ESTIMATION OF MEAN SPEED AND SPEED STANDARD DEVIATION FROM CFD PREDICTION

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Abstract

Mean velocity field predicted by CFD simulations on the basis of RANS equations differs from the mean (in time) magnitude of the velocity, i.e. the mean speed, existing in rooms and measured by low velocity thermal anemometers with omnidirectional (spherical) sensors. Similarly, velocity standard deviation differs from the standard deviation of the speed. In this paper the discrepancies are identified and discussed. A new method for estimation of mean speed and standard deviation of the speed based on CFD predictions of mean velocity and kinetic turbulence energy is suggested. Uncertainty of mean speed and standard deviation of the speed estimation is determined. A significant improvement can be expected in the determination of the *PMV* and *DR* indices by further processing of mean velocity and standard deviation of velocity, as predicted by CFD simulations.

Streszczenie

Średnia prędkość będąca wynikiem obliczeń CFD bazujących na równaniach RANS różni się od uśrednionego w czasie modułu wektora prędkości tj. od średniej szybkości, która mierzona jest w pomieszczeniach za pomocą termoanemometrów z czujnikami wielokierunkowymi (sferycznymi). Podobnie, odchylenie standardowe prędkości różni się od odchylenia standardowego szybkości. W artykule różnice te zostały zidentyfikowane i przedyskutowane. Nowa metoda estymacji średniej szybkości i odchylenia standardowego szybkości na podstawie uzyskanej z obliczeń CFD średniej prędkości i energii kinetycznej turbulencji została zaproponowana. Wyznaczona została niepewność takiej estymacji. Można oczekiwać znaczącej poprawy dokładności wyznaczania wskaźników komfortu termicznego *PMV* i wskaźnika dyskomfortu wywołanego zjawiskiem przeciągu *DR* z wyników obliczeń CFD.

Keywords: **CFD prediction; Speed; Velocity.**

1. PROBLEM DEFINITION

Computational Fluid Dynamics (CFD) based on Reynolds-Averaged Navier-Stokes (RANS) equations (time-averaged Navier-Stokes equations) is widely used today for predicting air movement in rooms. The predictions are used to calculate thermal comfort indices, such as *PMV* (Predicted Mean Vote) and *DR* (Draught Rating), recommended in the standards for assessment of indoor thermal environment (ISO Standard 7730, 1994, ASHRAE Standard 55 2004). Thus the reliability of the assessment depends on the accuracy of the CFD predictions. The quality control of CFD predictions and the factors important for modelling and numerical errors have been discussed in the literature (Sørensen and Nielsen 2003, ASHRAE Handbook Fundamentals 2005). In order to ensure that the important physical phenomena are correctly modelled and to quantify the error and uncertainty in the CFD simulations, the predictions are compared with experimental data, i.e. the CFD predictions are validated. Very often velocity measurements performed by low velocity thermal anemometer with omnidirectional velocity sensor (LVTA) are used for the validation. Present indoor climate standards recommend the use of these anemometers for measurements in spaces (ISO Standard 7726, 1998, ASHRAE Standard 55 2004, ASHRAE 113 1990). The experimental data may contain errors, which have to be considered during the validation. The uncertainty of velocity measurements by LVTA has been identified in detail by

Popiolek et al. (2007). However assessment of the indoor environment based on high quality, properly validated CFD predictions may still be incorrect due to improper interpretation and use of the predictions. Unfortunately this applies to most of the studies presently reported in the literature.

Figure 1 defines mean (in time) values of velocity components, $\overline{V}_x$, $\overline{V}_y$, $\overline{V}_z$, in an orthogonal coordinate system attached to the room geometry as predicted by CFD simulations and used to calculate the mean velocity, $\overline{V}$, in each point of the room:

$$\overline{V} = \left(\overline{V}_x^2 + \overline{V}_y^2 + \overline{V}_z^2\right)^{0.5} \quad (1)$$

