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Synteza i analiza złożonych układów piezoelektrycznych
z uwzględnieniem metod klasycznych i nieklasycznych

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Abstract

The practical objective of this work was to develop a method for the synthesis of mathematical models representing piezoelectric systems. In addition, the applicability of a non-classical method based on graphs and structural number algebra is verified in the context of analysis of piezoelectric systems. The synthesised models of mechatronic cascade systems constitute a mathematical representation of real piezoelectric stacks composed of wafers with different geometric parameters or material properties. The synthesis process is based on Cauer's method, where the transmittance function of the system is decomposed into a chain fraction. The transmittance polynomials were formed based on a set of resonant and anti-resonant frequencies which was assumed in the work. Coefficients of the chain fraction determined by Cauer's method correspond to parameters (stiffness and mass values) of elements in a mechanical cascade model. Damping values were determined using the Rayleigh equation. Interactions with the electrical part of a piezoelectric system were modelled as electromotive forces acting on the components of a mechanical system. Resonant frequencies of obtained models were validated through analysis with a matrix method. Additionally, a non-classical method involving structural number algebra was also used. A modal analysis was conducted using the matrix method by solving the differential equations of motion. Analogous analysis was also done using the graph method and structural number algebra. Obtained results were compared in terms of calculated resonant frequencies and model response. A set of computer algorithms was prepared based on both methods used in the work. Developed algorithms were analysed in terms of the computing power and memory required for the calculations. Mechanical models developed in the work were transformed into mechatronic systems through the application of constitutive equations. The goal of transformation was to fully simulate the behaviour of real piezoelectric systems. Using physical relationships that define the properties of piezoelectric materials, equations were established to calculate the real parameters of piezoelectric plates based on created models. Mathematical models were later introduced in an original computer application for the automatic synthesis of piezoelectric systems, made in Matlab R2019b. An analysis module based on the matrix method was also created. A laboratory test stand was also developed to study the behaviour of piezoelectric circuits excited by a signal with constant amplitude and variable frequency. The test stand was used to verify the accuracy of synthesised models. Piezoelectric elements with different geometric parameters were tested. The study included an analysis of a modal response of circuits made with single piezoelectric

wafers and with stacks. The results obtained with analytical and empirical methods have been compared. Conclusions concerning the effectiveness of proposed synthesis method were drawn on the basis of results obtained and suggestions for further direction of scientific work were also presented.

The method of synthesis developed by the author is based on methods of synthesis of complex mechanical and electrical systems, which are continuously developed by researchers of the Silesian University of Technology. The ongoing international research on methods of modelling piezoelectric systems using mechanical and electrical analogies was also included. The aim of this work is to enable synthesis and analysis of piezoelectric systems composed of elements with different geometric parameters and material properties by using introduced methods of mathematical modelling. Research in this domain is not yet sufficiently developed, and the author hopes to lay the foundation for new methods for the synthesis of complex piezoelectric systems using wafers with different technical parameters.

Scientific work included in this thesis can contribute to the development of technologies for active vibration damping of technical devices and the extension of applications of existing technologies to new industries. The developed synthesis method may allow the creation of more complex piezoelectric systems that can operate over a wider range of frequencies, which could extend the range of applications of piezoelectric systems in the automotive industry, industrial automation and precision positioning systems.