ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1980

Seria: AUTOMATYKA z. 53 Nr kol. 649

Ernest CZOGAŁA, Witold PEDRYCZ

Silesian Technical University

FUZZY SET THEORY AS A BASIS FOR COMPUTER AIDED FAULT DIAGNOSTICS

Summary. The paper deals with the method of the relational equations allowing for formal description of relationship between the symptoms $X \in F(\mathcal{X})$ and the faults $Y \in F(\mathcal{Y})$ in systems expressed as values of linguistic variables.

The considered equations have a following form:

$$Y = X \circ R,$$

where: $X \in F(\mathcal{X})$, $Y \in F(\mathcal{Y})$

The method of estimation of the fuzzy relation R and the condition of existence of the solution have been discussed. The numerical example makes up an illustration of the considerations of this paper.

1. Introduction

An early detection of abnormal states of the process, device (or system) is very important in any case. Some symptoms are detected not automatically e.g. by automatic detectors, but by a human operator because of his experience and skill.

In order to estimate the possible faults by using a linguistc kind of information, the methods of fuzzy set theory are employed [1,2,3,6,7]. The main purpose of this work is to construct a framework of a fault-diagnosing method, using the mathematical tool mentioned above.

2. Statement of the problem

For formalizing the problem we discuss the following. Let us introduce the notion:

$\mathcal{X}$ - the space of symptoms,

$\mathcal{Y}$ - the space of faults.

The relationship between symptoms of abnormal work of the process device and possible faults are given by the fuzzy relation R defined on the Cartesian product of $\mathcal{X}$ and $\mathcal{Y}$ i.e. $R \in F(\mathcal{X} \times \mathcal{Y})$ [2]. For every diagnosing procedure the set of symptoms (Fuzzy Pattern of Symptoms) treated as a fuzzy set $X \in F(\mathbb{X})$ is given and the faults (Fuzzy Pattern of Faults) defined as a fuzzy set $Y \in F(\mathbb{Y})$ must be chosen.

Generally in given symptoms-faults relational description the space of the condition of the environment, which we assign as $\mathbb{Z}$ should be taken into account as well. So we define the relation R as

$$R \in F(\mathbb{X} \times \mathbb{Y} \times \mathbb{Z}) \quad R \in F(\mathbb{X} \times \mathbb{Y}) \times P(\mathbb{Z}) \tag{1}$$

or

$$R \in F(\mathbb{X} \times \mathbb{Y}) \tag{2}$$

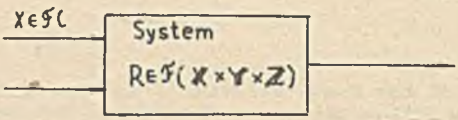

Fig. 1. Fuzzy system in variable conditions of environment $(Z \in F(\mathbb{Z})$ or $Z \in P(\mathbb{Z}))$

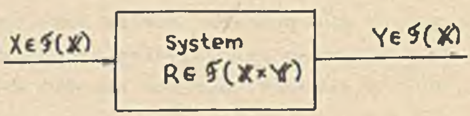

Fig. 2. Fuzzy system in fixed conditions of environment

where $F(\mathbb{X})$, $P(\mathbb{Y})$ stand for the family of fuzzy sets and ordinary sets, respectively, defined in the space $\mathbb{X}$. The notion introduced in such a way may be illustrated as in fig. 1 and fig. 2.

Thus fuzzy diagnosing system is described by the use of the fuzzy relational equations [4]:

$$Y = X \circ Z \circ R \tag{3}$$

or, when μ_Z is equal to δ_{z,z_0}, in the simplified form:

$$Y = X \circ R \tag{4}$$

Fuzzy relational equation (3) could be rewritten in the form:

$$\mu_Y(y) = \bigvee_{x \in \mathbb{X}} [\mu_X(x) \wedge \bigvee_{z \in \mathbb{Z}} \{\mu_Z(z) \wedge \mu_R(x,y,z)\}] \quad \forall_{y \in \mathbb{Y}} \tag{5}$$

where $\bigvee$, $\bigwedge$ stand for max and min operators, respectively, and similarly for (4) we put down:

$$\mu_Y(y) = \bigvee_{x \in \mathbb{X}} [\mu_X(x) \wedge \mu_R(x,y)] \quad \forall_{y \in \mathbb{Y}} \tag{6}$$

The equation given above could be illustrated in the Fig. 3.

