

Jacek WIDUCH<sup>1</sup>

## **1. MODERN VARIANTS OF VEHICLE ROUTING PROBLEMS**

### **1.1. Introduction**

The Vehicle Routing is a subject of research for more than 50 years when it was formulated by Dantzig and Ramser as an example of problem delivering gasoline to service stations<sup>2</sup>. It belongs to the group of optimization problems concerned with the determination of routes for the given fleet of vehicles which serve a set of customers satisfying defined constraints and objectives. It is realized in practice every day by thousands of companies over the world engaged in the delivery of different types of goods. It is used in areas influencing the modern economy and the cost of goods, such as logistics, communication, transport, manufacturing, civil and military systems. Over the years, the models and variants of the VRP have changed to reflect reality as accurately as possible.

This chapter focuses on some of the most popular contemporary research problems related to determining routes of vehicles. In recent years, new variants of delivery vehicles have been considered. The vast majority of vehicles run on combustion engines. Electrically powered vehicles are becoming more and more popular. New problems are associated with the use of electric vehicles, such as the availability of charging stations, longer charging time than refueling traditional combustion vehicles, shorter range. When designating travel routes and selecting the types of vehicles used, their impact on the surrounding environment and the generated pollution are more and more often taken into account. Ecological issues have become an important factor in transport planning. Another variant of the increasingly popular vehicles are unmanned vehicles.

---

<sup>1</sup> Silesian University of Technology, Faculty of Automatic Control, Electronics and Computer Science, Department of Algorithmics and Software, Gliwice, e-mail:jacek.widuch@polsl.pl.

<sup>2</sup> Dantzig G.B., Ramser J.H.: The truck dispatching problem, *Management Science*, 1959, Vol. 6, pp. 80–91.

The structure of this chapter is as follows. Section 2 presents the classical Vehicle Routing Problem (VRP). In subsection 2.1 the problem definition is presented and subsection 2.2 presents mathematical model. Section 3 presents selected modification of the VRP. In section 4 Green Vehicle Routing Problem is presented. Section 5 presents Electric Vehicle Routing Problem. In section 6 Unmanned Vehicle Routing Problem is presented. Finally, section 7 contains concluding remarks.

## 1.2. The classical vehicle routing problem

### 1.2.1. The problem definition

The classical Vehicle Routing Problem (VRP) is also known as Capacitated Vehicle Routing Problem (CVRP) and it was defined in 1959 by Dantzig and Ramser<sup>3</sup>. The VRP can be considered as combinatorial problem and it can be described as follows. Let  $G = (V, A)$  be a weighted graph with the weight function  $d: A \rightarrow R_{\geq 0}$ . The graph  $G$  contains a set of vertices  $V = \{0, 1, \dots, n\}$ , where vertex 0 represents the depot and vertices 1, ...,  $n$  represent customers to be served. Each customer  $i$  ( $i = 1, \dots, n$ ) is characterized by a demand  $q_i \geq 0$ , the depot 0 has a demand  $q_0 = 0$ . The set  $A = \{(i, j) : i, j \in V\}$  is the set of arcs linking vertices  $i$  and  $j$  with a weight  $d_{ij}$  which can be interpreted as a distance from  $i$  to  $j$ . The set of  $m$  vehicles, based at the depot 0, is available to serve customers 1, ...,  $n$  where each vehicle has the same maximum load capacity  $c$  (a homogenous fleet of vehicles is considered). The goal of VRP is to determine a set vehicle routes with the least total distance satisfying following conditions:

- each customer 1, ...,  $n$  is visited exactly once by exactly one vehicle,
- each vehicle starts and ends its route at the depot 0,
- the total load of each vehicle does not exceed the maximum load  $c$ .

A graphical illustration of the VRP is shown in Fig. 1.1. The number of customers is 14 and the solution contains three routes marked in blue, green and red.

---

<sup>3</sup> Dantzig G.B., Ramser J.H.: The truck dispatching problem, Management Science, 1959, Vol. 6, pp. 80–91.

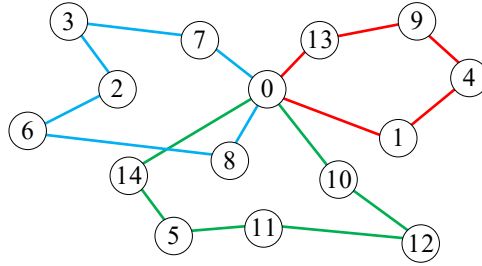


Fig. 1.1. A graphical presentation of the VRP

Rys. 1.1. Graficzna prezentacja problemu trasowania pojazdów

The VRP is an NP-hard problem<sup>4</sup>. Thus, exact solution of large problem is difficult to obtain. A new real-life challenges and practical applications require extensive modifications and new variants of the VRP. Thus the VRP is extended by including additional real-life aspects resulting many variants of the problem.

### 1.2.2. Mathematical model

Mathematical model of the VRP can be presented as follows<sup>5</sup>. Let  $x_{ij}^k$  be a binary decision variable which is equal to 1 only if arc  $(i, j) \in A$  belongs to the optimal solution and it belongs to the route served by vehicle  $k$  ( $k = 1, \dots, m$ ) and it is equal to 0 otherwise. The objective of the problem is to minimize the total distance defined as follows:

$$\text{minimize } \sum_{k=1}^m \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij}^k \quad (1.1)$$

subject to the following constraints:

$$\sum_{k=1}^m \sum_{i \in V} x_{ij}^k = 1 \quad (j = 1, \dots, n) \quad (1.2)$$

$$\sum_{k=1}^m \sum_{j \in V} x_{ij}^k = 1 \quad (i = 1, \dots, n) \quad (1.3)$$

$$\sum_{k=1}^m \sum_{j \in V} x_{0j} = m \quad (1.4)$$

$$\sum_{k=1}^m \sum_{i \in V} x_{0i} = m \quad (1.5)$$

$$\sum_{i \in V} \sum_{j \in V} q_j x_{ij}^k = c \quad (k = 1, \dots, m) \quad (1.6)$$

<sup>4</sup> Lenstra J.K., Rinnooy Kan A.H.G.: Complexity of vehicle and scheduling problems, *Networks*, 1981, Vol. 11, pp. 221–227.

<sup>5</sup> Borčinová Z.: Two models of the capacitated vehicle routing problem, *Croatian Operational Research Review*, 2017, Vol. 8, pp. 463–469.

Laporte G.: The vehicle routing problem: An overview of exact and approximate algorithms, *European Journal of Operational Research*, 1982, Vol. 59, pp. 345–358.

The constraints (1.2) and (1.3) ensure that each customer is visited by exactly one vehicle. The constraints (1.4) and (1.5) guarantee that each vehicle can leave the depot only once and returns to the depot. The constraints (1.6) ensure the capacity constraints are satisfied, i.e. the sum of the demands of the customers visited in a route does not exceed the vehicle capacity  $c$ .

