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ON FUZZY ŁUKASIEWICZ LOGIC IN DECISION-MAKING PROCESSES IN FUZZY SYSTEMS

Summary. The paper deals with the fuzzy Łukasiewicz logic in the problems of decision-making, especially in the control of complex, ill-defined processes. The method of control allowing to assign to each of the control rules a different grade of importance has been presented. The comparision of this algorithm with the one basing on the compositional rule of inference has been done. The numerical examples make up an illustration of the presented method.

1. Introduction

Recently a lot of papers on control and decision-making problems, mainly in so called ill-defined processes, based on the theory of fuzzy sets have been presented [3,4,7]. Their main advantage is the fact that they make it possible to formulate the subjective knowledge and experience of human operator [5], very commonly having a linguistic representation, in precise terms of fuzzy sets, relations and linguistic statements. Thus using these ideas qualitative information can be represented mathematically and handled in a completely rigorous manner [1,2,8,13,14].

It is interesting to have a look at a decision-making process (or fuzzy control) in terms of some other kind of logic from that used normally, the fuzzy Łukasiewicz logic, considering the applicability of this method to the design problem of a fuzzy controller at the control engineering point of view.

Because the method of fuzzy control is concerned with the notion of Łukasiewicz logic, we present shortly an evaluation of this logic, starting at the point of a classical two-valued logic (absolute truth and false), pointing out next the main ideas of a multiple-valued logic, fuzzy Łukasiewicz logic and the properties of truth qualification [2]. Next the method of control based on tis logic is considered, and numerical examples, illustrating this method and making it possible to compare it with the well-known method of fuzzy control given by Tong, Kickert, Mamdani [4,5,6,7] have been shown.

2. Łukasiewicz logic. Truth qualification and approximate reasoning

Let us put forward a proposition using capital letters e.g. P,Q,R,S,T, ... For every proposition P we could assign a value χ_P from the unit interval $V = [0,1]$:

$$0 \leq \chi_P \leq 1 \qquad \text{i.e. } \chi_P = P \rightarrow v \in V$$

defining the propositional connectives for negation, conjunction, disjunction and implication as follows:

$$\bar{P} \quad \chi_{\bar{P}} = 1 - \chi_P \tag{1}$$

$$P \vee Q \quad \chi_{P \vee Q} = \max(\chi_P, \chi_Q) = \chi_P \vee \chi_Q \tag{2}$$

$$P \wedge Q \quad \chi_{P \wedge Q} = \min(\chi_P, \chi_Q) = \chi_P \wedge \chi_Q \tag{3}$$

$$R = P \rightarrow Q \quad \chi_{P \rightarrow Q} = \min\,[1,\ (1 - \chi_P + \chi_Q)] \tag{4}$$

The logic defined as above is called an infinite valued Łukasiewicz logic $Ł_{\chi_1}$ [12] and forms an extension of a three-valued Łukasiewicz logic, interesting from the theoretical point of view e.g. pure mathematics [9,10] and having a wide practical interest mainly in digital techniques [13].

Taking into account V which consists of 3 elements $V = \{0, \frac{1}{2}, 1\}$ and using for example formulas (1) and (4) we get the basic logical connections:

P	$\bar{P}$
0	1
$\frac{1}{2}$	$\frac{1}{2}$
1	0

P \ Q	0	$\frac{1}{2}$	1
0	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	1	1
1	0	$\frac{1}{2}$	1

R

if V consists of 0 and 1 i.e. $V = \{0,1\}$ we obtain a well known two-valued logic.

Let us extend the notion of a multivalued logic assuming that the logical value of the proposition P is expressed as a fuzzy set defined on V i.e. we assing to each P a fuzzy set given by the membership function $\mu_P(v)$ $P \in F(V)$, denoting the logic formed in this way as $FŁ_{\chi_1}$ $(P \in FŁ_{\chi_1})$.

