop. The three-hop star network contains four nodes in the first hop, each node of the first hop is connected with three nodes of the second hop, and each nodes of the second hop is connected with seven nodes of the third hop. The four-hop network contains three nodes in the first hop, each node of the first hop is connected with two nodes of the second hop, each node of the second hop is connected with three nodes of the third hop, and each node of third hop is connected with four nodes of fourth-hop. The each node of hundred-hop star network has degree one. The calculations were done as a function of a probability of link being operative. The obtained results are shown in Tables 1, 2, 3. Figures 3, 4, 5 illustrate numerical results the obtained for reliability measures. Figures shown that the differences between networks' reliability decrease when the hop of the star network grows. The largest differences are for the probabilities of link failure equal to 0.3, 0.2, and 0.1. The differences for assumed measures are similar. For example for criterion *R* (*T*) the largest difference between one-hop and two-hop star network is for $p = 0.7$ and equals 22% (20%). The largest difference between two-hop and three- hop star network is for $p = 0.8$ and equals 12% (10%). The largest difference between three-hop and four-hop star network is for $p = 0.8$ and equals 8% (7%). The largest difference between one-hop and chain network is for $p = 0.9$ and equals 79% (65%). For the measure *S*, the largest difference between one-hop and two-hop star network is for $p = 0.5$ and equals 22%. The largest difference between two-hop and three-hop star network is for $p = 0.7$ and equals 13%. The largest difference between three-hop and four-hop star network is for *p*=0.7 and equals 8%. The largest difference between one-hop and chain network is for $p = 0.9$ and equals 80%.

Table 1

Obtained values for the expected number of nodes, which can communicate with the central node (*S*) (in percent)

p	*SG1(101,p)* one-hop star network	*SG2(101,p)* *two*-hop star network	*SG3(101,p)* three-hop star network	*SG4(101,p)* four-hop star network	*SG100(101,p)* hundred-hop star network
0	0.99	0.99	0.99	0.99	0.99
0.1	10.891	2.871	1.588	1.372	1.1
0.2	20.792	6.535	2.923	2.08	1.238
0.3	30.693	11.98	5.493	3.482	1.414
0.4	40.594	19.208	9.798	6.12	1.65
0.5	50.495	28.218	16.337	10.705	1.98
0.6	60.396	39.01	25.608	18.128	2.475
0.7	70.297	51.584	38.111	29.447	3.3
0.8	80.198	65.941	54.345	45.898	4.95
0.9	90.099	82.079	74.808	68.888	9.901
1	100	100	100	100	100

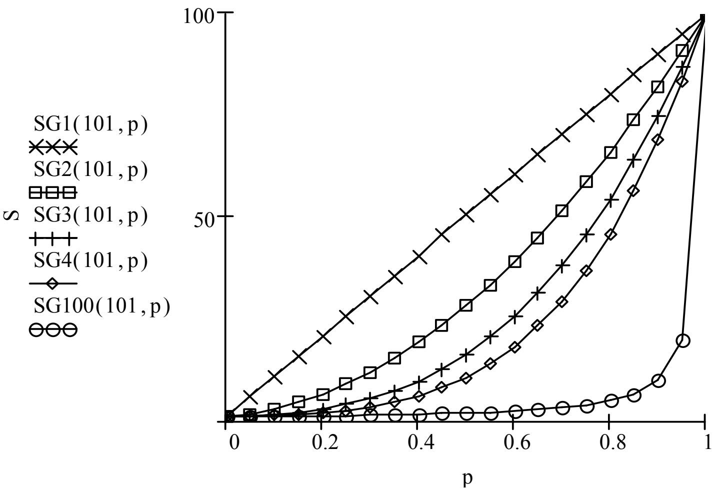

Fig. 3. The obtained results for measure *S* as a function of the probability of the link being operative
Rys. 3. Otrzymane wartości dla miary *S* w funkcji prawdopodobieństwa sprawności kanałów

Table 2

Obtained values for the expected number of nodes, which are connected by a path though the central node (*R*) (in percent)

p	*RG1(101,p)* one-hop star network	*RG2(101,p)* *two*-hop star network	*RG3(101,p)* three-hop star network	*RG4(101,p)* four-hop star network	*RG100(101,p)* hundred-hop star network
0	0	0	0	0	0
0.1	1.178	0.095	0.022	0.011	2.0005
0.2	4.317	0.519	0.122	0.052	6.0002
0.3	9.416	1.671	0.457	0.191	0.012
0.4	16.475	4.123	1.362	0.606	0.022
0.5	25.495	8.62	3.48	1.721	0.04
0.6	36.475	16.081	7.922	4.457	0.074
0.7	49.416	27.599	16.473	10.672	0.154
0.8	63.317	44.436	31.83	23.856	0.396
0.9	81.178	68.033	57.876	50.18	1.782
1	100	100	100	100	100