### 1.3. Selected modifications of the classical vehicle routing problem

The VRP model introduced in 1959<sup>6</sup> was extended to incorporate many realistic aspects like time-dependent travel times, time windows for pickup and delivery, input data that changes dynamically over time, multi depots, type of goods transported and type of vehicles, etc. These additional aspects bring along substantial complexity. Selected considered modifications of the VRP are following:

- Vehicle Routing Problem with Time Windows (VRPTW): given is the time windows constraints, i.e. customers opening and closing hours, to ensure that vehicles visit customers within a specific intervals. If vehicle arrives outside time window it waits until opening the window and there is waiting cost<sup>7</sup>.
- Open Vehicle Routing Problem (OVRP): the vehicle is not required to return to the depot after visiting customers it can terminate at one of the customers or other points<sup>8</sup>.
- Hazardous Vehicle Routing Problem (HVRP): transportation of hazardous materials (HazMat). HazMat include substances, solids or gases that can be flammable, toxic, explosive or radioactive and are the most threatening to the environment and to the human health even if they are transported under rules. There are areas where the transportation of HazMat is forbidden and it must be considered in route planning<sup>9</sup>.
- Large-Scale Vehicle Routing Problem (LSVRP): it is related to the scale of the problem instances with more than 200 customers<sup>10</sup>.

---

<sup>6</sup> Dantzig G.B., Ramser J.H.: The truck dispatching problem, *Management Science*, 1959, Vol. 6, pp. 80–91.

<sup>7</sup> Villalba A.F.L., La Rotta E.C.G.: Clustering and heuristics algorithm for the vehicle routing problem with time windows, *International Journal of Industrial Engineering Computations*, 2022, Vol. 13, pp. 165–184.

<sup>8</sup> Dasdemir E., Testik M.C., Öztürk D.T., Sakar C.T., Güleriyüz G., Testik Ö.M.: A multi-objective open vehicle routing problem with overbooking: Exact and heuristic solution approaches for an employee transportation problem, *Omega*, 2022, Vol. 108, pp. 102587.

<sup>9</sup> Ouertani N., Ben-Romdhane H., Krichen S.: A decision support system for the dynamic hazardous materials vehicle routing problem, *Operational Research*, 2022, Vol. 22, pp. 551–576.

<sup>10</sup> Costa J.G.C., Mei Y., Zhang M.: Adaptive search space through evolutionary hyper-heuristics for the large-scale vehicle routing problem, 2020 IEEE Symposium Series on Computational Intelligence (SSCI), 2020, pp. 2415–2422.

- Vehicle Routing Problem with Stochastic Demands (VRPSD): the problem where customer demands are defined by stochastic variables<sup>11</sup>.
- Heterogeneous Vehicle Routing Problem (HVRP): the customers are served by heterogeneous fleet of vehicles. The vehicles differ in their capacity and cost per day for using them<sup>12</sup>.
- Time Dependent Vehicle Routing Problem (VRP): the travel time is not constant and it is a function of the congestion at peak hours and weather changes<sup>13</sup>.
- Vehicle Routing Problem with Simultaneous Delivery and Pickup (VRPSDP): the goods need to be picked up from a customer and must be served to the other customers<sup>14</sup>.
- Vehicle Routing Problem with Backhauls (VRPB): modification of VRPSDP, in the each route all deliveries must be made before any pickups of the goods<sup>15</sup>.
- Multi Compartment Vehicle Routing Problem (MCVRP): a variant of the problem in which several product types must be transported separately and the vehicle capacity is split or can be split into several zones. It considers the transportation of dangerous goods, liquid or bulk products and the transportation of food products in different temperature zones<sup>16</sup>.
- Dynamic Vehicle Routing Problem (DVRP): the input data is revealed or updated continuously. The customers that have been already visited are removed from the network and the customers that placed a new order are added to the network. These changes are used to update routes<sup>17</sup>.
- Split Delivery Vehicle Routing Problem (SDVRP): the customer can be visited more than once by the vehicles<sup>18</sup>.

---

<sup>11</sup> Omori R., Shiina T.: Solution algorithm for the vehicle routing problem with stochastic demands, 2020 Joint 11th International Conference on Soft Computing and Intelligent Systems and 21st International Symposium on Advanced Intelligent Systems (SCIS-ISIS), 2020, pp. 1–6.

<sup>12</sup> Dang Y., Allen T.T., Singh M.: A heterogeneous vehicle routing problem with common carriers and time regulations: Mathematical formulation and a two-color ant colony search, *Computers & Industrial Engineering*, 2022, Vol. 168, pp. 108036.

<sup>13</sup> Ke-Wei J., San-Yang L., Xiao-Jun S.: A hybrid algorithm for time-dependent vehicle routing problem with soft time windows and stochastic factors, *Engineering Applications of Artificial Intelligence*, 2022, Vol. 109, pp. 104606.

<sup>14</sup> Ning T., Wang J., Han Y.: Logistics distribution de-carbonization pathways and effect in China: a systematic analysis using VRPSDP model, *International Journal of Low-Carbon Technologies*, 2021, Vol. 16, pp. 1404-1411.

<sup>15</sup> Subramanian A., Queiroga E.: Solution strategies for the vehicle routing problem with backhauls, *Optimization Letters*, 2020, Vol. 14, pp. 2429–2441.

<sup>16</sup> Heßler K.: Exact algorithms for the multi-compartment vehicle routing problem with flexible compartment sizes, *European Journal of Operational Research*, 2021, Vol. 294, pp. 188–205.

<sup>17</sup> Bonilha I.S., Mavrovouniotis M., Müller F.M., Ellinas G., Polycarpou M.: Ant Colony optimization with heuristic repair for the dynamic vehicle routing problem, 2020 IEEE Symposium Series on Computational Intelligence (SSCI), 2020, pp. 313–320.

<sup>18</sup> Ji B., Zhou S., Yu S.S., Wu G.: An enhanced neighborhood search algorithm for solving the split delivery vehicle routing problem with two-dimensional loading constraints, *Computers & Industrial Engineering*, 2021, Vol. 162, pp. 107720.

- Multi Depot Vehicle Routing Problem (MDVRP): variant of the problem which includes multiple depots<sup>19</sup>.
- Clustered Vehicle Routing Problem (CluVRP): variant of the problem where customers are grouped into different clusters. The vehicle which serves clients in a given cluster cannot leave it until all customers in the current cluster have been visited<sup>20</sup>.
- Consistent Vehicle Routing Problem (ConVRP): the extension of the problem by imposing that the same vehicle visits the same customers at approximately the same time on each day<sup>21</sup>.
- Vehicle Routing Problem with Arrival Time Diversification (VRPATD): the modification of the VRP which aims to the efficiency and security. These objectives are the major concerns in cash-in-transit transportation. The efficiency is achieved by determining shortest routes but the security can be improved by generating dissimilar plans of visit<sup>22</sup>.
- Periodic Vehicle Routing Problem (PVRP): the problem where each customer is visited many times and given is the minimum frequency of visits. Each customer is visited at every time period according to given schedule<sup>23</sup>.
- Multi Fleet Feeder Vehicle Routing Problem (MFFVRP): the customers are served by heterogeneous fleet of vehicles containing small and large vehicles. For example, the small vehicles can be represented by bicycles or motorcycles. The large vehicles have more capacity but the cost of using them is much greater than smaller vehicles. The small vehicles are reloaded as much as they are capacity or customer orders. The reloading is done in special points using large vehicles<sup>24</sup>.