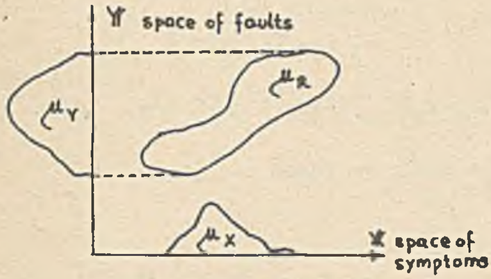

Fig. 3. Graphical illustration of fuzzy relational equation
$Y = X \circ R$

Under the assumption (accepted in every practical case) $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ have a finite number of elements:

$$\begin{aligned} \text{card}\ (\mathcal{X}) &= n_{\mathcal{X}} \\ \text{card}\ (\mathcal{Y}) &= n_{\mathcal{Y}} \\ \text{card}\ (\mathcal{Z}) &= n_{\mathcal{Z}} \end{aligned} \tag{7}$$

the membership functions of fuzzy set X could be expressed in vector form:

$$\mu_X = [\mu_X(x_i)] \quad i = 1,2,\dots,n_X \tag{8}$$

and fuzzy relation R has a matrix representation:

$$\mu_R = [\mu_R(x_i, y_j, z_l)] \quad \begin{aligned} i &= 1,2,\dots,n_{\mathcal{X}} \\ j &= 1,2,\dots,n_{\mathcal{Y}} \\ l &= 1,2,\dots,n_{\mathcal{Z}} \end{aligned} \tag{9}$$

Considering relational symptoms-faults description we present two main problems interesting at the theoretical and practical point of view. There are: estimation a symptoms-faults relation and inverse problem.

3. The method of estimating the symptmms-faults relation

For the purpose of estimating the mentioned relation a set of fuzzy symptoms and faults has been given:

$$\begin{matrix} X_1 & Y_1 \\ X_2 & Y_2 \\ \vdots & \vdots \\ X_n & Y_n \end{matrix} \tag{10}$$

We can estimate the symptoms-faults relation as:

$$\hat{R} = \bigcap_{l=1}^{n} X_l \,\textcircled{\small$\alpha$}\, Y_l \tag{11}$$

where α stand for α-operation [4] defined as

$$\mu_{X_1 \alpha Y_1}(x_i, y_m) \overset{\text{def}}{=} \begin{cases} 1, & \text{if } \mu_{X_1}(x_i) \leqslant \mu_{Y_1}(y_m) \\ \mu_{Y_1}(y_m); & \text{if } \mu_{X_1}(x_i) > \mu_{Y_1}(y_m) \end{cases} \tag{12}$$

and $R \in \hat{R}$ holds true. More details could be found in [5].

4. Inverse problem

Another problem is the task of the construction of possible symptoms which may occure when concrete faults are observed.

Using the same α-operation introduced, above, it is easy to prove, the least upper fuzzy set of symptoms, if we assume that it exists, is given as fuzzy set [4]

$$\hat{X} = R \alpha Y \tag{13}$$

with the membership function:

$$\mu_{\hat{X}}(x_i) = \bigvee_{y_j \in \mathcal{Y}} [\mu_R(x_i, y_j) \alpha \mu_Y(y_j)] \quad \forall_{x_i \in \mathcal{X}} \tag{14}$$

From the practical point of view it is interesting and important to construct not only such solution as it stated before, but the family of lower bound solutions i.e. such fuzzy sets of symptoms $X_1, X_2, \ldots, X_k$, that:

$$\forall_{x_i \in F(\mathcal{X})} \quad \text{if} \quad X_i \leqslant X \leqslant \hat{X} \quad \text{then} \quad X \circ R = Y \quad \text{holds.}$$

Now we briefly mention general results.

Let I_X I_Y define sets of values of membership functions of fuzzy sets X and Y:

$$I_X = \{\mu_X(x_1), \mu_X(x_2), \ldots, \mu_X(x_{n_{\mathcal{X}}})\}$$
$$I_Y = \{\mu_Y(y_1), \ldots, \mu_Y(y_{n_{\mathcal{Y}}})\} \tag{15}$$

The following theorem holds:

THEOREM 1.

If the solution of equation $X \circ R = Y$ exists, then exists such solution $\check{X}$ that

$$I_{\check{X}} \subset I_Y \tag{16}$$

Proof. Taking into account the assumption of theorem we get:

$$\max_{1 \le i \le n_X} [\min(\mu_X(x_i), \mu_R(x_i, y_i))] = \mu_Y(y_j) \qquad j=1,2,\dots,n_Y \tag{17}$$

Thus there exist such indexes i_j $j = 1,2,\dots,n_Y$ that:

$$\min(\mu_X(x_{i_j}), \mu_R(x_{i_j}, y_j)) = \mu_Y(y_i) \tag{18}$$

holds.

