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ON RELIABILITY OF SYSTEMS OPERATING IN RANDOM ENVIRONMENT

Summary. Markov reliability model of a system, operating in random environment is introduced and investigated. The model is described by multidimensional Markov process with block-wise infinitesimal matrix. Algorithms for calculation of time-dependent and steady state probabilities as well as Quality of Service (QoS) characteristics of the system based on this special form of infinitesimal matrix are given. Hybrid system of information transmission as an example of such approach is studied
Keywords: reliability models, random environments

O NIEZAWODNOŚCI SYSTEMÓW DZIAŁAJĄCYCH W ŚRODOWISKU LOSOWYM

Streszczenie. W artykule przedstawiono i zbadano model Markowa niezawodności systemu działającego w środowisku losowym. Model jest opisany przez wielowymiarowy proces Markowa z infinitezymalnym generatorem o strukturze blokowej. Na podstawie tej szczególnej postaci macierzy podane są algorytmy obliczania czasowo zależnych i stacjonarnych prawdopodobieństw oraz charakterystyk jakości usług (QoS) systemu. Jako przykład takiego podejścia przebadany został hybrydowy system transmisji informacji.
Słowa kluczowe: modele niezawodności, środowiska losowe

1. Introduction and motivation

Most of complex technical systems and biological objects are usually operating in changing environment, which can have a regular (seasons etc.) as well as random (weather, rein, smog, etc.) character. At that the mean time of environment changing can be co-measured with that failures and repair or to be both smaller or grater it. As an example of such systems information transmission systems can be considered. The influence of different environment conditions to the different type of channels leads to the necessity to construct hybrid information transmission systems (see [1], where detailed review of the problem has been done). Therefore the influence of this circumstances to their reliability characteristics is an important problem.

There are some papers, devoted to queueing systems in random environment investigation. One of the first it was the paper of Eisen and Tainiter, who investigated the system $M/M/1(ME)$ under assumption that the random environment takes only two states. Here and later the notation (ME–Markov Environment in commas) denoted that the system operates in Random Environment. The same system has been considered by Naor and Yehiali, and than generalized for the case of any finite number of environment's states by Yechiali. Newts used matrix-analytical method for investigation of one and multi-channel queueing systems in random environment. Than the models $M/M/1(ME)$ and $M/M/\infty(ME)$ have been considered in papers of Purdue and O'Cinneide & Purdue. For bibliography see, for example, [2]. In further these investigations developed in different directions connected with generalization of input flows, service mechanisms and environment processes. For the contemporary results and the bibliography see, for example, [3, 4]. However the problem of renewable systems reliability operating in random environment is not enough studied yet. One of aspects of the paper consists in description and studying of this problem.

In this paper a Markov reliability model of a system, operating in Markov random environment, is proposed and studied. The time-dependent as well as stationary and quasi-stationary probabilities are considered. As an example of the proposed approach Quality of Service (QoS) characteristics of a Hybrid Information Transmission System (HITS) are calculated.

2. The model

Consider a general reliability system, with n heterogeneous units, which operates in random environments that can takes m different states. The states of such a system can be described by $n+1$-dimensional vectors $\{x = (i, \mathbf{j}) = (i, j_1, j_2, \ldots, j_n)\}$, which first component i describes the environment states and take the values $i = \{1, 2, \ldots m\}$, and binary components j_k $(k = 1, 2, \ldots n)$ of the vector $\{\mathbf{j} = (j_1, j_2, \ldots, j_n)\}$ describe the states of appropriate units of the system and take the values $j_k = \{0, 1\}$ if k-th element of the system is in "down" or "up" state. Denote by

$$E = \{x = (i, \mathbf{j}) = (i, j_1, \dots j_n)), (i = \overline{1,m}), \ (j_k = \{0,1\}), \ (k = \overline{1,n})\} \tag{1}$$

the full system set of states, consisting of $|E| = m \times 2^n$ states, and by E_0 and E_1 the subsets of its up and down states.

The above considered systems can be modelled by multidimensional process

$$X(t) = (I(t), \mathbf{J}(t)), \tag{2}$$

where the first component describes the environment and takes m different values, and the second one $\mathbf{J}(t) = (J_1(t), \dots J_n(t))$ describes the multi-dimensional multi-state reliability system. Accordingly to the system structure the transitions from state x to state y with the same $\mathbf{j}(y) = \mathbf{j}(x)$ and any $i(y)$ or to the state y with the same $i(y) = i(x)$ and $\mathbf{j}(y) = \mathbf{j}(x) \pm \mathbf{e}_k$, where $\mathbf{e}_k$ is a vector all component of which are zeros except of the k-th one which is equal to one.