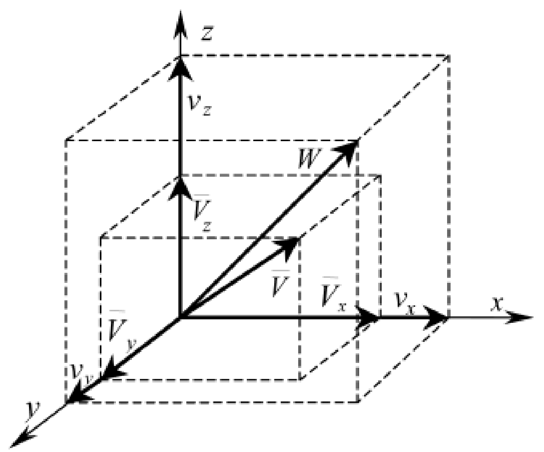

Figure 1.
Components of velocity vector and speed (magnitude of velocity vector)

The CFD simulations give also the turbulence kinetic energy, k, based on the variances of the velocity fluctuations, $\overline{v_x^2}$, $\overline{v_y^2}$, $\overline{v_z^2}$:

$$k = 0{,}5 \cdot \left(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\right) \quad (2)$$

The average variance of velocity fluctuation, $\overline{v^2}$, can be calculated from the turbulence kinetic energy:

$$\overline{v^2} = \left(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\right)/3 = 2k/3 \quad (3)$$

The velocity turbulence intensity is defined by the turbulent kinetic energy and the mean velocity as:

$$Tu_v = \left[\left(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\right)/3\right]^{0.5} / \overline{V} = \left(2k/3\right)^{0.5} / \overline{V} \quad (4)$$

In reality however, the magnitude and direction of the instantaneous velocity in each point of room airflow fluctuate in time. The magnitude of the instantaneous velocity, i.e. the instantaneous speed, W, can be defined by the mean velocity components, $\overline{V}_x$, $\overline{V}_y$, $\overline{V}_z$, and the components of the velocity fluctuation, v_x, v_y, v_z, :

$$W = \left[\left(\overline{V}_x + v_x\right)^2 + \left(\overline{V}_y + v_y\right)^2 + \left(\overline{V}_z + v_z\right)^2\right]^{0.5} \quad (5)$$

The instantaneous speed, can be presented as a sum of the mean, $\overline{W}$, and stochastic, w, component:

$$W = \overline{W} + w \quad (6)$$

The standard deviation of the speed can be defined as:

$$w^* = \left(\overline{w^2}\right)^{0.5} = \overline{\left(W - \overline{W}\right)^2}^{\,0.5} \quad (7)$$

The standard deviation of the speed and the mean speed can be used to define the turbulence intensity:

$$Tu_w = w^* / \overline{W} \quad (8)$$

Determination of the two indices, *PMV* and *DR*, requires knowledge on the air movement around occupants. The occupants' sensation of the air movement will depend on the extent to which the room airflow disturbs the free convection flow around the body, i.e. on the speed of the room airflow in the vicinity of the occupants. ISO standard 7726 (1998) and ASHRAE standard 55 (2004) refer to the mean speed and the standard deviation of speed. LVTA measures the instantaneous speed, which averaged in time gives the mean speed and the standard deviation of speed (equations 6 and 7).

If it is assumed that the mean flow direction is in *x*-direction, i.e. $\overline{V}_x = \overline{V}_y$, $\overline{V}_y = 0$, $\overline{V}_z = 0$ equation 5 can be rewritten as:

$$W = \left(\overline{V}^2 + 2\,\overline{V} v_x + v_x^2 + v_y^2 + v_z^2\right)^{0.5} = \left(\overline{V}^2 + \Delta\right)^{0.5} \quad (9)$$

where

$$\Delta = 2\,\overline{V} v_x + v_x^2 + v_y^2 + v_z^2$$

and then expanded in Taylor series:

$$W = \overline{V} + \frac{1}{2}\overline{V}^{-1}\Delta - \frac{1}{8}\overline{V}^{-3}\Delta^2 + \frac{1}{16}\overline{V}^{-5}\Delta^3 - \frac{5}{128}\overline{V}^{-7}\Delta^4 + \cdots \quad (10)$$

