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ON IDENTIFICATION IN FUZZY SYSTEMS AND ITS APPLICATIONS IN CONTROL PROBLEMS

Summary: The problem of identification in fuzzy systems described by the use of fuzzy equation is considered. The identification method and its performance index is also presented. The formal procedure of identification algorithm is illustrated by means of a numerical example. The possibility of use of the proposed algorithm for the solution of control problem is given as well.

1. Introduction

In the case of widespread methods of describing industrial processes as for example the input-output description, it has been assumed that there exists a functional relation between the input variables and the output variables of the process. In order to determine such an accepted mathematical model of the process, statistical methods are widely used, e.g. linear regression, stochastic approximation and also correlational analysis [1,2]. The model thus obtained forms the basis of control processes, usually by means of a digital computer.

In the case of many complex processes control algorithms may be set up, using qualitative and quantitive information concerning the given object and basing on the theory of fuzzy sets [3] In such cases a model which takes into account the input-output relations is more adequate than the description applied above; in the later model the input and output variables are treated as fuzzy sets, whereas the relations existing between them are described by means of a fuzzy relation.

The present paper formulates the problem of identification for such a description; its solution has been provided and some questions strictly connected with it are being discussed.

Taking into account the simultaneous identification and control it is possible to obtain an adaptive system, which has been illustrated in the paper, too.

2. Statement of the problem

Let us consider the fuzzy system described by means of a fuzzy relation R, whose behaviour (temporal evolution) is given by the fuzzy equation:

$$X_{k+1} = X_k \circ R \tag{1}$$

where X_k, X_{k+1} are element of the class $F(\mathcal{X})$ i.e. $X_k, X_{k+1} \in F(\mathcal{X})$ $R \in F(\mathcal{X} \times \mathcal{X})$ and $\circ$ stands for maxmin operation [4].
In equation (1) X_k, X_{k+1} are treated as fuzzy sets describing the state of a fuzzy system in instant time moments, the fuzzy relation R represents the relationships existing in the system. Using the concept of membership functions, equation (1) can be expressed in the following form:

$$\mu_{X_{k+1}}(x) = \bigvee_{y \in \mathcal{X}} [\mu_{X_k}(y) \wedge \mu_R(y,x)] \tag{2}$$

where μ_{X_k}, $\mu_{X_{k+1}}$, μ_R denote the membership functions of the fuzzy sets X_k, X_{k+1} and the fuzzy relation R respectively, $\vee$, $\wedge$ stand for Max and Min operators.
For the sake of convenience we can rewrite (1) in the form:

$$Y_k = X_k \circ R$$

thus the relation R transforms X_k into X_{k+1}, denoted here by Y_k.
The identification problem for fuzzy systems given by equation (1) is to estimate the unknown fuzzy relation R describing the considered system by means of an appropriate sequence of input and output "measurements" represented by the fuzzy sets X_k and Y_k for $k = 1,2,\dots K$ x).

3. Solution of a fuzzy equation

Before solving the fuzzy equation (1) let us introduce α-operation [5,6] defined as follows:

DEFINITION 1.

For every $A \in F(\mathcal{X})$ we define the α-operations as:

$$\mu_A(x) \alpha \mu_A(y) = \begin{cases} 1 & \text{if } \mu_A(x) \le \mu_A(y) \\ \mu_A(y) & \text{if } \mu_A(x) > \mu_A(y) \end{cases} \quad \forall_{x,y \in \mathcal{X}} \tag{3}$$

x) Such a problem statement is in its main idea similar to an active experiment.