In this paper it is supposed that the first component $I(t)$ is a continuous time finite state Markov process with infinitesimal matrix $\Lambda = [\lambda_{i,j}]$, and the second one $\mathbf{J}(t))$ is an (binary) alternative process with intensities $\alpha_{i,k}$ and $\beta_{i,k}$ for k-th component $(k = \overline{1,n})$ being the environment in its state i $(i = \overline{1,m})$. At that it is also supposed that the system simultaneously changes their characteristics when the environment change its state. In further the following notations are used:

- $\Lambda = [\lambda_{i,j}]$ is an infinitesimal matrix of the environment process changing;
- $\vec{\lambda}'_{i,\cdot} = (\lambda_{i,1}, \lambda_{i,2}, \cdot, \lambda_{i,m})$ is a vector-row of transition intensities of the environment from i-th state $(i = \overline{1,m})$;
- $\lambda_i = \sum_{1 \le k \le m} \lambda_{i,k}$ is the i-th environment state changing intensity;
- diag$\vec{\lambda}'_{i,\cdot}$–diagonal matrix with components of vector $\vec{\lambda}'_{i,\cdot}$ at the main diagonal.
- A_i– infinitesimal matrix of reliability component of the process operating in i-th state of environment, it is three diagonal block-wise matrix of complex structure that should be done for special cases separately.

Numerate the system states in lexicographical order:

$$E = \{(1, |0|), (1, |1|), \cdots (1, |n|), \cdots (m, |0|), (m, |1|), \cdots (m, |n|)\}, \tag{3}$$

where $|k|$ denotes dimension of vector $\mathbf{j}$ (number of units in the vector $\mathbf{j}$).

Under the given assumptions the process $X(t) = (I(t), J(t))$ is a multidimensional Markov process under states space E and a block-wise infinitesimal matrix $Q = [Q_{x,y}]$, which diagonal blocks $Q_{x,x}$ for $i(y) = i(x)$ are three-diagonal block-wise matrices of the form $Q_{x,x} = A_{i(x)} - \lambda_{i(x)} I$, and non-diagonal blocks $Q_{x,y}$ have a form $Q_{x,y} = \lambda_{j(x),j(y)} I$, where I is a unit matrix of appropriate dimensional,

$$Q = [Q_{x,y}] = \begin{bmatrix} A_1 - \lambda_1 I & \lambda_{1,2} I & \cdots & \lambda_{1,m} I \\ \lambda_{2,1} I & A_2 - \lambda_2 I & \cdots & \lambda_{2,m} I \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{m,1} I & \lambda_{m,2} I & \cdots & A_m - \lambda I \end{bmatrix}. \tag{4}$$

As an example consider *hybrid transmission information system* that consists of $n = 3$ heterogeneous channels: 1–laser channel, 2–radio-channel of millimeter radiocast diapason (hot reserve), and 3–IEEE 802.11n protocol radio-cannel. In [1] the system is considered as a non-reliable 3-units system for which the influence of environment is modelled as failure and repair of channels. We'll return to this example in the last section of the paper.

3. Kolmogorov equations

Special structure of infinitesimal matrix will be used for the Kolmogorov equations decision. In the following usual matrix notations will be used. At that the vectors is considered as vectors-columns and transpose operation is denoted by prime whereas the derivative is denoted by upper dot. Denote by $\vec{\pi}(t)$ the states probability vector,

$$\vec{\pi}'(t) = (\vec{\pi}'_1(t), \ldots, \vec{\pi}'_m(t)) \tag{5}$$

where sub-vectors $\vec{\pi}'_i(t)$ mean the states probabilities of a system operating in i-th state of environment. Denote also by $\vec{a}' = (a_1, \ldots, a_m)$ the initial environment distribution, and by $\vec{e}'_0 = (1, 0, \ldots, 0)$ $n{+}1$-dimensional vector, which the first (with number zero) component equals to one, all others being zeros. In these notations the Kolmogorov system of equations for time-dependent process $X(t)$ state probabilities with initial condition

$$\dot{\vec{\pi}}'(t) = \vec{\pi}'(t) Q, \quad \vec{\pi}'(0) = (a_1 \vec{e}'_0, \cdots, a_m \vec{e}'_0) \tag{6}$$

taking into account infinitesimal matrix Q structure can be represented in the form

$$\dot{\vec{\pi}}'_k(t) = \vec{\pi}'_k(t)(A_k - \lambda_k I) + \textstyle\sum_{i \neq k} \vec{\pi}'_i(t) \lambda_{i,k} I, \quad \vec{\pi}'_k(0) = a_k \vec{e}'_0, \quad (k = \overline{1, m}). \tag{7}$$