---

<sup>19</sup> Khairy O.M., Shehata O.M., Morgan E.I.: Meta-heuristic algorithms for solving the multi-depot vehicle routing problem, 2020 2nd Novel Intelligent and Leading Emerging Sciences Conference (NILES), 2020, pp. 276–281.

<sup>20</sup> Anisul Islam Md., Gajpal Y., ElMekkawy T.Y.: Hybrid particle swarm optimization algorithm for solving the clustered vehicle routing problem, *Applied Soft Computing*, 2021, Vol. 110, pp. 107655.

<sup>21</sup> Stavropoulou F.: The consistent vehicle routing problem with heterogeneous fleet, *Computers & Operations Research*, 2022, Vol. 140, pp. 105644.

<sup>22</sup> Soriano A., Vidal T., Gansterera M., Doerner K.: The vehicle routing problem with arrival time diversification on a multigraph, *European Journal of Operational Research*, 2020, Vol. 286, pp. 564–575.

<sup>23</sup> Vega-Figueroa S.E., López-Becerra P.A., López-Santana E.R.: Hybrid algorithm for the solution of the periodic vehicle routing problem with variable service frequency, *International Journal of Industrial Engineering Computations*, 2022, Vol. 13, pp. 277–292.

<sup>24</sup> Salehi Sarbijan M., J. Behnamian J.: Multi-fleet feeder vehicle routing problem using hybrid metaheuristic, *Computers & Operations Research*, 2022, Vol. 141, pp. 105696.

The current most popular variants of the VRP are Green VRP, Electric VRP and Unmanned VRP described in more detail in Sections 4–6. A review of other studied variants of the VRP is presented in Bhuvanewari et al. (2018), Braekers et al. (2016), Gupta and Saini (2018), Widuch (2020)<sup>25</sup>.

#### 1.4. Green vehicle routing problem

The transportation sector is one of the sources of various types of emissions that have a negative influence on the environment. Thus over the last decade, it is becoming more and more popular to determine routes taking into account the influence of vehicles on the environment. The model of VRP with environmental concern is named as the Green Vehicle Routing Problem (GVRP). The GVRP includes different environmental issues when determining routes, such as Green House Gas (GHG) emission, fuel consumption, noise, pollution, using fleet of Alternative Fuel Vehicles (AFVs), i.e., vehicles that use alternative fuel like methanol, electricity and natural gas, etc.

The GVRP was proposed in 2012<sup>26</sup> where a homogeneous fleet of AFVs is considered and the total traveled distance is minimized. In the paper a difficulties that exists as a result of limited infrastructure of Alternative Fueling Stations (AFSs) are considered. The vertices of the graph represent depot, customers and AFSs. Determined paths include plan of refueling and stops at AFSs to eliminate the risk of running out of fuel while passing the routes. Thus each route may include a stop at one or more AFSs. Refueling policy requires an AFV to leave a refueling station with a full tank. The experiments were carried out using a medical textile supply company depot location in Virginia (VA). A customers were created based on hospital locations in Virginia (VA), Maryland (MD) and the District of Columbia (DC) using Google Earth.

---

<sup>25</sup> Bhuvanewari M., Eswaran S., Rajagopalan S.P.: A survey of vehicle routing problem and its solutions using bio-inspired algorithms, *International Journal of Pure and Applied Mathematics*, 2018, Vol. 118, pp. 259–264.  
 Braekers K., Ramaekers K., Nieuwenhuysse I.V.: The vehicle routing problem: state of the art classification and review, *Computers & Industrial Engineering*, 2016, Vol. 99, pp. 300–313.  
 Gupta A., Saini S.: On solutions to vehicle routing problems using swarm optimization techniques: A review. [in:] *Advances in Computer and Computational Sciences*, Singapore, 2018, pp. 345–354.  
 Widuch J.: Current and emerging formulations and models of real-life rich vehicle routing problems. [in:] *Smart Delivery Systems*, 2020, pp. 1–35.

<sup>26</sup> Erdoğan S., Miller-Hooks E.: A Green vehicle routing problem, *Transportation Research Part E: Logistics and Transportation Review*, 2012, Vol. 48, No. 1, pp. 100–114.

A fleet of AFVs is also considered in other papers. In Bruglieri et al.<sup>27</sup> (2019) two Mixed Integer Linear Programming (MILP) formulations to the model are proposed. In the first model, only single visit to AFS between two customers is allowed but in the second model two consecutive visits to AFS are permitted. The goal of the problem is to determine at most  $m$  routes of AFV minimizing the total distance. In Bruglieri et al.<sup>28</sup> (2019) a more realistic variant of the GVRP is introduced. The problem with capacitated AFSs is considered where only a limited number of fueling pumps are available at AFS. Thus, the number of refueling vehicles simultaneously at AFS is limited. A heterogeneous AFSs are considered, i.e. the limit may be different for each AFS. In addition, AFSs are divided into two groups: public and private. The private AFSs are owned by the company that owns the AFVs.

A mixed fleet of vehicles containing conventional Gasoline or Diesel Vehicles (GDVs) and AFVs is also considered<sup>29</sup>. There is considered AFVs' limited driving ranges, a sporadic number of AFSs and lengthy refueling. The GDVs represent unrestricted vehicles with respect to AFS's aspects. The AFV may visit one or more AFS but the GDVs visit only customers. The goal of presented problem is to determine set of routes minimizing the total cost of travel.

Since industrial revolution we observe an increase in atmospheric CO<sub>2</sub> emissions. A global emissions of CO<sub>2</sub> since 1980 is shown in chart (Fig. 1.2). In last 20 and 10 years, the increment has been around 12% and 6% respectively growing from 370.57 ppm (part per million) in 2001 and 390.63 ppm in 2011 and to 414.73 ppm in 2021.

---

<sup>27</sup> Bruglieri M., Mancini S., Pisacane O.: More efficient formulations and valid inequalities for the Green Vehicle Routing Problem, *Transportation Research Part C*, 2019, Vol. 105, pp. 283–296.

<sup>28</sup> Bruglieri M., Mancini S., Pisacane O.: The green vehicle routing problem with capacitated alternative fuel stations, *Computers and Operations Research*, 2019, Vol. 112, pp. 104759.

<sup>29</sup> Koyuncu I., Yavuz M.: Duplicating nodes or arcs in green vehicle routing: A computational comparison of two formulations, *Transportation Research Part E*, 2019, Vol. 122, pp. 605–623.