We can define the $\textcircled{\alpha}$- compositions of the fuzzy set and fuzzy relation in the following way:

$$B = A \textcircled{\alpha} R \qquad R \in F(X + X) \qquad A,B \in F(X)$$

where B has its membership function in the form:

$$\mu_B(x) = \bigvee_{y \in X} [\mu_A(y) \alpha \ \mu_R(x,y)] \qquad \forall_{x \in X} \tag{4}$$

Similarly we can define the $\textcircled{\alpha}$-composition of two fuzzy sets:

$$G = A \textcircled{\alpha} B \qquad A,B \in F(X)$$

as the fuzzy relation $G \in F(X + X)$ with the membership function:

$$\mu_G(x,y) = \mu_A(x) \alpha \ \mu_B(y) \qquad \forall_{x,y \in X} \tag{5}$$

Let us point out a few useful properties of the α-operation which next form the base of lemmas and theorem connected with the solution of the fuzy equation:

$$\mu_A(x) \wedge (\mu_A(x) \alpha \ \mu_A(y)) \leqslant \mu_A(y) \tag{6}$$

$$\mu_A(x) \alpha \ \mu_A(y) \geqslant \mu_A(y) \tag{7}$$

$$\mu_A(x) \alpha \ (\mu_A(x) \alpha \ \mu_A(y)) \geqslant \mu_A(y) \tag{8}$$

$$\mu_A(x) \alpha \ (\mu_A(y) \vee \mu_A(z)) \geqslant \mu_A(x) \alpha \ \mu_A(y) \qquad \forall_{x,y,z \in X} \tag{9}$$

The truthfulness of the three first relations is obvious. Let us verify the inequality (9), considering the following cases:

a) $\mu_A(x) \leqslant \mu_A(y)$ and $\mu_A(x) \leqslant \mu_A(z)$ hence we have:

$$\mu_A(x) \alpha \ (\mu_A(y) \vee \mu_A(z)) = 1$$

b) $\mu_A(x) > \mu_A(y)$ and $\mu_A(x) > \mu_A(z)$ thus:

$$\mu_A(x) \alpha \ (\mu_A(y) \vee \mu_A(z)) = \mu_A(y) \vee \mu_A(z) \geqslant \mu_A(x) \alpha \ \mu_A(y) = \mu_A(y)$$

c) $\mu_A(y) \leqslant \mu_A(x)$ and $\mu_A(x) \leqslant \mu_A(z)$ which leads to $\mu_A(x) \alpha \ (\mu_A(y) \vee \mu_A(z)) = 1$

Now the following lemmas are evident:

LEMMA 1.

For every $X_k \in F(\mathcal{X})$ and $R \in F(\mathcal{X} \times \mathcal{X})$ we have:

$$R \subset X_k \circledast (X_k \circ R) \tag{10}$$

LEMMA 2.

For every $X_k, X_{k+1} \in F(\mathcal{X})$:

$$X_k \circ (X_k \circledast X_{k+1}) \subset X_{k+1} \tag{11}$$

We can prove the following theorem:

THEOREM 1.

For the fuzzy equation $X_{k+1} = X_k \circ R$, $X_k, X_{k+1} \in F(\mathcal{X})$, $R \in F(\mathcal{X} \times \mathcal{X})$ the least upper bound relation $\hat{R} \in F(\mathcal{X} + \mathcal{X})$ satisfying equation (1) is given by the following equation:

$$\hat{R} = X_k \circledast X_{k+1} \tag{12}$$

Proof. From Lemma 1 we obtain

$$R \subset X_k \circledast X_{k+1} \quad \text{i.e.} \quad R \subset \hat{R}$$

We also have $X_k \circ R \subset X_k \circ \hat{R}$ which is equivalent to $X_{k+1} \subset X_k \circ \hat{R}$

From Lemma 2 follows $X_k \circ \hat{R} \subset X_{k+1}$, thus $\hat{R}$ satisfies the equation $X_{k+1} = X_k \circ \hat{R}$.

4. Identification method in the case of finite space $\mathcal{X}$

Validation of an estimated model by the use of a performance index and truth qualification method.

For many practical purposes the space $\mathcal{X}$ could be considered as finite i.e. $\mathcal{X} = \{x_1, x_2, \ldots, x_N\}$ therefore the membership functions of fuzzy sets X_k, Y_k and R could be treated as vectors and a matrix respectively:

$$\mu_{X_k} = [\mu_{X_k}(x_1) \; \mu_{X_k}(x_2) \; \ldots \; \mu_{X_k}(x_N)] = [\mu_{X_k}(x_i)] \quad i = 1,2,\ldots,N$$

$$\mu_{Y_k} = [\mu_{Y_k}(x_1) \; \mu_{Y_k}(x_2) \; \ldots \; \mu_{Y_k}(x_N)] = [\mu_{Y_k}(x_i)] \quad i = 1,2,\ldots,N$$

$$R = \begin{bmatrix} \mu_R(x_1,x_1) & \mu_R(x_1,x_2) & \dots & \mu_R(x_1, x_N) \\ \mu_R(x_2,x_1) & & \dots & \\ \vdots & & & \\ \mu_R(x_N,x_1) & & \dots & \mu_R(x_N,x_N) \end{bmatrix} = [\mu_R(x_i,x_j)]_{i,j=1,2,..N}$$