Multiplying these equations to vector-column of units $\vec{1} = (1, \cdots, 1)'$ from the right using the notation $\vec{\pi}'_k(t)\vec{1} = p_k(t)$ and taking into account the equality $A_k \vec{1} = \vec{0}$, one get the system of equations for the time-dependent probability state of environment,

$$\dot{p}_k(t) = -\lambda_k p_k(t) + \textstyle\sum_{i \neq k} p_i(t) \lambda_{i,k}, \quad p_k(0) = a_k, \quad (k = \overline{1, m}). \tag{8}$$

In terms of Laplace transform $\tilde{\pi}_j(s) = \int e^{-st}\pi_j(t)dt$ the system (7) with initial condition takes the form of algebraic equations,

$$s\tilde{\vec{\pi}}'(s) - a_k\vec{e}_0' = \tilde{\vec{\pi}}'(s)(A_k - \lambda_k I) + \sum_{i\neq k} \tilde{\vec{\pi}}_i'(t)\lambda_{i,k} I, \tag{9}$$

which solution is

$$\tilde{\vec{\pi}}_k'(s) = a_k\vec{e}_0'((s+\lambda_k)I - A_k)^{-1}. \tag{10}$$

The last expression has a form of ration of rational vector-functions of variable s, which inversion allows to find time-dependent states probability distribution. The same also possible to find with direct solution of the equations (7).

The block-wise structure of the matrices A_k can be used for numerical solution of the last equation. It will be used in the last section when some special Example will be studied.

4. Stationary characteristics

Due to irreducibility and finite state of the process the steady state probabilities exist and coincide with the limiting ones $\pi_x = \lim_{t\to\infty} \pi_x(t)$. They satisfy to the system of global balance equations that for the probability distribution vector $\vec{\pi}' = \{\pi_x, x \in E\}$ jointly with normalizing condition in matrix form is

$$\vec{\pi}'Q = 0, \quad \vec{\pi}'\vec{1} = 1. \tag{11}$$

Analogously to (7) this equation can be represented in the form

$$\vec{\pi}_k'(A_k - \lambda_k I) + \sum_{i\neq k} \vec{\pi}_i'\lambda_{i,k} I = 0, \quad (k = \overline{1,m}), \quad \sum_{1\leq k\leq m} \vec{\pi}_k'\vec{1} = 1. \tag{12}$$

For the environment steady state probabilities $\vec{\pi}_k'\vec{1} = p_k$ it gives by multiplying by vector of units from the right the system of equations

$$-\lambda_k p_k + \sum_{i\neq k} p_i\lambda_{i,k} = 0, \tag{13}$$

that under the normalizing condition $\sum_{1\leq i\leq m} p_i = 1$ has an unique solution.

Taking into account the expression $\lambda_i = -\lambda_{i,i}$ for numerical solution system of equations (12) under stationary regime of environment it should be represented in the form

$$\vec{\pi}_k' A_k + \sum_{1\leq i\leq m} \vec{\pi}_i'\lambda_{i,k} = 0, \quad \vec{\pi}_k'\vec{1} = p_k, \quad (k = \overline{1,m}). \tag{14}$$

The steady state probabilities are used for the system QoS characteristics, such as:

- steady state probabilities of environments $p_i = \vec{\pi}_i'\vec{1}$,

- the system failures in i-th environment state $\pi_{\text{fail. } i} = \sum_{x\in E_1:\, i(x)=i} \pi_x$,
- the full system failure $\pi_{\text{fail.}} = \sum_{x\in E_1} \pi_x$,
- system availability $\pi_{\text{av.}} = 1 - \pi_{\text{fail.}} = \sum_{x\in E_0} \pi_x$.