The calculus of the propositions in $FL\chi_1$ is based on the extension principle [2]. Generally $T,P,Q \in FL\chi_1$ are given and $T = f(P,Q)$. Thus the membership function of the fuzzy set T is equal to: $T = \varphi(P,Q)$

$$\mu_T(v) = \sup_{(w,u)\in\varphi^{-1}(v)} \mu_{\varphi(P,Q)}(w,u) \qquad w,u,v\in V \tag{5}$$

and, where φ stands for the logical sum, disjunction, negation and implication, we put down:

$$\mu_{\bar{P}}(v) = \sup_{w=1-v} \mu_P(w) \tag{6}$$

$$\mu_{P\wedge Q}(v) = \sup_{(w,u)\in\varphi_1^{-1}(v)} \min(\mu_P(w),\mu_Q(u)) \qquad \varphi_1(w,u) = \min(w,u) \tag{7}$$

$$\mu_{P\vee Q}(v) = \sup_{(w,u)\in\varphi_2^{-1}(v)} \min(\mu_P(w),\mu_Q(u)) \qquad \varphi_2(w,u) = \max(w,u) \tag{8}$$

$$\mu_{P\to Q}(v) = \sup_{(w,u)\in\varphi_3^{-1}(v)} \min(\mu_P(w),\mu_Q(u)) \qquad \varphi_3(w,u) = \min(1,\ 1-w+u) \tag{9}$$

It is worth to be noticed that the last formula tends towards inequality:

$$0 \vee [\,-1 + \mu_P(v) + \mu_R(v)] \leq \mu_Q(v) \leq 1 \tag{10}$$

Rewriting it in terms of α-cut of fuzzy sets [2,3] we obtain:

$$Q^\alpha = [-1 + p_1(\alpha) + r(\alpha)] \vee 0$$

where $P^\alpha = [p_1(\alpha),\ p_2(\alpha)]$ $\qquad R^\alpha = [r(\alpha),1]$

are ordinary sets with a characteristic function:

$$P^\alpha : \chi_{P^\alpha} = \begin{cases} 1, & v\in[p_1(\alpha),\ p_2(\alpha)] \\ 0, & \text{otherwise,} \end{cases}$$

and $P = \bigcup_{\alpha\in[0,1]} P^\alpha$, $\quad R = \bigcup_{\alpha\in[0,1]} \alpha R^\alpha$ holds true.

Another concept, strictly connected with the fuzzy logic, is the truth qualification [2], by means of which it is easy to handle some results concerning the linguistic relations between sentences in approximate reasoning.

Let us consider the propositions:

$$P_1 : X \text{ is } A$$

$$P_2 : X \text{ is } D,$$

where A,D are fuzzy sets defined in the space $\mathbb{U}$, given by its membership functions $\mu_A(u)$ and $\mu_D(u)$ respectively.
According to the truth qualification the propositions P_1, P_2 might be compared:

$$P_2 \text{ is } \tau \equiv P_1$$

where τ is called the truth value of the sentence P_1 with respect to the sentence P_2, and is expressed as a fuzzy set $\tau \in F(\mathbb{V})$ with the membership function: $\mu_\tau : V \to V$, so that

$$\mu_A(u) = \mu_\tau(\mu_D(u)) \tag{11}$$

is satisfied.
The above condition could be treated as a formula to find the truth value of the proposition P_1 with respect to the fuzzy proposition P_2. For a given A and D, τ could be evaluated as:

$$\mu_\tau(v) = \mu_A(\mu_D^{-1}(v)), \tag{12}$$

if μ_D is a one-to-one corrspondence from $\mathbb{U}$ to $\mathbb{V}$. Otherwise we can approximate the linguistic truth value of these propositions in the following form:

$$\mu_\tau u(v) = \sup_{u \in \mu_D^{-1}(v)} \mu_A(\mu_D^{-1}(v)) \tag{13}$$

$$\mu_\tau l(v) = \inf_{u \in \mu_D^{-1}(v)} \mu_A(\mu_D^{-1}(v)) \tag{14}$$

i.e. we calculate the upper and lower bound of the membership function. It is to been seen that formula (13) corresponds to the one used for the evaluation of the truth value given in [2].