For the identification purpose a set of "measurements" - fuzzy sets $\{X_k\}_{k=1,2,...,K}$ and $\{Y_k\}_{k=1,2,...,K}$ are given.

Using formula (12) for every pair of fuzzy sets, we calculate $\hat{R}_k = X_k \textcircled{\alpha} Y_k$, taking as the final result the relation:

$$\bar{R} = \bigcap_{k=1}^{k} \hat{R}_k \tag{13}$$

i.e. $\mu_{\bar{R}}(x_i,x_j) = \underset{1 \le k \le K}{\text{Min}} \; \mu_{\hat{R}_k}(x_i,x_j) \qquad i,j = 1,2,...,N$

Now we introduce the following definitions which will be useful in the considerations of the performance index of identification.

DEFINITION 2

We call a fuzzy set $A \in F(\mathcal{X})$, where card $(\mathcal{X}) = n$, k - normal, $k\in[1,N]$ iff

$$\mu_A(x_k) = 1 \tag{14}$$

DEFINITION 3

The degree of fuzziness of the normal fuzzy set is a non-negative number

$$\varphi_A = \sum_{i=1}^{N} \mu_A(x_i) \tag{15}$$

DEFINITION 4

l - normal and m - normal fuzzy sets are called independent iff $l \neq m$. Generally l_m - normal fuzzy sets $m = 1,2,..,K$ are mutually indapendent if $l_1 \neq l_2 \neq l_3,...,l_K$.

As the performance index of the identification procedure we can use every metric $\rho_{R,\bar{R}}$, especially the Hamming distance between R and $\bar{R}$ which the form:

$$\rho_H(R, \hat{R}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \left| \mu_R(x_i,x_j) - \hat{R}(x_i,x_j) \right| \tag{16}$$

The following theorem gives a sufficient condition for choosing the fuzzy sets $\{X_k\}_{k=1,2,...,K}$ minimizing the performance index of indentification.

THEOREM 2.

If k = 1,2,... exists, the following conditions are satisfied:

$$(i) \quad X_k' \subseteq X_k$$

$$(ii) \; \mu_{X_k}(x_i) \leq \mu_{Y_k'}(x_j) \qquad i,j = 1,2,\ldots,N$$

then

$$\varrho_{R,\hat{R}'} \leq \varrho_{R,\hat{R}} \tag{17}$$

$$\hat{R}' = X_k' \textcircled{\alpha} Y_k' \qquad \hat{R} = X_k \textcircled{\alpha} Y_k$$

Proof. A.a. Let us assume that the following conditions are satisfied:

$$\exists_{i_0, j_0} \quad \mu_{X_k'}(x_{i_0}) \leq \mu_{Y_k'}(x_{j_0}) \quad \mu_{X_k}(x_{i_0}) > \mu_{Y_k}(x_{j_0})$$

Hence $\mu_{Y_k'}(x_{j_0}) \geq \mu_{X_k}(x_{i_0})$ and $\mu_{X_k}(x_{i_0}) > \mu_{Y_k}(x_{j_0})$

So $\mu_{Y_k'}(x_{j_0}) > \mu_{Y_k}(x_{j_0})$ which leads to a contradiction with (i).