Let us denote:

$$K = \{ i_1, i_2, \dots, i_{n_Y} \} \tag{19}$$

We fix k $1 \le k \le n_Y$. Let us consider two cases:

a) $\mu_X(x_{i_k})$ $\mu_Y(y_k)$, then we get:

$$\mu_{\tilde{X}}(x_{i_k}) = \mu_Y(y_k) \in I_Y \tag{20}$$

b) $\mu_X(x_{i_k}) > \mu_Y(y_k)$, then we get:

$$\mu_{\tilde{X}}(x_{i_k}) = \max_{j \in J(k)} \mu_Y(y_j) \tag{21}$$

where

$$J(k) = \{ j : i_j = i_k \} \tag{22}$$

In both cases we obtain:

$$\min(\mu_{\tilde{X}}(x_{i_k}), \mu_R(x_{i_k}, Y_k)) = \mu_Y(y_k) \tag{23}$$

Now let $i \notin K$, $1 \le i \le n_X$ then

$$\min(\mu_X(x_i), \mu_R(x_i, y_i)) \le \mu_Y(y_j) \qquad j=1,2,\dots,n_Y \tag{24}$$

holds. Assuming

$$\mu_{\tilde{X}}(x_i) = \min_{1 \le j \le n_Y} \mu_Y(y_j) \in I_Y \tag{25}$$

we get similarly:

$$\min(\mu_{\tilde{X}}(x_i), \mu_R(x_i,y_j)) \leq \mu_Y(y_j) \quad j = 1,2,\ldots,n_Y \tag{26}$$

In such way we defined $\tilde{X} \in F(X)$, that (23) and (26) hold.

From eq. (23) we have:

$$\max_{1 \leq i \leq n_X} (\min(\mu_{\tilde{X}}(x_i), \mu_R(x_i,y_j))) =$$

$$= \max (\max_{i \in K} (\min(\mu_{\tilde{X}}(x_i), \mu_R(x_i,y_j)), \max_{i \notin K} (\min(\mu_{\tilde{X}}(x_i),$$

$$= \mu_R(x_i,y_j))) = \max (\mu_Y(y_j), \max_{i \notin K} (\min (\mu_{\tilde{X}}(x_i), \mu_R(x_i,y_j)) = \mu_Y(y_j) \tag{27}$$

according to (26). Fuzzy set $\bar{X}$ could have values of membership function created as the variation with repetitions of values of membership function of fuzzy set Y. Thus we obtain the following criterion of existence of solution of fuzzy relational equation.

CRITERION.

If there is no fuzzy set with the membership,function created as the variation with repetition of I_y, then the solution does not exist.

The following theorem describes an algebraic property of set of solutions of fuzzy relational equation.

THEOREM 2.

The set of solution of fuzzy relational equation (4) forms a commutative semigroup.

Proof. Let X, $Z \in F(X)$ satisfy the equation (4). We prove $X \cup Z$ also satisfies (4).

We have:

$$\mu_Y(y_j) = \bigvee_{i=1}^{n} [\mu_X(x_i) \wedge \mu_R(x_i,y_j)] \tag{28}$$

$$\mu_Y(y_j) = \bigvee_{i=1}^{n} [\mu_Z(x_i) \wedge \mu_R(x_i,y_j)] \tag{29}$$

so we obtain:

$$\bigvee_{i=1}^{n} [(\mu_X(x_i) \vee \mu_Z(x_i)) \wedge \mu_R(x_i,y_j)] =$$

$$= \bigvee_{i=1}^{n} [(\mu_X(x_i) \wedge \mu_R(x_i,y_j)) \vee (\mu_Z(x_i) \wedge \mu_R(x_i,y_j))] =$$

$$= \bigvee_{i=1}^{n} [\mu_X(x_i) \wedge \mu_R(x_i,y_j)] \vee \bigvee_{i=1}^{n} [\mu_Z(x_i) \wedge \mu_R(x_i,y_j)] =$$

$$= \mu_Y(y_j) \qquad \forall_{1 \leq j \leq n_Y} \tag{30}$$

Commutativity of this semigroup is a consequence of the properties of the lattice of fuzzy set [2].

Because of the fact, that the least upper bound fuzzy set is given by the formula (13), we will try to find all lower bound solutions of equation (4) as fuzzy sets with membership functions given as permutations with repetitions of $I_Y \cup \{0\}$.