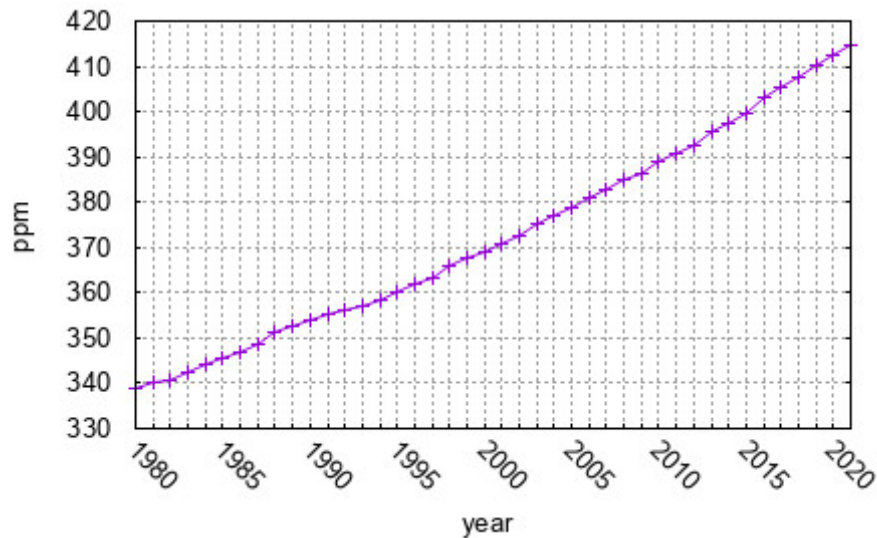


Fig. 1.2. Global CO<sub>2</sub> emission since 1980  
 Rys. 1.2. Globalna emisja CO<sub>2</sub> od roku 1980<sup>30</sup>

The most popular objective in the GVRP is to reduce CO<sub>2</sub> emissions. In Ferreira et al.<sup>31</sup> (2021) three variants of 2L-CVRP are researched: allowing split delivery (2L-SDVRP), green variant (G2L-CVRP) and hybrid of split delivery and green requirements (G2L-SDVRP). The goal of green variants of the problem is to reduce the CO<sub>2</sub> emissions, the total distances of the routes are not considered. The amount of CO<sub>2</sub> emissions is determined based on following parameters: the CO<sub>2</sub> emission rate per liter of fuel consumed, the fuel consumption rate with the vehicle fully loaded and the fuel consumption with the empty vehicle. Mentioned parameters depend on travel speed, type of vehicles and terrain conditions.

The GVRP is also considered as a Multi-Objective Optimization (MOO) problem where three objectives are minimized simultaneously<sup>32</sup>. The first criterion defines the difference between the largest and shortest distance of all routes. The second criterion represents to total costs and the last criterion is related to the CO<sub>2</sub> emissions to the fuel consumption of vehicles. Another MOO problem is presented in paper Dukkanci et al.<sup>33</sup> (2019), where the Green Location-Routing Problem (GLRP) is introduced. In the GLRP

<sup>30</sup> Source: <https://gml.noaa.gov/ccgg/trends/global.html>.

<sup>31</sup> Ferreira K.M., Queiroz T.A., Toledo F.M.B.: An exact approach for the green vehicle routing problem with two-dimensional loading constraints and split delivery, *Computers and Operations Research*, 2021, Vol. 136, pp. 105452.

<sup>32</sup> Londono J.F.C., Rendon R.A.G., Ocampo E.M.T.: Iterated local search multi-objective methodology for the green vehicle routing problem considering workload equity with a private fleet and a common carrier, *International Journal of Industrial Engineering Computations*, 2021, Vol. 12, pp. 115–130.

<sup>33</sup> Dukkanci O., Kara B.Y., Bektaş T.: The green location-routing problem, *Computers and Operations Research*, 2019, Vol. 105, pp. 187–202.

given is a set of potential depots with operating costs and a set of customers with known demands. The problem consists of locating depots from where vehicles will be dispatched to serve the customers. The objective is to minimize three objectives: a total cost, fuel consumption and CO<sub>2</sub> emissions. Presented algorithm was tested using a data containing selected cities of the United Kingdom with real road distances but client demands were randomly generated. In Kirci<sup>34</sup> (2019) the GVRP with Time Windows (GVRPTW) is considered. Presented solutions are tested using real scenarios in Turkey.

In Wang et al.<sup>35</sup> (2022) the Multi Depot GVRPTW (MDGVRPTW) is presented where the objective is to minimize carbon emissions. The carbon emission reduction potential of the multi depot model over the single depot model is considered. The algorithms were tested using the Solomon benchmark instances<sup>36</sup>. The results of tests shown that using multi depot in the VRP can reduce carbon emissions by at most 37.6%. The paper Qin et al.<sup>37</sup> (2019) proposes vehicle routing optimization model with adaptive vehicle speed. Presented model is a hybrid of path optimization and speed optimization. The objective is to select optimal speed of vehicle and minimize distribution cost which consists of driver salary, transportation cost, carbon emissions cost and penalty cost when the vehicle violates the time window.

The paper Qin et al.<sup>38</sup> (2019) introduces the carbon tax into the VRP to calculate carbon emissions cost. Other cost items consist of fixed cost of vehicles, transportation cost and penalty cost when the vehicle violates the time windows required by the customer. The objective of the problem is to minimize the total cost. The test were carried out based on a real case data from a company Yanjing Beer Co. Ltd (Shunyi District, Beijing, China) which recycles empty beer bottles.

The impact of route optimization of heavy vehicles on fuel consumption and GHG emissions is also considered<sup>39</sup>. The regular routes of two trucks of Blacktown City (Australia) were extracted and compared with the optimized distances from the simulated model. The results shown that the distance can be reduced by 60%, fuel consumption by 62% and GHG emissions by 62% per month.

---

<sup>34</sup> Kirci P.: A novel model for vehicle routing problem with minimizing CO<sub>2</sub> emissions, 3rd International Conference on Advanced Information and Communications Technologies (AICT), 2019, pp. 241–243.

<sup>35</sup> Wang S., Han C., Yu Y., Huang M., Sun W., Kaku I.: Reducing carbon emissions for the vehicle routing problem by utilizing multiple depots, *Sustainability*, 2022, Vol. 14, No. 3, pp. 1264.

<sup>36</sup> <https://www.sintef.no/projectweb/top/vrptw/solomon-benchmark/>

<sup>37</sup> Qin G.Y., Tao F.M., Li L.X.: A Green vehicle routing optimization model with adaptive vehicle speed under soft time window, 2019 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM), 2019, pp. 1–5.

<sup>38</sup> Qin G., Tao F., Li L., Chen Z.: Optimization of the simultaneous pickup and delivery vehicle routing problem based on carbon tax, *Industrial Management & Data Systems*, 2019, Vol. 119, No. 9, pp. 2055–2071.

<sup>39</sup> Karimipour H., Tam V.W.Y., Le K.N., Burnie H.: Routing on-road heavy vehicles for alleviating greenhouse gas emissions, *Cleaner Engineering and Technology*, 2021, Vol. 5, pp. 100325.