Selecting the term true as a unitary truth value, the following holds: P is true ≡ P. The term true might be represented by its membership function, the same as given in [2] or simply as:

$$\mu_{\tau}(v) = v \qquad \forall_{v \in V} \tag{15}$$

For the purpose of approximate reasoning, linguistic hedges can be used, for both the type of concentration:

τ_2 : very true $\quad \mu_{\tau_2}(v) = \mu_{\tau}(v)^2$

τ_3 : very very true $\quad \mu_{\tau_3}(v) = \mu_{\tau}(v)^3$

τ_n very very .... very true $\quad \mu_{\tau_n}(v) = \mu_{\tau}(v)^n$

and the type of fuzzification:

τ_{-2} : slightly true $\quad \mu_{\tau_{-2}}(v) = \mu_{\tau}(v)^{\frac{1}{2}}$

τ_{-3} : a little true $\quad \mu_{\tau_{-3}}(v) = \mu_{\tau}(v)^{\frac{1}{3}}$

τ_{-n} a little ... little true $\quad \mu_{\tau_{-n}}(v) = \mu_{\tau}(v)^{\frac{1}{n}}$

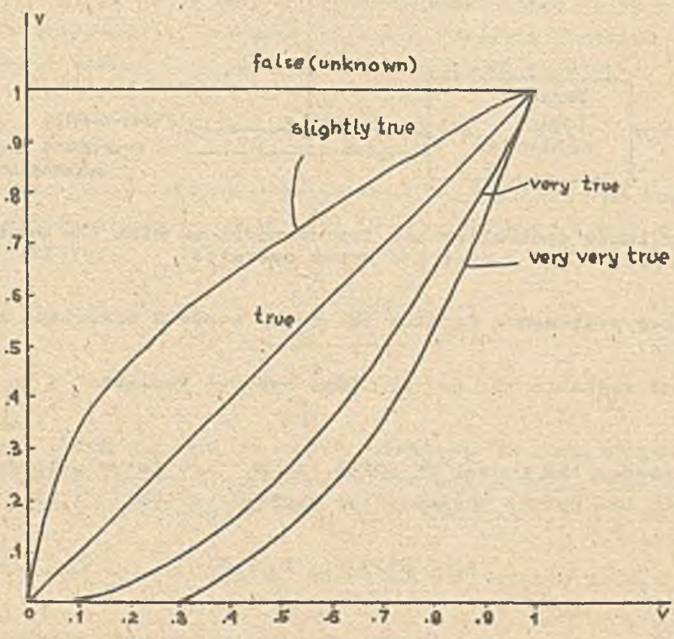

Fig. 1. Membership function of the term "true" and results of linguistic hedges used for it

For the basic truth value defined by (15) $\tau_n(n\to\infty)$ tends to one-point membership function (total true), $\tau_{-n}(n\to\infty)$ tends to the total false (false in two valued logics). The result of these calculations is illustrated by Fig. 1.

3. The method of fuzzy control

Now we will shortly present the method of fuzzy control [6,7]. The basis of it is formed by a collection of implication statements which casually link the input and output of the controlled process (where the input and output are expressed as linguistic variables, with a fuzzy set representation), and may be treated as a formalization of the control rules used by a skilled operator.
The connections betweeen the three main components, of the system, process-skilled operater-fuzzy logic controller, are schematically represented in Fig. 2.

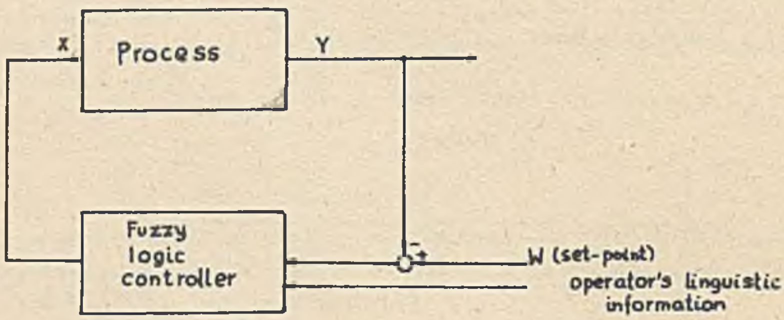

Fig.2. Fuzzy logic controller and its connections with the controlled process and human operator

Implication statements forming the fuzzy control algorithm are as follows:
If the output variable Y is A_i, then control variable X is B_i

$$i = 1,2,\ldots,N$$

where N denotes the number of rules, A_i B_i are fuzzy sets defined on the spaces of the output variable and control variable i.e.

$$A_i \in F(\mathcal{Y}), \quad B_i \in F(\mathcal{X})$$

For the purpose of computer implementation, the spaces $\mathcal{X}$ and $\mathcal{Y}$ are considered to be discrete ones

$$\mathcal{X} = \{x_1, x_2, \ldots, x_L\} \qquad \mathcal{Y} = \{y_1, y_2, \ldots, y_M\}$$