When the terms with velocity fluctuations to power above 4 can be ignored (as these terms will be rather small) equation 10 gives:

$$W \cong \overline{V} + v_x + \frac{v_y^2 + v_z^2}{2\overline{V}} - \frac{v_x v_y^2 + v_x v_z^2}{2\overline{V}^2} - \frac{v_y^4 + v_z^4}{8\overline{V}^3} + \frac{v_x^2 v_y^2 + v_x^2 v_z^2}{2\overline{V}^3} - \frac{v_y^2 v_z^2}{4\overline{V}^3} \tag{11}$$

Equation 11 is averaged in time and the following equation is obtained:

$$\overline{W} \cong \overline{V}\left[1 + \frac{\overline{v_y^2} + \overline{v_z^2}}{2\overline{V}^2} - \frac{\overline{v_x v_y^2} + \overline{v_x v_z^2}}{2\overline{V}^3} - \frac{\overline{v_y^4} + \overline{v_z^4}}{8\overline{V}^4} + \frac{\overline{v_x^2 v_y^2} + \overline{v_x^2 v_z^2}}{2\overline{V}^4} - \frac{\overline{v_y^2 v_z^2}}{4\overline{V}^4}\right] \tag{12}$$

It can be seen from equation 12 that for correct calculation of the mean speed $\overline{W}$ information on higher order correlations between the fluctuations of velocity components is needed. CFD predictions do not provide such information. The mean speed will be different than the mean velocity predicted by CFD simulation (higher). A method should be found to estimate mean speed on the basis of mean velocity and turbulent kinetic energy. The estimation has its own uncertainty which should be assessed.

The relation between speed standard deviation and mean velocity and velocity turbulence intensity can be derived from equations 5 and 6. Substituting equation 6 for equation 5 gives:

$$\left(\overline{W} + w\right)^2 = \left(\overline{V}_x + v_x\right)^2 + \left(\overline{V}_y + v_y\right)^2 + \left(\overline{V}_z + v_z\right)^2 = \tag{13}$$
$$\overline{W}^2 + 2\overline{W}w + w^2 = \overline{V}_x^2 + 2\overline{V}_x v_x + v_x^2 + \overline{V}_y^2 + 2\overline{V}_y v_y + v_y^2 + \overline{V}_z^2 + 2\overline{V}_z v_z + v_z^2$$

The above equation after averaging in time yields:

$$\overline{W}^2 + \overline{w^2} = \overline{V}_x^2 + \overline{V}_y^2 + \overline{V}_x^2 + \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = \overline{V}^2 + 2k \tag{14}$$

The speed standard deviation can be found as:

$$\overline{W}^2 + \overline{w^2} = \overline{V}_x^2 + \overline{V}_y^2 + \overline{V}_x^2 + \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = \overline{V}^2 + 2k \tag{15}$$

Thus, the speed standard deviation can be directly calculated if the mean velocity and the turbulent kinetic are known (CFD predictions provides these values) and if the mean speed is properly estimated.

Determination of PMV and DR indices based on mean velocity and standard deviation of velocity (equations 1 and 3) as obtained from CFD predictions is in general incorrect. Furthermore, the CFD predictions cannot be validated by direct comparison with measurements performed with LVTA. In other words, the presently used assumptions that:

$$\overline{W} \cong \overline{V} \tag{16}$$

and

$$w^* \cong \left(\frac{2}{3}k\right)^{0.5} \tag{17}$$

are incorrect.

The discrepancies identified above raise an important question: how to correct the velocity predictions obtained with the CFD simulations based on RANS equations in order to validate them by measurements performed with LVTA and use them for correct thermal comfort assessment, i.e. correct determination of *PMV* and *DR* indices. In other words, it is important to find a method for estimation of mean speed $\overline{W}_e$ and standard deviation of speed w_e^* on the basis of CFD predictions of mean velocity, $\overline{V}$, and turbulence kinetic energy, k.