Authors of the paper Soon et al.<sup>40</sup> (2019) propose optimization of traffic light control to balance traffic flow. This strategy is developed to minimize GHG emissions and urban congestion. When the traffic congestion is detected then the traffic lights on the congested roads are coordinated to generate green wave scenarios. To avoid the spread of congestion towards upstream roads, a cooperative routing scheme is used to probabilistically distribute vehicles away from the congested road. A fuel consumption and GHG emissions depend on acceleration and speed of vehicles therefore proposed method reduces frequency of acceleration. Proposed methods were tested based on Singapore traffic data. The results shown reducing CO<sub>2</sub> by 37.7%, fuel consumption by 37.6%, travel time by 47.5% and waiting time by 57.3%. The number of arrived vehicles at designated destination were increased by 62.6%.

The emissions depend on fuel consumption. The objective of problem presented in Zarouk et al.<sup>41</sup> (2022) is minimizing fuel consumption and maximizing customer satisfaction derived from the provision of sufficient and timely requests. Both objectives must be simultaneously satisfied thus the problem is solved as MOO problem. Considered model considers a set of heterogeneous vehicles where each vehicle is characterized by a fuel consumption rate lit/km. A similar MOO is researched in Erdođdu and Karabulut<sup>42</sup> (2022). The objective is to simultaneously minimize the total distance and the total fuel consumption of all vehicles.

In Hou et al.<sup>43</sup> (2021) a problem of energy consumption is considered in the time dependent networks. In the network the vehicle travel time depends on the characteristics of the road network speed. The objective is to minimize the sum of vehicle fixed cost, fuel consumption cost and time window penalty cost. The influence of time dependent networks on routes optimization and the relationship between fuel consumption and vehicle load is researched. In Zhu and Hu<sup>44</sup> (2019) the situation of waiting at customers to avoid bad traffic is proposed. The objective is to find set of optimal routes and the optimal waiting times at each customer nodes with minimum total cost which includes fuel consumption.

---

<sup>40</sup> Soon K.L., Lim J.M.-Y., Parthiban R.: Coordinated traffic light control in cooperative green vehicle routing for pheromone-based multi-agent systems, *Applied Soft Computing Journal*, 2019, Vol. 81, pp. 105486.

<sup>41</sup> Zarouk Y., Mahdavi I., Rezaeian J., Santos-Arteaga F.J.: A novel multi-objective green vehicle routing and scheduling model with stochastic demand, supply, and variable travel times, *Computers & Operations Research*, 2022, Vol. 141, pp. 105698.

<sup>42</sup> Erdođdu K., Karabulut K.: Bi-objective green vehicle routing problem, *International Transactions in Operational Research*, 2022, Vol. 29, pp. 1602–1626.

<sup>43</sup> Hou D., Fan H., Ren X.: Multi-depot joint distribution vehicle routing problem considering energy consumption with time-dependent networks, *Symmetry*, 2021, Vol. 13, pp. 2082.

<sup>44</sup> Zhu L., Hu D.: Study on the vehicle routing problem considering congestion and emission factors, *International Journal of Production Research*, 2019, Vol. 57, No. 19, pp. 6115–6129.

When vehicles passing through long distances to serve client demands it may halt at refuelling stations. The time and the cost associated with these visits have an influence on the total cost of the route. The Green VRP with Queues (GVRP-Q) is introduced in paper Poonthalir and Nadarajan<sup>45</sup> (2019). The problem aims to serve a set of customers but includes refuelling time in the refuelling stations where refuelling stations are modelled as M/M/1 model. The impact of refuelling time on the total cost of the route is researched and the objective is to minimize the total cost.

### **1.5. Electric vehicle routing problem**

The variant of VRP where the customers are visited by a fleet of electric vehicles (EVs) is named as the Electric VRP (EVRP). It uses a eco-friendly green vehicles therefore it can be viewed as a variant of the GVRP. The EVs have limited driving range and long charging time therefore when planning routes for EVs it is necessary to predict the energy consumption and plan visits at charging stations in the route. The EVs have shorter driving range than traditional combustion engine vehicles (CEVs) and the availability of charging stations is limited compared to the petrol stations. The charging time of an EV is much longer than the time of refuelling a CEV. In contrast to CEVs, the time of charging cannot be omitted.

The charging of EVs is an important issue and it determines their use. It is widely studied and many constraints are assumed. In Euchti and Yassine<sup>46</sup> (2022) the EVRP with fixed heterogeneous fleet is considered. A recharging of the battery at any customer for a certain period is allowed. A vehicle can leave a depot with fully charged battery. The objective is to minimize total cost which contains the travelling cost proportional to the travelled distance and the recharging cost which is proportional to the period of recharging the battery. Another objective is considered in Hesam Sadati et al.<sup>47</sup> (2022). The paper presents the EVRP with homogenous fleet of EVs which serve customers with known demands and alternative delivery points and time windows. The order of customer is delivered to exactly one of defined locations. A recharging of EVs only at

---

<sup>45</sup> Poonthalir G., Nadarajan R.: Green vehicle routing problem with queues, *Expert Systems With Applications*, 2019, Vol. 138, pp. 112823.

<sup>46</sup> Euchti J., Yassine A.: A hybrid metaheuristic algorithm to solve the electric vehicle routing problem with battery recharging stations for sustainable environmental and energy optimization, *Energy Systems*, 2022.

<sup>47</sup> Hesam Sadati M.E., Akbari A., Çatay B.: Electric vehicle routing problem with flexible deliveries, *International Journal of Production Research*, 2022, pp. 1–27.

the company owned depot is allowed but the EV can leave the depot with 90% of battery capacity. The goal of the problem is to minimize the total travelled distance and number of used vehicles. Computational experiments were carried out based on data of U.K. retailer serving 40 customers in East Midlands area. The results shown a reduction of the total distance by 23% and the fleet size by 20%.

A new generation of charging batteries is Wireless Charging Technology (WCT). The system uses innovative technology developed by KAIST in South Korea<sup>48</sup>. The battery can be wirelessly charged from the transmitters using the noncontact charging mechanism. Each transmitter consists of an inverter and inductive cables which generate a magnetic field to charge the battery. When the vehicle runs in the area where inductive cables are installed then the battery is charged until it reach its maximum capacity. In the paper Elbaz and Alaoui<sup>48</sup> (2022) is considered a problem how to install the power transmitters to guarantee to each vehicle a battery capacity between its minimal and maximal level. In Wang et al.<sup>49</sup> (2022) a hybrid charging method is discussed which combines traditional Plug-in Charging Technology (PCT) and WCT. The availability of charging stations both types may vary over time due to the allocation or equipment maintenance. Therefore while planning routes it is not only need to determine optimal routes with minimal travel cost but also to determine the most efficient charging strategy.