Basing on these rules, which form a mathematical formalization of experience, the knowledge and skill of human operator, we can evaluate the value of the control variable for a given numerical or linguistic value of the output variable using the equation:

$$B' = A' \circ R \tag{16}$$

$$R = \bigcup_{i=1}^{N} A_i \times B_i \tag{17}$$

o stands for max-min operator and

$$\mu_{B'}(x) = \bigvee_{y \in \mathcal{Y}} [\mu_{A'}(y) \wedge \mu_R(y,x)] \quad \forall_{x \in \mathcal{X}} \tag{18}$$

where $\vee$, $\wedge$ denotes max and min operators respectively.
The final value of control x_o is estimated from the obtained membership function $\mu_{B'}(x)$ of fuzzy set B'. One obvious method is to choose the value which corresponds to the peak of the membership function or to compute the average value if $\mu_{B'}(x)$ is not a unimodal function:

$$\mu_{B'}(x_o) = \max_{1 \le i \le L} \mu_{B'}(x_i) \tag{19}$$

Another method is to form an average based on the shape of the membership function (centre of area method [5]) i.e. x_o chosen in such a way satisfies the condition:

$$x_o = \frac{\sum_{i=1}^{L} \mu_{B'}(x_i) x_i}{\sum_{i=1}^{L} \mu_{B'}(x_i)} \tag{20}$$

The method of fuzzy control is widely discussed and the computer implementation problem has been also solved in the papers of Tong and Mamdani. Implication statements discussed by these authors have a slightly different form:

if the error variable e ist C_i then control variable is B_i

$i = 1, 2, \ldots, N$

which is in fact, taking into account relationship

$$e = w - y = f(w,y) \tag{21}$$

identical to that presented above. For given A_i and W, C_i may be obtained as

$$C_i = W - A_i$$

where - denotes an algebraic operation of the fuzzy sets W and A_i

$$\mu_{C_i}(e) = \sup_{(w,y)\in f^{-1}(e)} [\mu_W(w) \wedge \mu_{A_i}(y)] \tag{22}$$

or in a very special case when W is a fuzzy singleton with the membership function $\mu_W(w) = \delta_{w, w_o}$

$$\mu_{C_i}(e) = \mu_{A_i}(w_o - e) \tag{23}$$

The following algorithm might be extended in a natural way to a multidimensional control problem.

The above considerations are proper as a base of control algorithms under two assumptions:

- the spaces of the output and control variables are completely "covered" by fuzzy sets A_i and B_i i.e.

$$\forall_{y_i} \exists_j \mu_{A_j}(y_i) \neq 0 \tag{24}$$

$$\forall_{x_i} \exists_j \mu_{B_j}(x_i) \neq 0 \tag{25}$$

- the control rules are formed with respect to one or several noncompetitive criteria.

In order to illustrate the importance of the second assumption let us set the following example, considering two statements of fuzzy control as seen from different points of view:

rule no 1.

if Y is big then X is - big,

stated with respect to the criterion of accuracy-keeping up the value of the output variable on the desired level with high accuracy,

rule no 2.

if Y is big then X is zero,

which is created with respect to the minimal energy criterion

The terms: big, -big, zero are fuzzy sets with the membership functions:

	y_1	y_2	y_3	y_4	y_5
μ_{big}	0	.2	.5	.8	1

	x_1	x_2	x_3	x_4	x_5	x_6
μ_{-big}	1	.6	.3	.2	0	0
μ_{zero}	0	.5	.8	1	.6	0

R computed as $R = R_1 \cup R_2$ is equal to:

$$\mu_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ .2 & .2 & .2 & .2 & .2 & 0 \\ .5 & .5 & .5 & .5 & .5 & 0 \\ .8 & .6 & .5 & .5 & .5 & 0 \\ 1 & .6 & .8 & 1 & .6 & 0 \end{bmatrix}$$

When the output variable is less than big and given by the membership function

	y_1	y_2	y_3	y_4	y_5
$\mu_{less\ than\ big}$	.2	.2	.8	1	.9

the algorithm gives a solution in the form:

	x_1	x_2	x_3	x_4	x_5	x_6
$\mu_{B'}$	.9	.6	.9	.9	.9	0

where B' is very much fuzzified and is not satisfactory neither from the accuracy point of view nor economy (minimal control values of control variable).