Koskela et al. (2001) already demonstrated that due to fluctuations of magnitude and direction of instantaneous velocity at each point in a room, the speed, when averaged in time, i.e. the mean speed $\overline{W}$, will be different from the mean velocity $\overline{V}$, resulting from CFD predictions. The correlation between mean speed and mean velocity was studied by Koskela et al. (2001) both theoretically and with measurements in a laboratory test case. The theoretical correlation was developed by statistical calculations with the assumption of normally distributed and uncorrelated turbulent velocity components. The measurement data supported the trend of the calculated correlation curves, but the spread of measured values was large. For the air velocity measurements Koskela et al. (2001) used 3-dimensional ultrasonic anemometer, of the accuracy ± 0.02 m/s and a 50 mm spacing between the sensor elements. The size is larger than the smallest eddies in room flows. Therefore, the sensor averages the velocity fluctuations spatially. They did not provide information on uncertainty of the mean speed estimation when using the above mentioned technique.

In this paper the differences between mean velocity and standard deviation of velocity from CFD simulations and respectively mean speed and standard deviation of the speed are identified on the basis of Laser Doppler Anemometer measurements performed in the occupied zone of two full-scale test rooms at different combinations of ventilation system, air supply devices, airflow rate, air temperature and occupancy. A method for estimation of the mean speed and the standard deviation of the speed based on the predictions from CFD simulations is presented, the uncertainty of estimation is found.

2. THE METHOD

A comprehensive database of velocity measurements performed by 3-D Laser Doppler Anemometer (LDA) collected in a previous study (Melikov 1997) was analyzed. The database comprises 291 instantaneous velocity measurements in two full-scale test rooms at different combinations of ventilation system, air supply devices, airflow rate, air temperature and occupancy. The LDA anemometer was calibrated in a traceable test stand prior to the measurements. The expanded uncertainty of the LDA was: $\hat{U}(\overline{V}) = 0.052$ (m/s). The LDA measurements of the three instantaneous velocity components in x, y and z direction of an orthogonal coordinate system attached to the room geometry were used to identify accurately the instantaneous velocity vector, which was used to calculate parameters defined by equations 5, 6, 7 and 8, namely, the instantaneous speed, W, the mean speed, $\overline{W}$, the standard deviation of speed, w^*, and the speed turbulence intensity, Tu_w, measured by low velocity thermal anemometer. The LDA data also allow for calculation of the parameters defined by equations 1, 2 and 3, namely, the mean velocity, $\overline{V}$, and its three components in an orthogonal coordinate system and the three components of standard deviation of velocity, which were used to calculate the turbulent kinetic energy, k.

The analyses of the results, shown in Figure 2, identified large differences between the mean velocity and the mean speed. The differences decrease with the increase of the mean speed, but remain large for the range of mean velocity 0.1 m/s – 0.3 m/s, which is important for thermal comfort. In fact 95% lines (dotted lines) in the figure identify differences up to 0.08 m/s, revealing that under identical conditions mean velocity predicted by CFD simulation can be up to 100% lower than the mean speed measured by LVTA.

Large differences between the standard deviation of velocity, $(2k/3)^{0.5}$, and the standard deviation of speed, w^*, were identified as well. The results in Figure 3 show that the standard deviation of velocity as predicted by CFD simulations, can differ by more than 40% from the standard deviation of speed measured by LVTA.

The LDA database was analysed in order to determine the mean speed, $\overline{W}$, as a function of the mean velocity, $\overline{V}$. A clear dependence of the correlation between the mean speed and the mean velocity on the turbulence intensity, $Tu_v = (2k/3)^{0.5}/\overline{V}$, was observed. This can be seen in Figure 4, where the ratio of the mean speed and the mean velocity as a function of the turbulence intensity, Tu_v. The physical explanation of this dependence is the definition of the speed, i.e. fluctuating flow with mean velocity equal to zero will have a mean speed different from zero.

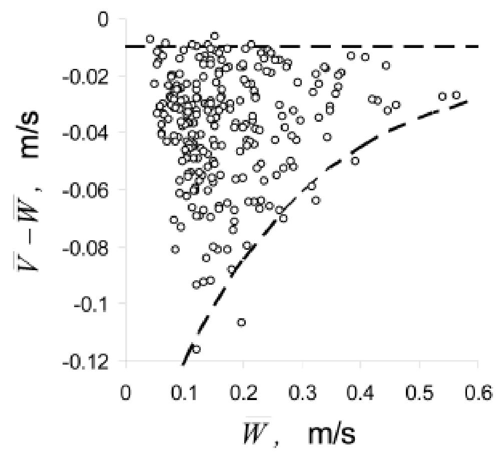

Figure 2.
Difference between mean velocity $\overline{V}$ and mean speed $\overline{W}$, based on the analyses of velocity measurement database performed by 3-D LDA in rooms under realistic conditions, Melikov (1997)