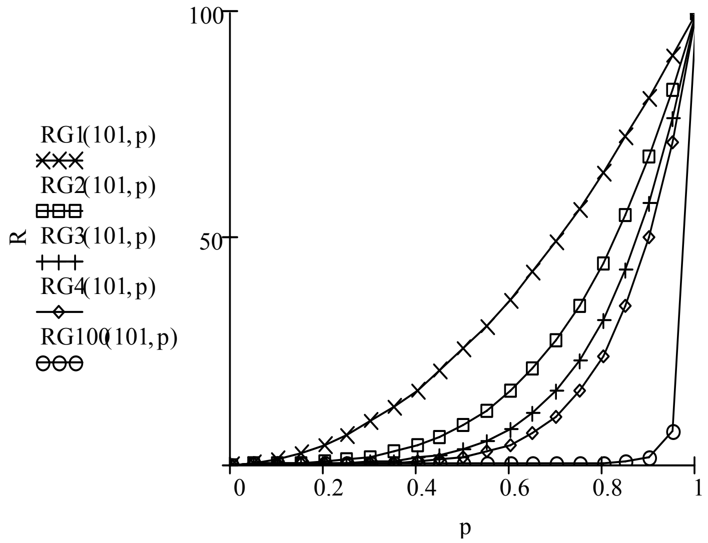

Fig. 4. The obtained results for measure *R* as a function of the probability of the link being operative

Rys. 4. Otrzymane wartości dla miary *R* w funkcji prawdopodobieństwa sprawności kanałów

The values of the reliability measures for one-hop star network were counted for different number of nodes. The obtained results for eleven, hundred and one, and three hundred and one nodes are shown in table 4. Figures 6, 7 illustrate numerical results obtained for reliability measures.

Table 3

Obtained values for the expected number of node pairs communicating (*T*) (in percent)

p	*TG1(101,p)* one-hop star network	*TG2(101,p)* *two*-hop star network	*TG3(101,p)* three-hop star network	*TG4(101,p)* four-hop star network	*TG100(101,p)* hundred-hop star network
0	0	0	0	0	0
0.1	1.178	0.319	0.278	0.251	0.22
0.2	4.317	1.032	0.768	0.643	0.494
0.3	9.416	2.494	1.618	1.274	0.845
0.4	16.475	5.235	3.138	2.339	1.311
0.5	25.495	9.957	5.908	4.249	1.96
0.6	36.475	17.536	10.932	7.847	2.926
0.7	49.416	29.021	19.824	14.795	4.513
0.8	64.317	45.634	35.034	28.192	7.604
0.9	81.178	68.771	60.11	53.518	
1	100	100	100	100	100

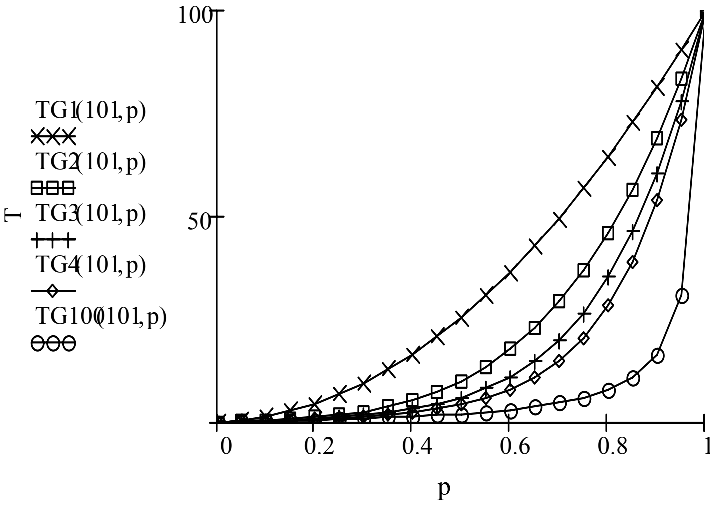

Fig. 5. The obtained results for measure *T* as a function of the probability of the link being operative

Rys. 5. Otrzymane wartości dla miary *T* w funkcji prawdopodobieństwa sprawności kanałów

Figures 6, 7 show that the differences between the networks' reliability decrease when number of nodes grows. For example for criterion *T* largest difference between eleven-nodes network and hundred and one-nodes network is for $p = 0.4, 0.5, 0.6$ and equals 4%. The largest difference between hundred and one-nodes network and three hundred and one-nodes network equals 0.3%. For criterion *S* the largest difference between eleven-nodes network and hundred and one-nodes network is about 7%. The largest difference between hundred and one-nodes network and three hundred and one-nodes network equals 0.6%.

Table 4

Obtained values for the expected number of node pairs communicating (*T*) and values for the expected number of nodes, which can communicate with the central node (*S*) (in percent)

p	*TG1(11,p)*	*TG1(101,p)*	*TG1(301,p)*	*SG1(11,p)*	*SG1(101,p)*	*SG1(301,p)*
0	0	0	0	9.091	0.99	0.332
0.1	2.636	1.178	1.06	18.182	10.891	10.299
0.2	6.909	4.317	4.106	27.273	20.792	20.266
0.3	12.812	9.416	9.14	36.364	30.693	30.233
0.4	20.364	16.475	16.159	45.455	40.594	40.199
0.5	29.545	25.495	25.166	54.545	50.495	50.166
0.6	40.364	36.475	36.159	63.636	60.396	60.133
0.7	52.818	49.416	49.14	72.727	70.297	70.1
0.8	66.909	64.317	64.106	81.818	80.198	80.066
0.9	82.636	81.178	81.06	90.909	90.099	90.033
1	100	100	100	100	100	100