4. A fuzzy control algorithm based on fuzzy Łukasiewicz logic

Let us assume we have N control rules in the form:

- if the output variable is A_i then input control variable is B_i.

The truth value of each control rule denoted by R_i specifies the degree of importance and reliability assigned to it. Thus for every output variable treated as a fuzzy set $A' \subset \mathcal{F}(\mathcal{Y})$ the truth value of the propositions

$$A_i \text{ is } \tau_{A_i} = A' \tag{26}$$

according to (12) is evaluated as

$$\mu_{\tau_{A_i}}(v) = \mu_{A_i}(\mu_{A'}^{-1}(v)) \tag{27}$$

$$i = 1,2,\ldots,N$$

If $\mu_{\tau_{A_i}}(v)$ and $\mu_{R_i}(v)$ are given, then using (5) the linguistic truth value $\mu_{\tau_{B_i}}(v)$ can be derived. In this way the input (control) variable asserted by the i-th rule is equal to:

$$\mu_{B_i'}(x) = \mu_{\tau_{B_i}}(\mu_{B_i}(x)) \tag{28}$$

and as a final result B' which takes into account the possibility of existence of competitive criteria we state:

$$\mu_{B'}(x) = \bigcap_{i=1}^{N} \mu_{B_i'}(x) \tag{29}$$

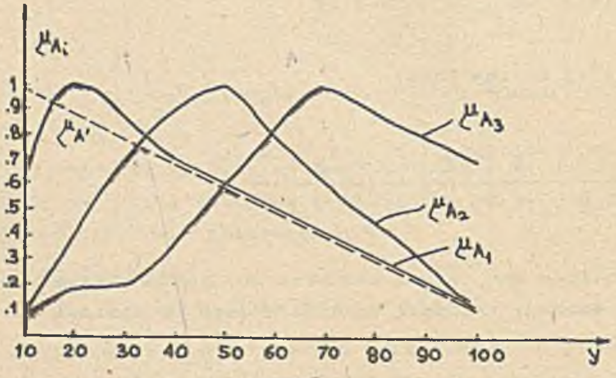

Fig. 3. Fuzzy sets of the output variable A_i

Now, introducing a performance index we investigate the properties of the described algorithm, pointing out its most characteristic features. Let the control algorithm consist of 3 implication statements:

$$\text{if } Y \text{ is } A_i \text{ then } X \text{ is } B_i \quad i = 1,2,3 \tag{30}$$

where A_i, B_i are fuzzy sets defined on the spaces $\mathcal{Y}$ and $\mathcal{X}$ respectively $A_i \subset \mathcal{F}(\mathcal{Y})$, $B_i \subset \mathcal{F}(\mathcal{X})$ with the membership functions depicted in Fig 3 and 4. The truth value R of each control rules has been assumed in the form given by eq (15).

The output variable A' treated as a fuzzy set defined on Y is expressed by its membership function $\mu_{A'}(y)$ which is also depicted in Fig 3. Using the algorithm described above we obtain results which are clearly shown in Fig 5. It is useful to introduce a measure of uncertainty of every rule with respect of the output variable A'. We propose to use the following grade of fuzziness:

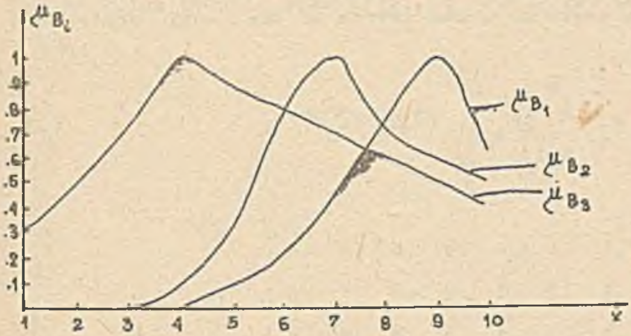

Fig 4. Fuzzy sets of the input control variable B_i

$$\nu(B_i') = \frac{1}{\operatorname{Sup}\mu_{B_i'}(x)} \sum \mu_{B_i'}(x) \tag{31}$$

which in the case of the normal fuzzy set B_i' could be called a cardinal number of fuzzy set B_i'.