The alternative method of charging EVs is battery swapping<sup>50</sup>. It is the only way of supplement energy of EVs. There are two kinds of vehicles and two kinds of routes are determined. For the EVs determined are routes to deliver demands to the set of customers. The second kind of routes are determined to the special Battery Swap Vehicles (BSVs) to swap depleted batteries on EVs with a fully charged at a designated place and time. The BSVs are electric vehicles and are only used to swap battery in EVs, they are not allowed to swap battery in another BSV. In addition, the BSVs cannot be used to serve demands to customers. There is challenging problem of synchronization between two kinds of vehicles, i.e. EVs and BSVs. The problem is considered as MOO problem, the goal is to minimize a total cost of using EVs and BSVs and the total

---

<sup>48</sup> Elbaz H., Alaoui A.E.: Optimal installation of the power transmitters in the dynamic wireless charging for electric vehicles in a multipath network with the round-trip case, *International Journal of Intelligent Transportation Systems Research*, 2022, Vol. 20, pp. 46–63.

<sup>49</sup> Wang Z., Ye K., Jiang M., Yao J., Xiong N.N., Yen G.G.: Solving hybrid charging strategy electric vehicle based dynamic routing problem via evolutionary multi-objective optimization, *Swarm and Evolutionary Computation*, 2022, Vol. 68, pp. 100975.

<sup>50</sup> Raeesi R., Zografos K.G.: The electric vehicle routing problem with time windows and synchronised mobile battery swapping, *Transportation Research Part B*, 2020, Vol. 140, pp. 101–129.

distance travelled by them. Another problem is considered in Zhou and Zhao<sup>51</sup> (2022). The BSVs are not available and the EV needs to visit the Battery Swap Station (BSS) to swap its battery. There is only one BSS but its capacity is unlimited. It is assumed that each EV can swap the battery at most once during passing a route and the impact of the load of EV on the battery power consumption is omitted. The objective of the problem is to minimize the total distribution costs and maximize utilization of batteries. In Li et al.<sup>52</sup> (2020) is considered a problem of battery swapping where many BSSs are available. The EV is allowed to visit the BSS one or more times. The objective is to minimize the total cost which contains the cost of using EV and power consumption.

In Iwańkiewicz<sup>53</sup> (2021) a battery lifetime is considered while planning the routes. The vehicle can leave a charging station when batteries are charged to 80%. A battery charging time is nonlinear. It has been shown that the time of charging from 80% to 100% is so long and it is more effective to charge it more frequently to the 80% level than charge it less frequently to the 100% level. Presented methods were tested based on real data containing points located in north-western Poland: single depot located in Stargard City, four charging points located in small cities and 28 customers.

The paper Lin et al.<sup>54</sup> (2021) presents the EVRP where the scheduling of charging the EVs from the charging stations and discharging to the charging stations are considered. The EV can stop at the station to charge the batteries but it can also inject stored energy to the grid. The objective is to determine an optimal schedule of charging and discharging based on the energy price which depend on time-of-use. The energy demands can be shifted from peak hours to off-peak hours when the cost of energy is lower. At high price periods the EVs can recover cost of energy by injection it to the grid. The tests were carried out using a real data of the fleet which serve demands to the customers at the region of Kitchener-Waterloo in Ontario (Canada).

In Macrina et al.<sup>55</sup> (2019) the problem with a mixed fleet containing conventional CEVs and EVs is presented. Since a full battery charging requires a long time the possibility of recharging batteries partially at any available recharging station is allowed.

---

<sup>51</sup> Zhou B., Zhao Z.: Multi-objective optimization of electric vehicle routing problem with battery swap and mixed time windows, *Neural Computing and Applications*, 2022.

<sup>52</sup> Li J., Wang F., He Y.: Electric vehicle routing problem with battery swapping considering energy consumption and carbon emissions, *Sustainability*, 2020, Vol. 12, pp. 10537.

<sup>53</sup> Iwańkiewicz R.: Effective permutation encoding for evolutionary optimization of the electric vehicle routing problem, *Energies*, 2021, Vol. 14, pp. 6651.

<sup>54</sup> Lin B., Ghaddar B., Nathwani J.: Electric vehicle routing with charging/discharging under time-variant electricity prices, *Transportation Research Part C*, 2021, Vol. 130, pp. 103285.

<sup>55</sup> Macrina G., Di Puglia Pugliese L., Guerriero F., Laporte G.: The green mixed fleet vehicle routing problem with partial battery recharging and time windows, *Computers and Operations Research*, 2019, Vol. 101, pp. 183–199.

The objective is to minimize a total cost which consists of four components: the cost of recharging at charging stations, a vehicle activation cost which depends on the battery capacity, the cost of routes travelled by EVs and CEVs. In Chen et al.<sup>56</sup> (2021) the problem with mixed fleet is also presented but the only way of supplement energy of EVs during passing the route is battery swapping. A depot is only one place where the vehicles are recharged at the night. The vehicle starts its route with a fully charged battery.

In the paper Basso et al.<sup>57</sup> (2021) an energy consumption estimation is considered. Proposed estimation model considers vehicle speed, topography, powertrain efficiency and the effect of braking and acceleration. Presented methods were tested using real road network from Gothenburg (Sweden) and the results of estimation were compared with real data from public transport route. The maximum error of estimation was less than 5% while the average error was 2.28%.

A real-world charging infrastructure is also analysed as EVRP. In Hecht et al.<sup>58</sup> (2021) a charging infrastructure in Germany, a leading adopter of EVs in Europe, is evaluated by analysing travel time for five EVs on 60 routes passed through Germany. Based on evaluation the final conclusions are formulated. Total travel time is an average of 8% more due to battery recharging. On average, a single stop for charging takes about 14 minutes and the average time between two charges is 90 minutes. Range optimization by 30% makes possible to obtain the travel time comparable to traditional CEV. It can be reached by optimising driving behaviour. A charging infrastructure makes possible to reach destination points without running out of battery power and violating defined constraints.

The battery life of EV decreases with the increase of charging times and at some point it reaches its end of life. There is a problem of used batteries utilization. Recycling and reuse of the batteries reduce its the influence to the environmental pollution. Therefore the challenge is to building a recycling network and it is a critical for using the EVs. In Hu et al.<sup>59</sup> (2022) three strategies to deal with used batteries are analysed: remanufacturing, reuse and recycling materials. A model of recycling network is

---

<sup>56</sup> Chen Y., Li D., Zhang Z., Wahab M.I.M., Jiang Y.: Solving the battery swap station location-routing problem with a mixed fleet of electric and conventional vehicles using a heuristic branch-and-price algorithm with an adaptive selection scheme, *Expert Systems With Applications*, 2021, Vol. 186, pp. 115683.

<sup>57</sup> Basso R., Kulcsár B., Egardt B., Lindroth P., Sanchez-Diaz I.: Energy consumption estimation integrated into the Electric Vehicle Routing Problem, *Transportation Research Part D*, 2019, Vol. 69, pp. 141–167.

<sup>58</sup> Hecht Ch., Victor K., Zurmühlen S., Sauer D.U.: Electric vehicle route planning using real-world charging infrastructure in Germany, *eTransportation*, 2021, Vol 10, pp. 100143.