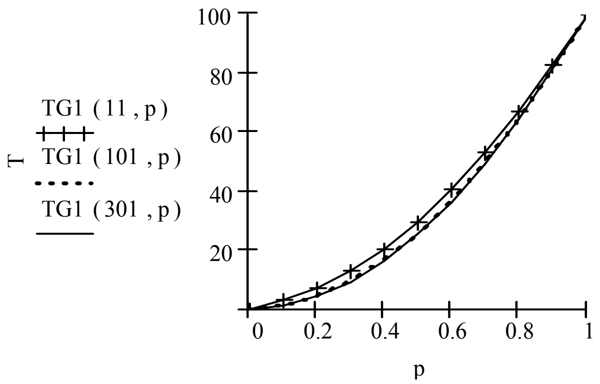

Fig. 6. The obtained results for measure *T* for different number of the nodes of the network

Rys. 6. Otrzymane wartości dla miary *T* dla sieci o różnej liczby węzłów

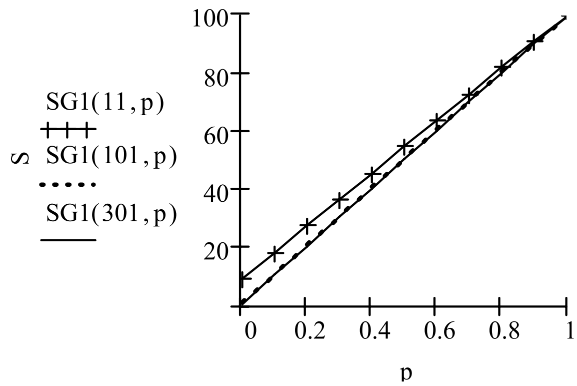

Fig. 7. The obtained results for measure *S* for different number of the nodes of the network

Rys. 7. Otrzymane wartości dla miary *S* dla sieci o różnej liczbie węzłów

4. Conclusions

This paper investigated the reliability of star and tree networks, whose nodes and links fail independently. Several measures of reliability have been considered. The recurrence method is used in analyzing networks. In the numerical calculations the network reliability measures were considered as a function of a probability of link being operative. The comparison of the reliability of the considered networks shows that differences between them decrease when the hop number and number of the nodes of the star network grows. The differences all the considered measures behave similarly. The values of reliability measures were counted for one, two, three, four and hundred- hop star networks.

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Omówienie

W pracy rozważono sieci o strukturze gwiaździstej oraz drzewiastej. Jako ocenę niezawodności tych sieci przyjęto: średnią liczbę sprawnych węzłów, które mogą komunikować się z węzłem centralnym (*S*); średnią liczbę sprawnych par węzłów, które mogą komunikować się przez węzeł centralny (*R*); średnią liczbę sprawnych par węzłów, które mogą komunikować się wzajemnie (*T*). Przyjmuję się, że dowolny sprawny węzeł może komunikować się z innym sprawnym węzłem, jeżeli kanały i węzły tworzące trasę pomiędzy tymi węzłami są sprawne. Założono, że: 1) węzły i kanały sieci ulegają uszkodzeniom z określonym prawdopodobieństwem; 2) uszkodzenia elementów sieci są od siebie statystycznie niezależne. Wyznaczenie niezawodności sieci o dowolnej strukturze jest problemem klasy NP [1, 5]. Dla sieci gwiaździstych problem upraszcza się, ponieważ w sieciach gwiaździstych dowolnego stopnia pomiędzy każdą parą węzłów istnieje dokładnie jedna trasa. Zatem, średnia liczba sprawnych węzłów, które mogą komunikować się z węzłem centralnym, średnią liczba sprawnych par węzłów, które mogą komunikować się przez węzeł centralny oraz średnia liczba par węzłów, które mogą komunikować się wzajemnie, równa jest sumie prawdopodobieństw sprawności odpowiednich tras (1). Dla sieci wysokiego stopnia z dużą liczbą węzłów, do wyznaczenia wartości parametru niezawodnościowego wygodniej jest zastosować metodę rekurencyjną [2, 3, 4]. W pracy metodę rekurencyjną zastosowano do wyznaczenia wartości miar niezawodności dla jedno-, dwu-, trzy-, cztero- i stustopniowej sieci gwiaździstej zawierającej sto jeden węzłów. Wyznaczono niezawodność sieci jako funkcję prawdopodobieństwa sprawności kanałów. Otrzymane wyniki przedstawiono na rys. 3, 4, 5. Dla jednostopniowej sieci gwiaździstej wyznaczono wartości miar dla różnej liczby węzłów, otrzymane wyniki ilustrują rys. 6, 7. Przeprowadzone badania wykazały, że różnice pomiędzy niezawodnością rozważanych struktur maleją wraz ze wzrostem stopnia oraz liczby węzłów sieci. Największe różnice występują dla prawdopodobieństw sprawności kanałów równych 0,6, 0.7, 0,8.

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