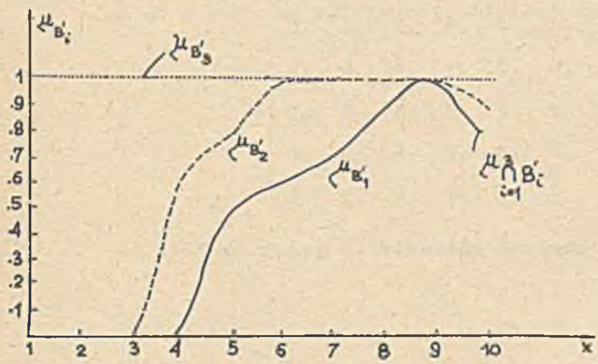

Fig 5. Result of a fuzzy control algorithm ($\mu_{R_i}(v) = V$, for every control rule)

Hence if $\nu(B_2')$ grows this fact may be interpreted as a decrease of the informational value of the considered rule e.g. the third rule in above example with respect to A' is quite useless. This leads to the estimation of the quality of the control algorithm with respect to an assertion of the control rules. The design problem (or verification problem) of a fuzzy controller may be stated as a construction problem of control rules satisfying the following condition:

$$\forall_{A \in \mathcal{F}(Y)} \quad \exists_{1 \leq i \leq N} \quad \nu(B_i') < \varepsilon \tag{32}$$

where ε - given positve number,

i.e. in every situation the algorithm should be sufficiently informative.

It is interesting to compare the obtained results with those given by the algorithm described in section 3. The matrix of the fuzzy controller defined by the expression:

$$\mu_R(y_i, x_j) = \bigvee_{k=1}^{N} [\mu_{A_k}(y_i) \wedge \mu_{B_k}(x_j)] \tag{33}$$

has the following form:

$$\mu_R = \begin{bmatrix} .1 & .1 & .1 & .1 & .1 & .2 & .5 & .6 & .6 & .6 \\ .2 & .2 & .2 & .2 & .3 & .4 & .5 & .8 & 1 & .6 \\ .2 & .2 & .2 & .2 & .3 & .7 & .7 & .8 & .9 & .6 \\ .3 & .4 & .4 & .4 & .4 & .8 & .9 & .7 & .7 & .6 \\ .3 & .5 & .6 & .6 & .6 & .8 & 1 & .7 & .6 & .6 \\ .3 & .5 & .7 & .8 & .8 & .8 & .8 & .7 & .6 & .5 \\ .3 & .5 & .7 & 1 & .9 & .8 & .7 & .6 & .6 & .5 \\ .3 & .5 & .7 & .9 & .9 & .8 & .7 & .6 & .5 & .5 \\ .3 & .5 & .7 & .8 & .8 & .8 & .7 & .6 & .5 & .4 \\ .3 & .5 & .7 & .7 & .7 & .7 & .7 & .6 & .5 & .4 \end{bmatrix}$$

Thus for A' as in Fig 3 the control variable is given in Fig. 6.

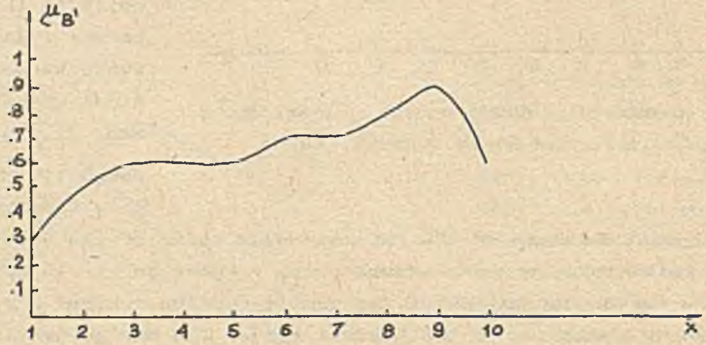

Fig. 6. Result of a fuzzy control algorithm

The shape of the fuzzy set of the input (i.e. control variable) is similar to that depicted in Fig. 5 although an analysis similar to that described above is impossible.