<sup>59</sup> Hu X., Yan W., Zhang X., Feng Z., Wang Y., Ying B., Zhang H.: LRP-based design of sustainable recycling network for electric vehicle batteries, *Processes*, 2022, Vol. 10, pp. 273.

proposed where the goal is to minimize the total cost and carbon emissions. The model was validated based on GEM company (a Chinese company of recycling and reuse) as an example.

In Erdem<sup>60</sup> (2022) the Electric Waste Collection Problem (EWCP) is introduced. In EWCP a heterogeneous fleet of EVs is used to visit customers where the waste bins are located. The problem is researched on the real-world case in Samsun (Turkey) where the collection of recycling wastes (plastic, metal, paper and glass) and used cooking oil are considered.

## 1.6. Unmanned vehicle routing problem

Over the past few years Unmanned Aerial Vehicles (UAVs), or drones, have become more and more popular and find application in many fields. This type of vehicles have the potential to reduce the cost and the time of delivery. UAVs are not limited by roads infrastructure therefore delivering with UAVs may be faster than delivering with traditional vehicles. It can be used to deliver parcels, food or medical supplies over dangerous areas thus it has a potential to save lives in critical situations like war or time of natural disaster. In 2013 Amazon revealed plans for Prime Air which assumed delivering by UAVs packages under 2.5 kg in less than 30 min<sup>61</sup>.

The paper Rajan et al.<sup>62</sup> (2022) presents a problem of using UAVs in patrolling missions. Given are two kinds of targets: primary targets and supplemental targets where a given number of supplemental targets correspond to each primary target. The main goal is to determine a path which visits each primary target. After visiting the primary target the UAV sends collected information to the operator. If the operator deems that the information is insufficient then UAV visits all supplemental targets corresponding to visited primary target and collects additional information before it visits the next primary target. The objective is to minimize the total cost of traversing the path.

---

<sup>60</sup> Erdem M.: Optimisation of sustainable urban recycling waste collection and routing with heterogeneous electric vehicles, *Sustainable Cities and Society*, 2022, Vol. 80, pp. 103785.

<sup>61</sup> Deng X., Guan M., Ma Y., Yang X., Xiang T.: Vehicle-assisted UAV delivery scheme considering energy consumption for instant delivery, *Sensors*, 2022, Vol. 22, pp. 2045.

<sup>62</sup> Rajan S., Sundar K., Gautam N.: Routing problem for unmanned aerial vehicle patrolling missions – a progressive hedging algorithm, *Computers & Operations Research*, 2022, Vol. 142, pp. 105702.



Another problem is studied in Shen et al.<sup>63</sup> (2022). The paper focuses on a multi-drone path planning problem where a given set of drones collect a data from a set of points in a determined area. The goal of the problem is to maximize the number of visited point and eliminate flight path intersections to avoid drone collisions in flight. There are two solutions to avoid collision. In the first case a set of paths is determined and all possible collisions are eliminated. In the second case during planning paths possible collisions are detected and eliminated and a set of collision-free paths is determined. The authors focuses on the second method.

In Wang et al.<sup>64</sup> (2020) a cooperative path planning of UAV swarm with dynamic customer arrivals in a set of potential locations and waiting deadlines is presented. The objective is to determine optimal paths for set of UAVs maximizing the total number of successfully served customers. The cooperation of UAVs is an important part of path planning. The UAV needs to decide whether to wait in the current location or chase a new location, with a coordination with the other available UAVs.

In Ragab et al.<sup>65</sup> (2022) the UAVs are researched as a component of smart city. A drone has a various smart devices which consists of different sensors like time-of flight sensor, magnetic field change sensor, thermal and chemical sensor and orientation sensor. Drones are remotely controlled and often move without human control. Proposed conception of Internet of Drones (IoD) is a technique where a set of drones, where each of them serves a given flight area and executes a special procedure which communicate and sends a data via sensors to dedicated control server. It assumes that the users can monitor any drones via own mobile phone and trusted control server in a flying area. Drones have limited battery resources that have influence to the efficiency of data transmission and communication in IoD. Therefore a significant problem is the development of an effective data transmission which is a challenging process. There is proposed a hierarchal routing also named clustering where the network is divided into subgroups. A cluster contains cluster members and cluster head (CH), which is selected among all members. The CH controls the performance of the cluster. In the paper an efficient grouping drones in the cluster enabling optimal data transmission via shortest routes is studied.

---

<sup>63</sup> Shen K., Shivgan R., Medina J., Dong Z., Rojas-Cessa R.: Multi-depot drone path planning with collision avoidance, IEEE Internet of Things Journal, 2022, pp. 1–1.

<sup>64</sup> Wang K., Zhang X., Duan L.: Cooperative path planning of a UAV swarm to meet temporal-spatial user demands, GLOBECOM 2020 - 2020 IEEE Global Communications Conference, 2020, pp. 1–6.

<sup>65</sup> Ragab M., Altalbe A., Al-Malaise ALGhamdi A.S., Abdel-khalek S., Saeed R.A.: A drones optimal path planning based on swarm intelligence algorithms, Computers, Materials & Continua, 2022, Vol 72, No. 1, pp. 365–380.

The paper Chen et al.<sup>66</sup> (2022) presents delivery with mixed fleet containing traditional vehicles and drones. Clients make delivery requests over the day and the operator dynamically dispatches drones and vehicles to serve clients before defined delivery deadline. Vehicles can deliver multiple packages but due to the road traffics the deliver is slowly. Drones have limited capacity and require charging the battery but the time of delivery is faster. The operator must determine whether a delivery request can be served and, if so, it decides which type of vehicle, i.e. traditional or drone, will serve it. The goal of the problem is to maximize the number of served clients per day. A similar problem is presented in Nguyen et al.<sup>67</sup> (2022) but the objective is to minimize a total transportation cost. It assumes that a traditional vehicle can serve many customers passing a single trip. A drone serves only single customer in the trip and returns to the depot. It should be pointed out that not all delivers can be served by drones and some must be served by traditional vehicle. In Coindreau et al.<sup>68</sup> (2021) is considered a problem of delivery using mixed fleet of vehicles where drones are embedded into the truck but the truck is embedded with a maximum one drone. Drones are loaded with a parcels and launched directly from the truck. Drones serve clients and autonomously return to the truck where are charged and reloaded. Drone can transport only one parcel at a time. If parcel is too heavy or if drone landing is impossible then a parcel is delivered by a truck. Presented model was inspired by a large European logistic provider<sup>69</sup>. The tests were carried out based on real data of the provider. The results shown that a total cost of fuel consumption by the truck can be reduced at least 15% when the percentage number of clients reachable by drone is above 50%. For instances containing 50 or 100 clients where each client was reachable by drone the cost reduction was between 20% and 35%.

Another cooperation traditional vehicles and UAVs is presented in Deng et al.<sup>70</sup> (2022). Customers are divided into clusters where each cluster has single stopping point. The vehicle carrying multiple UAVs from distribution center and traverses determined route which contains stopping points. When vehicle reaches stopping point multiple UAVs take-off at the same time and deliver parcels to the customers and return to the

---

<sup>66</sup> Chen X., Ulmer M.W., Thomas B.W.: Deep Q-learning for same-day delivery with vehicles and drones, *European Journal of Operational Research*, 2022, Vol. 298, pp. 939–952.