Let us now illustrate the proposed method by a numerical example.
We have the fuzzy relation R defined by the following matrix:

$$R = \begin{bmatrix} 1 & .8 & .9 & .6 & .5 & .7 \\ 0 & .3 & .5 & 1 & .2 & 0 \\ .7 & .9 & 1 & .8 & .6 & 0 \\ .2 & 1 & .7 & .5 & .4 & .2 \\ 0 & 0 & .1 & .2 & 1 & 0 \\ 0 & .2 & .3 & .5 & .6 & 1 \end{bmatrix}$$

and the fuzzy measurements of input and output of the fuzzy system $\{X_k\}$, $\{Y_k\}$ k = 1,2,...,6 are as follows:

$\{X_k\}$						$\{Y_k\}$					
[1	.3	.2	.1	0	0]	[1	.8	.9	.6	.5	.7]
[.3	1	.2	.1	0	0]	[.3	.3	.5	1	.3	.3]
[.2	.3	1	.1	0	0]	[.7	.9	1	.8	.6	.3]
[.1	.2	.3	1	0	0]	[.3	1	.7	.5	.4	.2]
[0	.1	.2	.3	1	0]	[.2	.3	.3	.3	1	.2]
[0	0	.1	.2	.3	1]	[.2	.2	.3	.5	.6	1]

$\varphi_{X_k} = 1.6$

Using formula (13) we obtain:

$$\mu_{\hat{R}} = \begin{bmatrix} .1 & .8 & .9 & .6 & .5 & .7 \\ .3 & .3 & .5 & 1 & .3 & .2 \\ .7 & .9 & 1 & .8 & .6 & .2 \\ .2 & 1 & .7 & .5 & .4 & .2 \\ .2 & .2 & .3 & .3 & 1 & .2 \\ .2 & .2 & .3 & .5 & .6 & 1 \end{bmatrix}$$

Similarly, using fuzzy data:

$$\begin{array}{l} [1 \;\; .6 \;\; .5 \;\; .3 \;\; .2 \;\; .1] \\ [.6 \;\; 1 \;\; .5 \;\; .3 \;\; .2 \;\; .1] \\ [.5 \;\; .6 \;\; 1 \;\; .3 \;\; .2 \;\; .1] \\ [.3 \;\; .5 \;\; .6 \;\; 1 \;\; .2 \;\; .1] \\ [.2 \;\; .3 \;\; .5 \;\; .6 \;\; 1 \;\; .1] \\ [.1 \;\; .2 \;\; .3 \;\; .5 \;\; .6 \;\; 1] \end{array} \qquad \begin{array}{l} [1 \;\; .8 \;\; .9 \;\; .6 \;\; .5 \;\; .7] \\ [.6 \;\; .6 \;\; .6 \;\; 1 \;\; .5 \;\; .6] \\ [.7 \;\; .9 \;\; 1 \;\; .8 \;\; .6 \;\; .5] \\ [.6 \;\; 1 \;\; .7 \;\; .6 \;\; .6 \;\; .3] \\ [.5 \;\; .6 \;\; .6 \;\; .5 \;\; 1 \;\; .2] \\ [.3 \;\; .5 \;\; .5 \;\; .5 \;\; .6 \;\; 1] \end{array} \qquad \varphi_{X_k} = 2.7$$

$$\vdots$$

$$\begin{array}{l} [1 \;\; .9 \;\; .9 \;\; .9 \;\; .9 \;\; .9] \\ [.9 \;\; 1 \;\; .9 \;\; .9 \;\; .9 \;\; .9] \\ [.9 \;\; .9 \;\; 1 \;\; .9 \;\; .9 \;\; .9] \\ [.9 \;\; .9 \;\; .9 \;\; 1 \;\; .9 \;\; .9] \\ [.9 \;\; .9 \;\; .9 \;\; .9 \;\; 1 \;\; .9] \\ [.9 \;\; .9 \;\; .9 \;\; .9 \;\; .9 \;\; 1] \end{array} \qquad \begin{array}{l} [1 \;\; .9 \;\; .9 \;\; .9 \;\; .9 \;\; .9] \\ [.9 \;\; .9 \;\; .9 \;\; 1 \;\; .9 \;\; .9] \\ [.9 \;\; .9 \;\; 1 \;\; .9 \;\; .9 \;\; .9] \\ [.9 \;\; 1 \;\; .9 \;\; .9 \;\; .9 \;\; .9] \\ [.9 \;\; .9 \;\; .9 \;\; .9 \;\; 1 \;\; .9] \\ [.9 \;\; .9 \;\; .9 \;\; .9 \;\; .9 \;\; 1] \end{array} \qquad \varphi_{X_k} = 5.5$$

and calculating the Hamming distance between the identified relation $\hat{R}$ and the relation R with respect to the degree of fuzziness, we obtain the following results:

φ_{X_k}	1	1.6	2.7	4	4,7	5.1	5.5
$\varphi_H(R, \hat{R})$	0	1.9	4.5	8.8	11.1	12.5	15.6

Thus taking for identification the minimal identification set of independent fuzzy measurements equal to N, we see that $\varrho_H(R,\hat{R})$ is an increasing function of φ_{X_k}. For a special case when $\varphi_{X_k} = 1$ i.e. the minimal degree of fuzziness, for such series of fuzzy sets:

$$\begin{array}{l} [1 \;\; 0 \;\; 0 \ldots \;\; 0 \;\; \ldots \;\; 0] \\ [0 \;\; 1 \;\; 0 \qquad\qquad\quad 0] \\ \qquad\quad \vdots \\ [0 \;\; 0 \;\; 0 \qquad 0 \qquad\; 1] \end{array}$$

we obtain $\varrho_{R,\hat{R}}$ equal to zero.

Now using each of the obtained models and a concrete input fuzzy set we can prove their validity.
Let us take as an input measurement the fuzzy set defined by the membership function:

$$\mu_X = [.5 \quad .6 \quad 1 \quad .4 \quad .2 \quad 0]$$

In this case Output fuzzy set is given as:

$$\mu_Y = [.7 \quad .9 \quad 1 \quad .8 \quad .6 \quad .5]$$

Bor the above relations the Hamming distance between the output fuzzy set of an object and the model is shown on the fig. 1. It is covenient to express the validity of a fuzzy model in terms of fuzzy linguistic truth, considering the following statement:

"the output is Y" is τ, which is equivalent to:

"the output is $\hat{Y}$".

So $$\mu_\tau(v) = \sup_{x_i \in \mu_Y^{-1}(v)} \mu_{\hat{Y}}(x_i) \qquad (18)$$

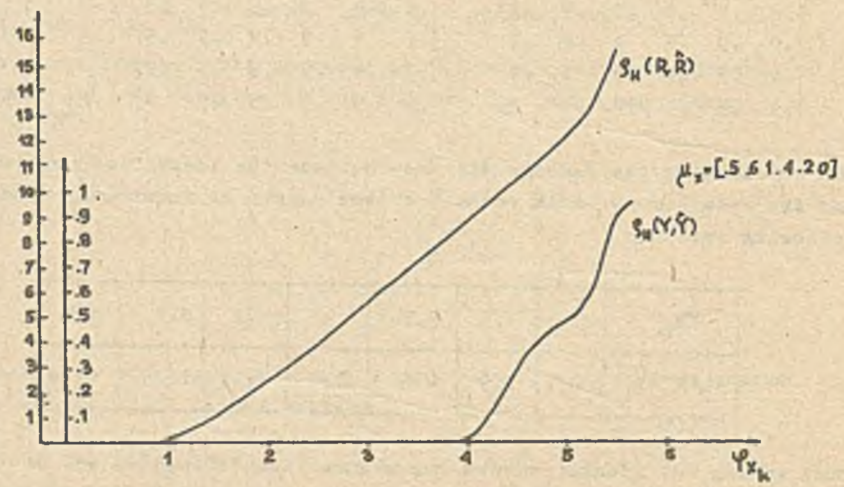

Fig. 1. The Hamming distance between the model and the process vs. degree of fuzziness of identifying fuzzy sets φ_{X_k}

where the fuzzy set τ defined on $v = [0,1]$ stands for the linguistic truth value:

(19)

We assume a model of the term "truth" is given by the fuzzy set, in the case of the discretized space $V = \{0, .1, .2 \ldots .1\}$, with the membership function:

v	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$\mu_{true}(v)$	0	0	0	0	0	.5	.6	.7	.8	.9	1