We examine now the effect of the variation of the importance of one of the control rules which is described by varying the linguistic truth value of R_i. For this purpose we fix the truth values of the first and third rules changing the reliability of the second one, taking into consideration sequentially $\mu^2_{R_2}$, $\mu^3_{R_2}$, $\mu^4_{R_2}$, $\mu^5_{R_2}$ which is equivalent to the increasing truth value of this rule (Fig. 7). The grade of fuzziness of B_2' decreases (Fig. 8), clearly pointing out the increasing importance of the considered rule. Thus in this case analysing the values of the grade of fuzziness one might clearly point out the influence of the concrete truth value of the considered control rule upon the final result.

If the condition (12) is not satisfied the upper and lower value of control variables can be computed basing on eqs (13) and (14). Such control (fuzzy sets) may be treated as optimistic and pessimistic controls. For A′ given in Fig. 9 using (13), (14) the upper and lower control variables are illustrated in Fig. 10a and 10b, which gives and idea on the range of control value adequate in this situation.

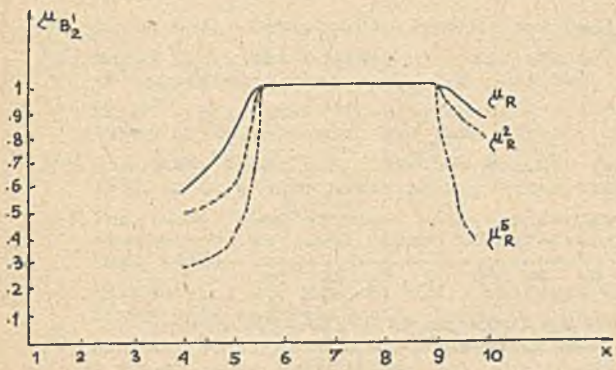

Fig. 7. Result of a fuzzy control algorithm in the case of a varying reliability of the control rule

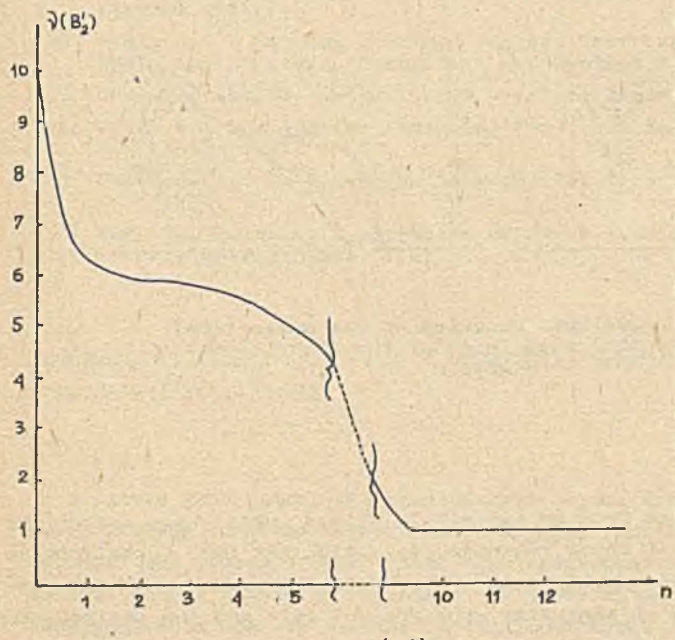

Fig. 8. Grade of fuzziness (B_2') of a fuzzy control set versus the varying importance of the second control rule

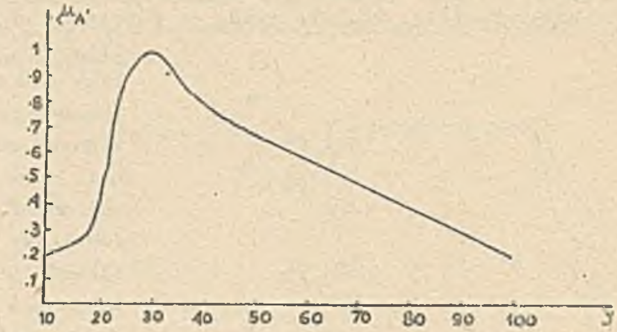

Fig. 9. Membership function of the output fuzzy set (variable) A´

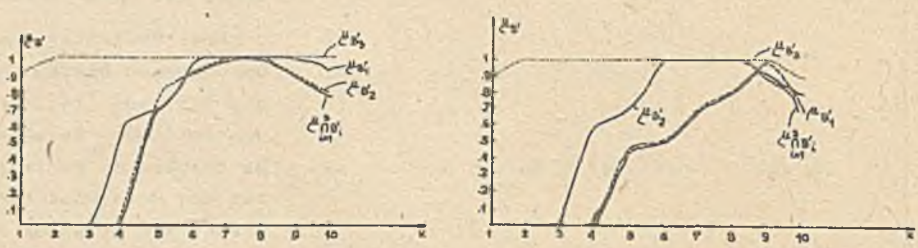

Fig. 10. Membership function of the upper (10a) and lower (10b) fuzzy sets of the control variable B´