<sup>67</sup> Nguyen M.A., Thi-Huong Dang G., Hoàng Hà M., Minh-Trien Pham: The min-cost parallel drone scheduling vehicle routing problem, *European Journal of Operational Research*, 2022, Vol. 299, pp. 910–930.

<sup>68</sup> Coindreau M.-A., Gallay O., Zufferey N.: Parcel delivery cost minimization with time window constraints using trucks and drones, *Networks*, 2021, Vol. 78, pp. 400–420.

<sup>69</sup> The name of the provider is not published as per a nondisclosure agreement.

<sup>70</sup> Deng X., Guan M., Ma Y., Yang X., Xiang T.: Vehicle-assisted UAV delivery scheme considering energy consumption for instant delivery, *Sensors*, 2022, Vol. 22, pp. 2045.

vehicle. In presented model single UAV can deliver multiple parcels to many customers in single flight. The vehicles are only used as a mobile warehouses and charging stations.

The paper Qin et al.<sup>71</sup> (2021) focuses on the charging problem of UAVs. The given fleet of UAVs start from base and conduct missions by visiting a set of target points. Proposed charging model bases on Mobile Charging Vehicles (MCVs). Simultaneously with UAVs the MCVs travel to special charging points located at the routes of the UAVs where UAVs are charged wirelessly. In the paper optimization routes of MCVs is researched where the objective is to minimize the total cost power supply and maximize completion quality of UAVs missions.

Using UAVs is also studied as a green routing model<sup>72</sup>. The UAV yield lower energy consumption than traditional vehicles and its using reduces greenhouse gas emissions. The paper focusses on the impact of cooperating UAVs with traditional combustion vehicles on CO<sub>2</sub> emission. As the number customers increases, the number of required vehicles and CO<sub>2</sub> emissions also increase. The results of experiments shown that the 300, 400 and 500 customers instances require one fewer traditional vehicle when UAVs are used. On average, the emission of CO<sub>2</sub> was reduced about 20%. For the 200 customers instance the emission of CO<sub>2</sub> was reduced about 16%.

In Wang et al.<sup>73</sup> (2022) a global path planning for Unmanned Surface Vehicles (USVs) is presented. The USV has high speed, intelligence, low cost and no risk of casualty. It can carry different loads thus it can be used to serve clients in civilian fields. It has also good concealment and resistance to extreme conditions therefore it is useful in military area. Global path planning is applied in the USV self-navigation to determine a safe and anti-collision path between source and destination points. Presented methods were tested in a 3 × 3 km maritime environment with 30 × 30 grid environment model.

The UAVs are applied in diverse environments. The paper Ribeiro et al.<sup>74</sup> (2020) presents using the UAVs in the mining industry. The UAVs are used to handle periodic inspections of the belt conveyors that transport iron ore. The goal is to determine the best route for inspections minimizing to total cost of operation. The tests were carried

---

<sup>71</sup> Qin W., Shi Z., Li W., Li K., Zhang T., Wang R.: Multiobjective routing optimization of mobile charging vehicles for UAV power supply guarantees, *Computers & Industrial Engineering*, 2021, Vol. 162, pp. 107714.

<sup>72</sup> Chiang W.-C., Lib Y., Shang J., Urban T.L.: Impact of drone delivery on sustainability and cost: Realizing the UAV potential through vehicle routing optimization, *Applied Energy*, 2019, Vol. 242, pp. 1164–1175.

<sup>73</sup> Wang H., Zhang J., Dong J.: Application of ant colony and immune combined optimization algorithm in path planning of unmanned craft, *AIP Advances*, 2022, Vol. 12, pp. 025313.

<sup>74</sup> Ribeiro R.G., Júnior J.R.C., Cota L.P., Euzébio T.A.M., Guimarães F.G.: Unmanned aerial vehicle location routing problem with charging stations for belt conveyor inspection system in the mining industry, *IEEE Transactions on Intelligent Transportation Systems*, 2020, Vol. 21, No. 10, pp. 4186–4195.

out based on a real data of conveyor belt system in Brazil. The system consists of the loading terminal and 120 km of belt conveyors leading to 230 inspection points. In Grogan et al.<sup>75</sup> (2021) using the UAVs with wireless sensors and cameras to exploring and preparing search and rescue operation on an area after the occurrence of a tornado. Presented method consists of two steps. The aim of the first step is to generate a set of waypoints that must be visited. The second step contains determining a route that visits set of waypoints minimizing the longest tour of the UAV. Presented method was tested based on a real data from the National Oceanic and Atmospheric Administration and Geographic Information System data from the states of Oklahoma and Texas, USA. The paper Liperda et al.<sup>76</sup> (2020) focuses on the use of drones to monitoring flooded areas. The objective is to minimizing the total time required to perform operations. The object of researching was Bekasi city in Indonesia, which was affected by flooding in February 2020. This city is categorized in the high-risk class with a score of 33.6 against flood disasters. The paper Ozkan<sup>77</sup> (2021) proposes a method of determining a route of UAVs to mitigate forest fire risks. The researching was carried out based on fire-risk maps countrywide generated every day by the Turkish State Meteorological Service. The maps make possible to predict fire risk 3 days later based on meteorological data. The author proposes a method of determining the risky regions based on these maps that will be visited by the UAVs.

## 1.7. Conclusions

This chapter presents the limited review on current variants of the VRP. The vast majority of the analyzed papers in the chapter come from the last 5 years. The studied problems are based on real transport systems. A real-life characteristics are often considered either individually or with a limited number of characteristics. New trends, which are the reduction of the negative impact on the environment, pollution and

---

<sup>75</sup> Grogan S., Pellerin R., Gamache M.: Using tornado-related weather data to route unmanned aerial vehicles to locate damage and victims, *OR Spectrum*, 2021, Vol. 43, pp. 905–939.

<sup>76</sup> Liperda R.I., Pewira Redi A.A.N., Sekaringtyas N.N., Astiana H.B., Sopha B. M., Maria Sri Asih A.: Simulated annealing algorithm performance on two-echelon vehicle routing problem-mapping operation with drones, 2020 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM), 2020, pp. 1142–1146.

<sup>77</sup> Ozkan O.: Optimization of the distance-constrained multi-based multi-UAV routing problem with simulated annealing and local search-based matheuristic to detect forest fires: The case of Turkey, *Applied Soft Computing*, 2021, Vol. 113, pp. 108015.

greenhouse gas emissions, are more and more often taken into account in the analyzed problems. The ecological aspects are resolved by using a vehicles with alternative energy source.

Future research could look at problems that take into account both new types of vehicles and new emerging features of transport systems. Taking into account more and more characteristics makes problem solving methods more and more complicated. The VRP and their variants are known as NP-hard problems and they are solved using various heuristics methods. Therefore, the research focuses on the development of more and more effective methods enabling the determination of the solution closest to the optimal one.