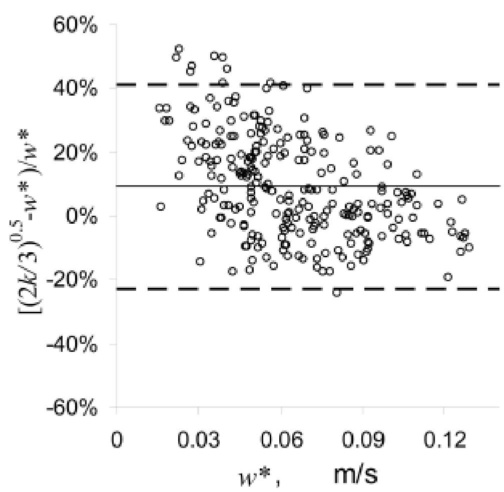

Figure 3.
Difference between the standard deviation of velocity, $(2k/3)^{0.5}$, and the standard deviation of speed, w^*, based on the analyses of velocity measurement database performed by 3-D LDA in rooms under realistic conditions, Melikov (1997)

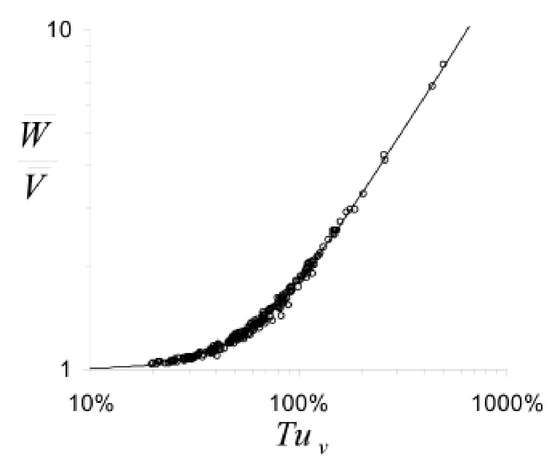

Figure 4.
Ratio of mean speed and mean velocity $\overline{W}/\overline{V}$ as a function of $Tu_v=(2k/3)^{0.5}/\overline{V}$

Computer optimization of the data was performed in order to estimate the mean speed, $\overline{W_e}$, as a function of the mean velocity, $\overline{V}$, and the turbulence intensity, Tu_V or the turbulent kinetic energy, k. In the following, $\overline{W_e}$ is referred to as the estimated mean speed. The best correlation was obtained using equations 6 and 7:

$$\overline{W}_e=\begin{cases}\overline{V}\left(1-0.044\,Tu_v+1.195\,Tu_v^{\,2}-0.329\,Tu_v^{\,3}\right) & \text{if } Tu_v\le 1.3\\ \overline{V}\left(0.287+1.502\,Tu_v\right) & \text{if } Tu_v>1.3\end{cases} \quad (6)$$

or

$$\overline{W}_e=\begin{cases}\overline{V}-0.036\,k^{0.5}+0.797\,k/\overline{V}-0.179\,k^{1.5}/\overline{V}^2 & \text{if } k^{0.5}/\overline{V}<1.592\\ 0.287\,\overline{V}+1.226\,k^{0.5} & \text{if } k^{0.5}/\overline{V}\ge 1.592\end{cases} \quad (7)$$

Equations 6 and 7 can be used to determine the estimated mean speed, $\overline{W_e}$, based on the mean velocity, $\overline{V}$, and the turbulent kinetic energy, k, as predicted by CFD simulations.

The results in Figure 5 illustrate accuracy of the estimation. The difference between the estimated mean speed (based on CFD prediction of mean velocity, turbulence kinetic energy and turbulence intensity, eq.17), $\overline{W_e}$, and the mean speed, $\overline{W}$, as obtained directly from the analyses of the LDA measurements, is shown in the figure. The results reveal that the mean speed can be estimated by equation 6 with uncertainty lower than 0.006 m/s. This is a significant improvement in comparison with the results shown in Figure 2.