5. Conclusion

A fuzzy logic derived from an infinitely valued Lukasiewicz logic may be treated as a means of fuzzy reasoning essential for the formalization and solution of decision problems in the case of ill-defined complex processes. The facilities of analysing each control rule and the easiness of modifying truth values of every one of them represented by a grade of fuzzines is interesting from the designing point of view. Morever, since the control algorithm can be used as a part of software for man-machine interactive system, it is important to prepare a sufficient tool which would make it possible to understand the notion of linguistic truth values and its circumenstances (e.g. the sensitivity of the control algorithm).

REFERENCES

[1] Zadeh L.A.: Fuzzy sets, Inform. and Control 8, 1965.

[2] Zadeh L.A.: The concept of a linguistic variable and its application to approximate reasoning, Inf. Sci 8-9, 1975.

[3] Zadeh L.A.; Tanaka K.S.Fu, Shimura M.: Fuzzy Sets and their Applications Academic Press, New York, 1975.

[4] van Nauta Lemke H.R., Kickert W.J.M.: The application of fuzzy controller in a warm water plant, Automatica 12, 1976.

[5] Tong R.H.: Some problems with the design and implementation of fuzzy controlers. Internal report CUED/F-CAMS/TR 127, Cambridge Engineering Dept 1976.

[6] Kickert W.J.M., Mamdani E.H.: Analysis of a fuzzy logic controller, Fuzzy Sets and Systems 1, 1978.

[7] Mamdani E.H., Assilian S.: An experiment in linguistic synthesis with a fuzzy logic controller, Int. J. Man-Machine Studies 7, 1975.

[8] Gaines B.R.: Foundations of fuzzy reasoning, Int. J.Man-Machine Studies 8, 1976.

[9] Rosser J.B., Turquette A.R.: Many - valued logics, North Holland,Amsterdam 1952.

[10] Skala H.J.: On many - valued logics, fuzzy sets, fuzzy logics and their applications, Fuzzy Sets and Systems 2, 1978.

[11] Computer science and multiple - valued logic, North Holland 1978.

[12] Giles R.: Łukasiewicz logic and fuzzy set theory, Int. J.Man-Machine Studies 8, 1976.

[13] Kaufmann A.: Introduction to Fuzzy Set Theory, Academic Press, New York 1975.

[14] Negoita, Ralescu: Application of fuzzy sets to systems analysis,Birkhauser-Verlag, Basel, 1975.

РАСПЛЫВЧАТАЯ ЛОГИКА ЛУКАСЕВИЧА В ПРОЦЕССАХ ПРИНЯТИЯ РЕШЕНИЯ В РАСПЛЫВЧАТЫХ СИСТЕМАХ

Р е з ю м е

В работе дано применение расплывчатой логики Лукасевича в вопросах принятия решения особенно касающихся управления сложными производственными процессами. Дан основной метод управления включающий правила управления, различной степени важности. Проведено сравнение метода с методами управления базирующими на правиле выводов. Приведен числовой пример.

ROZMYTA LOGIKA ŁUKASIEWICZA W PROCESACH PODEJMOWANIA DECYZJI W SYSTEMACH ROZMYTYCH

S t r e s z c z e n i e

W pracy przedstawiono zastosowanie rozmytej logiki Łukasiewicza w problemach podejmowania decyzji, w szczególności w zagadnieniach sterowania złożonymi procesami przemysłowymi. Została podana podstawowa metoda ste-

rowania, ujmująca reguły sterowania o różnym stopniu ważności. Dokonano też porównania niniejszej metody z metodami sterowania opartymi na złożeniowej regule wnioskowania. Ilustracją niniejszych rozważań są przykłady numeryczne.