5. Reliability function

Denote by T the lite time of the system. It is the time to the first destination by the process $X(t)$ failure subset E_1 of the system states,

$$T = \inf\{t:\ J(t) \in E_1\}. \tag{15}$$

To calculate its cumulative distribution function (c.d.f.) $F(t) = \mathbf{P}\{T \le t\}$ one should solve the Kolmogorov system of equations with absorbing set E_1. Remind that E_0 and E_1 denote the "up" and "down" states of then system and represent the infinitesimal matrix Q of the process and the probability state vector $\vec{\pi}'(t)$ in the form

$$Q = \begin{bmatrix} Q_{0,0} & Q_{0,1} \\ Q_{1,0} & Q_{1,1} \end{bmatrix}, \quad \vec{\pi}'(t) = (\vec{\pi}'_{E_0}(t), \vec{\pi}'_{E_1}(t)), \quad \vec{e}'_0 = (\vec{e}'_{0,E_0}, \vec{e}'_{0,E_1}), \tag{16}$$

where blocks of the matrix with indices 0 and 1 correspond to transition intensities from the states subsets E_0 to E_1 and back. Now putting $Q_{1,0} = \vec{\mathbf{e}}'_{0,E_1} = 0$, represent the system (7) in the form

$$\dot{\vec{\pi}}'_{E_0}(t) = \vec{\pi}'_{E_0}(t) Q_{0,0}, \quad \dot{\vec{\pi}}'_{E_1}(t) = \vec{\pi}'_{E_0}(t) Q_{0,1}. \tag{17}$$

In terms of LT this system taking into account initial condition take the form

$$s\tilde{\vec{\pi}}'_{E_0}(s) - \vec{e}'_{0,E_0} = \tilde{\vec{\pi}}'_{E_0}(s) Q_{0,0}, \quad s\tilde{\vec{\pi}}'_{E_1}(s) - \vec{e}'_{0,E_1} = \tilde{\vec{\pi}}'_{E_0}(s) Q_{0,1}, \tag{18}$$

and has the solution

$$\tilde{\vec{\pi}}'_{E_0}(s) = \vec{e}'_{0,E_0}(Is - Q_{0,0})^{-1}, \quad \tilde{\vec{\pi}}'_{E_1}(s) = \tfrac{1}{s}\vec{e}'_{0,E_0}(Is - Q_{0,0})^{-1} Q_{0,1}. \tag{19}$$

For the c.d.f. of the system life time it gives

$$F(t) = \mathbf{P}\{T \le t\} = \textstyle\sum_{j\in E_1} \pi_j(t) = \vec{\pi}'_{E_1}(t)\vec{1} \tag{20}$$

and therefore its LT is $\tilde{F}(s) = \tilde{\vec{\pi}}'_{E_1}(s)\vec{1}$. Thus, the moment generating function (m.g.f.)

$$\tilde{f}(s) = \mathbf{E}\left[e^{-sT}\right] = s\tilde{F}(s) \tag{21}$$

of lite time T can be represented as

$$\tilde{f}(s) = s\tilde{\vec{\pi}}'_{E_1}(s)\vec{1} = \vec{e}'_{0,E_0}(Is - Q_{0,0})^{-1} Q_{0,1}\vec{1}. \tag{22}$$

For the numerical calculation of this function as well as the moments of life time T the special structure of the infinitesimal matrix Q should be used.

6. An example

As an Example consider *hybrid transmission information system*, formulated in the section 2. In [1] the system is considered as a non-reliable 3-units system for which the influence of environment is modelled as failure and repair of channels. For analysis the real statistical data given in [5] have been used. These are given in the Table 1.

Table 1
Availability, non availability mean times and transmission rate of different channels

	Channels		
characteristics	optic channel	mm-diapason channel	IEEE channel
mean available time	26 (hours)	30(hours)	10000(hours)
mean non available time	23(hours)	15(hours)	5(hours)
transmission rate c_k	$\approx 1(Gbit/sec)$	$\approx 1(Gbit/sec)$	$\approx 300(Mbit/sec)$

These values have been uses for estimation of the failure and restoration rates of channels: $\alpha_k = m_{k,0}^{-1}$, $\beta_k = m_{k,1}^{-1}$, as well as the information transmission rates in any states of system $c_{\mathbf{j}} = c_1\, j_1 + c_2\, j_2 + c_3\, j_3$.

For the example the failure probabilities π_f, the mean life time of the system $\mathbb{E}T$ and the mean information transmission rate of hybrid system $\mathbb{E}c = \vec{\pi}'\vec{c}$ have been calculated for "hot" and "cold" optic channel redundancy, which in the following Table 2 are presented.

Table 2
QoS characteristics of hybrid transmission system in stable environment

	QoS characteristics		
redundancy	Failure probab. π_f	$\mathbb{E}T$	$\mathbb{E}\bar{c}$
hot	$7,87 \cdot 10^{-5}$	43033 (hours)	1493, 9 Mbit/sec.
cold	$5,33 \cdot 10^{-5}$	63542 (hours)	1494 Mbit/sec.

Comparison of the results for "hot" and "cold" redundancy systems shows that the only mean "up" time for the case of cold redundancy approximately into 1.5 times grater than for the case of the "hot" one, the other characteristics almost the same.