Next evaluation such n_0 which minimizes the distance:

$$\underset{n}{\mathrm{Min}} \sum_{i=1}^{11} \left| \mu_{\tau}^{n}(v_i) - \mu_{true}(v_i) \right| = \sum_{i=1}^{11} \left| \mu_{\tau}^{n_0}(v_i) - \mu_{true}(v_i) \right| \tag{20}$$

the truth value by using linguistic hedges of each model is stated. For example, for some values of the performance index we get:

model 1 $\quad \varrho_H(R,\hat{R}) = 11.1, \; n = .7$

model 2 $\quad \varrho_H(R,\hat{R}) = 12.5, \; n = .5$

model 3 $\quad \varrho_H(R,\hat{R}) = 15.5, \; n = .2$

the validity of each of them is expressed as:

τ(model 1) = about true

τ(model 2) = more or less true

τ(model 3) = about false

and the logical hedges are constitued according to the well known rules [7,3].

5. The use of the identification method in control problems

Let us consider now the process described by the use of the fuzzy equation:

$$Y_k = X_k \circ R$$

where R is an unknown fuzzy relation describing the system. The control problem is to choose the proper fuzzy input X_{opt} in order to obtain a given $Y_{opt}(k)$ (the process is assumed to be controlled i.e. $\forall_k \exists_{X_{opt}} \; Y_{opt}(k) = X_{opt} \circ R$ is satisfied). For this purpose let us use the iteration procedure with the starting point $\mu_{\hat{R}_0}(x_i, x_j) = 1$, which corresponds to the meaning of total indeterminacy.

1. k=1.
2. Compute an input X_k equal to $X_k = R_{k-1} \circledcirc\!\!\alpha \; Y_{opt}(k)$ and use it as a fuzzy control.

3. Estimate the fuzzy relation $\hat{R}_{k+n_0}$, using the set of measurements of fuzzy input and output X_{k+l}, Y_{k+l}, $l = 1,2,\ldots, n_0$

$$\hat{R}_{k+n_0} = \bigcap_{j=1}^{k+n_0} \hat{R}_j$$

4. Go to 2.

The convergent character of this procedure is worth to noticed. The method coult be slightly modified and used for processes described by the fuzzy equation for example having the form:

$$X_{k+1} = X_k \circ R_{u_i(k)}$$

where $u_i(k) \in \mathbb{U}$, $\mathbb{U}$ denotes the control space.

6. Concluding remarks

The idea of the identification of fuzzy systems and the identification method in the case of finite space $\mathcal{X}$ and its use for control problems are the main results of the paper.
It is based on the concept of the solution of a special type of fuzzy equations. The fundamental feature of such an approach is a close connection between the states of a physical process and the state sets by which its fuzzy representation is defined. The paper introduces also the performance index of identification. A numerical example clearly demonstrates the mechanism of the solution emphasizing the easiness of the proposed algorithms. A deeper analysis of identification and control problems will be the subject of the next papers.

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ИДЕНТИФИКАЦИЯ В РАСПЛЫВЧАТЫХ СИСТЕМАХ ОПИСАННЫХ УРАВНЕНИЯМИ ЗАВИСИМОСТЕЙ И ЕЁ ПРИМЕНЕНИЕ К ПРОБЛЕМАМ УПРАВЛЕНИЯ

Р е з ю м е

В работе рассмотрена проблема идентификации в расплывчатых системах описанных расплывчатыми уравнениями зависимостей. Дан метод идентификации и характеризирующий его указатель качества. Алгоритм идентификации иллюстрирован численным примером. Показана тоже возможность использования результатов идентификации к проблемам управления.

IDENTYFIKACJA W SYSTEMACH ROZMYTYCH (OPISANYCH RÓWNANIAMI RELACYJNYMI) I JEJ ZASTOSOWANIE DO PROBLEMÓW STEROWANIA

S t r e s z c z e n i e

W pracy rozważano problem identyfikacji w systemach rozmytych opisanych rozmytymi równaniami relacyjnymi. Przedstawiono metodę identyfikacji i wskaźnik jakości ją charakteryzujący. Algorytm identyfikacji zilustrowano przykładem numerycznym. Zaprezentowano też możliwość wykorzystania wyników identyfikacji do problemów sterowania.