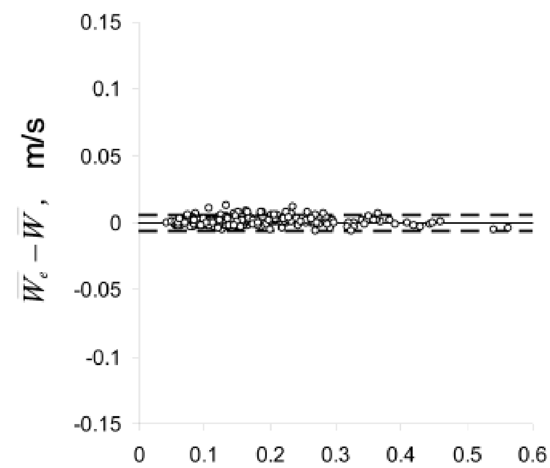

Figure 5.
Difference between the estimated mean speed, $\overline{W_e}$, calculated by means of eq.6 (or 7), and the mean speed, $\overline{W}$, obtained from the analyses of LDA measurements. The dotted lines define the area with 95% of the measurement points

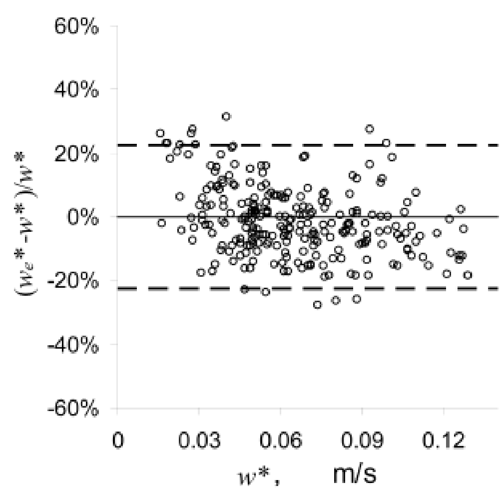

Figure 6.
Relative difference between w_e^* (estimated standard deviation of speed, calculated from eq. 15) and measured speed standard deviation w^*, data from measurement performed by Laser Doppler Anemometer in rooms under realistic conditions, Melikov (1997)

The results in Figure 6 illustrate the accuracy of the standard deviation of speed estimation using equation 15. The figure shows relative difference between the estimated standard deviation of speed, w_e^* and the standard deviation of the speed, w^*. The results show that the standard deviation of the speed can be predicted with uncertainty of 22%. This is also a significant improvement in comparison with the results shown in Figure 3.

3. DISCUSSION

The results presented and discussed show that further processing of the mean velocity and turbulent kinetic energy from CFD predictions leads to an accurate estimation of the mean speed and standard deviation of speed. This can be used for determination of *PMV-PPD* and *DR* indices and thus for realistic assessment of indoor thermal environment in compliance with the requirements of the indoor climate standards and guidelines.

The results also show that it is incorrect to compare directly CFD predictions with measurements performed with low velocity thermal anemometer with omnidirectional velocity sensor as in fact it is most often done and reported in the literature. In order to validate CFD predictions with such measurements the mean velocity and standard deviation of velocity need to be processed and estimated mean speed and estimated standard deviation of speed should be determined. A large database of LDA measurements was used in the present study to develop the equations suggested for further processing of CFD predictions. The formulae suggested in this paper can be used. The results reveal that the mean speed can be estimated by equation 6 with uncertainty lower than 0.006 m/s. This proves the validity of the equation suggested for further processing of CFD predictions.

4. CONCLUSIONS

Difference up to 100% between mean velocity and mean speed and difference up to 40% between standard deviation of velocity and standard deviation of speed have been identified. The results show that CFD predictions can be used for estimation of the mean speed, $\overline{W}_e$, with uncertainty 0.006 m/s and for estimation of the standard deviation of the speed with uncertainty of 22%. A significant improvement in the determination of *PMV* and *DR* indices by further processing of mean velocity and standard deviation of velocity predicted by CFD simulations, can be expected. Further processing of CFD predictions is also needed before validation by measurements performed with low velocity omnidirectional thermal anemometers.

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