Another approach consists in investigation the same system in the random environment, where changing of an environment leads to the changing of the transmission rates of channels. In the following modelling example it is supposed that the environment behavior follows to semi-Markov process with mean time in i-th state staying equals to m_i. The same rates c_j of channels is used and these characteristics contains in the Table 3 and transition matrix is

Table 3

Characteristics of random environment

	Characteristics of channels		
state of environment	mean time	channels are used	transmission rate
1 (rein, snow)	30(hours)	1-st, 3-rd	$c_1 = 1.3(Gbit/sec)$
2 (smog, fog)	15(hours)	2-nd, 3-rd	$c_2 = 1.3(Gbit/sec)$
3 (clear, sunny)	150(hours)	all	$c_3 = 2.3(Gbit/sec)$

$$P = [p_{i,j}] = \begin{bmatrix} 0.0 & 0.2 & 0.8 \\ 0.1 & 0.0 & 0.9 \\ 0.5 & 0.5 & 0.0 \end{bmatrix} \tag{23}$$

The system QoS depends on the channels that are used in appropriate state of environment. Denote by $u = (u_{i,j} : \ i = \overline{1,m}, \ j = \overline{1,n} \ u_{i,j} = \{0, 1\}$, indicator of the channels using in different states of environments,

$$u_{i,j} = \begin{cases} 0, & \text{if } j\text{-th channel "down" in } i\text{-th environment state,} \\ 1, & \text{if } j\text{-th channel "up" in } i\text{-th environment state,} \end{cases} \tag{24}$$

For given environment process $I(t)$ the process of the information transmitted $Y(t)$ can be presented as

$$Y(t) = \sum_{1 \le i \le m} \sum_{1 \le j \le n} \int\limits_0^t c_{i,j} u_{i,j} 1_{\{I(\tau)=i\}} \, d\tau. \tag{25}$$

The long ran mean value of the system transmission rate due to ergodic theorem is

$$\lim_{t\to\infty} \frac{1}{t} Y(t) = \frac{1}{m} \sum_{1 \le i \le m} \sum_{1 \le j \le n} c_j u_{i,j} m_i \tag{26}$$

where $m = \sum_{1 \le i \le 3} \pi_i m_i$ is mean environment cycle time and π_i is the stationary probability of i-th environment state that is solution of equation

$$\pi_j = \sum_{1 \le i \le 3} \pi_i p_{i,j}, \quad \sum_{1 \le i \le 3} \pi_i = 1. \tag{27}$$

The calculations gives the following results:

- the steady state environment: $\pi_1 = 0.258, \ \pi_2 = 0.281, \ \pi_3 = 0.461$;
- mean cycle value: $m = \sum_{1 \le i \le 3} \pi_i m_i = 100.2553 \, (hours)$;

- mean long ran rate $\sum_{1\leq i\leq m}\sum_{1\leq j\leq n} c_{i,j}u_{i,j}\frac{m_i}{\pi_i m_i} = 4026\,(Mbit/sec.)$

7. Conclusion

Two approaches for the QoS characteristics of data transmission hybrid systems investigation are considered. In the first one the channels accessability and non-accessability times are modelled as their up and down times. In the second approach accessability of different channels depends on environment states that is modelled with semi-Markov process. The absence of enough statistical data do not allow to investigate the system in details, therefore only simple numerical example has been calculated.

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Omówienie

Większość złożonych systemów technicznych i obiektów biologicznych działa zwykle w zmieniającym się środowisku, które może mieć regularny (pory roku itp.), a także przypadkowy (pogoda, deszcz, smog itp.) charakter. Średni czas zmian otoczenia może być mierzony wraz z upadkami i naprawami systemu, dla określenia jego wpływu na zawodność systemu. Jako przykład mogą służyć systemy przekazywania informacji. Wpływ zmiennych warunków środowiska na różnego typu kanały transmisji prowadzi do konieczności budowy hybrydowych systemów przesyłu informacji (patrz [1], gdzie dokonano szczegółowego omówienia problemu). Wpływ tych czynników na charakterystyki niezawodności rozważanych systemów jest istotnym problemem. W niniejszym artykule został zaproponowany i przebadany model Markowa niezawodności systemu, pracującego w losowym środowisku Markowa. Rozważane są sposoby określenia prawdopodobieństwa stanów niestacjonarnych, stacjonarnych i quasi-stacjonarnych. Jako przykład proponowanego podejścia obliczane są charakterystyki QoS hybrydowego systemu przesyłania informacji (HITS).

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