5. Numerical example

Now we illustrate these considerations by a numerical example. Let us consider the electronic system which consists of N components $\{x_1, x_2, x_3, x_4\}$ (N=4) on which the space of symptoms $\mathcal{X}$ is created. The space of faults $\mathcal{Y}$ consists of three elements $\mathcal{Y} = \{y_1, y_2, y_3\}$ Taking into account the following collection of fuzzy sets of faults and symptoms:

$$\begin{array}{cc} \text{Faults} & \text{Symptoms} \\ [1\ \ 0\ \ 0\ \ 0] & [.9\ \ .5\ \ .2] \\ [0\ \ 1\ \ 0\ \ 0] & [.6\ \ .9\ \ .8] \\ [0\ \ 0\ \ 1\ \ 0] & [.4\ \ 1\ \ .5] \\ [0\ \ 0\ \ 0\ \ 1] & [.6\ \ .8\ \ 1] \end{array} \tag{31}$$

and using the method described in section 3 we obtain the following fuzzy symptoms-faults relation:

$$\mu_R = \begin{array}{c} \text{f a u l t s} \\ \begin{bmatrix} .9 & .5 & .2 \\ .6 & .9 & .8 \\ .4 & 1 & .5 \\ .6 & .8 & 1 \end{bmatrix} \end{array} \text{ s y m p t o m s} \tag{32}$$

The (i,j)-element of the matrix μ_R expresses the degree of connection between the i-th symptom and j-th fault which may occur in the system described above.

Thus having a fuzzy set of symptoms:

$$\mu_X = [.3 \quad .9 \quad 1 \quad .2] \tag{33}$$

the fuzzy set of faults can be easily computed as:

$$\mu_Y = [.6 \quad 1 \quad .8] \tag{34}$$

Similarly, when faults are given, we can obtain a fuzzy set of symptoms, which can appear in such a situation.

Fuzzy set of faults has been defined as:

$$\mu_Y = [.9 \quad .8 \quad .8] \tag{35}$$

Using the method descussed before we get the least upper fuzzy set $\hat{X}$ of symptoms: equal to:

$$\mu_{\hat{X}} = [1 \quad .8 \quad .8 \quad .8] \tag{36}$$

Lower bound solutions are the following:

$$\mu_{X_1} = [.9 \quad .8 \quad 0 \quad 0] \tag{37}$$

$$\mu_{X_2} = [.9 \quad 0 \quad 0 \quad .8] \tag{38}$$

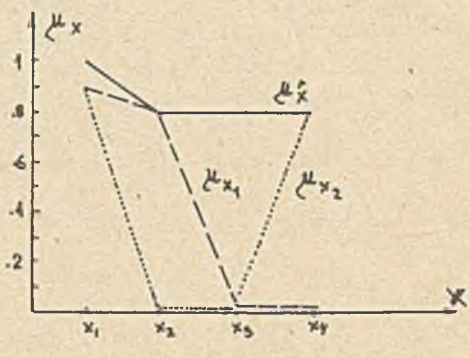

Fig. 4. Least upper solution and lower bound solutions of fuzzy equation

Y = X ∘ R

Obtained results are depicted in the Fig. 4.

We get every $X \subset F(\mathcal{X})$ such that $X_1 \subseteq X \subseteq \hat{X}$ or $X_2 \subseteq X \subset \hat{X}$ satisfies equation given above. It could be interpreted as follows: the least upper bound fuzzy set and each lower bound correspond to the worst state and the least necessary state among the possible states of symptoms that bring the same exactitudes of the faults, respectively.

4. Concluding remarks

Until now the theory of reliability has only been a tool for the security problem at the design stage of arbitrary electronical system. Practically, many damages of these systems are prevented by the suitable activity of the human operator.

The development of the multivalued logic and the theory of fuzzy sets makes it possible to built automatic preventing algorithms based on a very general kind of inexact information, namly information expressed in natural language used by the human operator.

Because of the linguistic gform of information, such algorith could form a basis of software for interactive computer systems. This paper presents an attempt of building and applying these methods given in mathematical categories of fuzzy sets.

REFERENCES

[1] Zadeh L.A.: Fuzzy sets, Inf. and Control, 8, 1965.

[2] Ralescu V., Negoita D.: Application of fuzzy sets theory to systems analysis, Birkhauser-Verlag, Basel, 1975.

[3] Computer Science and multiplevalued logic, North Holland, 1978.

[4] Sanchez E.: Resolution of composite fuzzy relation equations, Inf. and control, 30, 1976.

[5] Czogała E., Pedrycz W.: An approach to identification problem, Archiwum Automatyki i Telemechaniki (to appear).

[6] Czogała E., Pedrycz W.: Computer fault diagnostics by the use of multivalued logic and fuzzy set theory, II Int. Conf. on Fault Tolerant Systems and Diagnostics, Brno 1979.

[7] Terano T., Tsukamoto Y.: Diagnosis of engine trouble by fuzzy logic, VII Triennal Congress IFAC, Helsinki 1978.

ТЕОРИЯ РАСПЛЫВЧАТЫХ МНОЖЕСТВ КАК ОСНОВАНИЕ ДИАГНОСТИКИ ОТКАЗОВ НА БАЗЕ ПРИМЕНЕНИЯ ЭВМ

Р е з ю м е

В работе представлено метод уравнений зависимостей дающими зависимости между последствами $X \in F(\mathcal{X})$ и отказами $Y \in F(\mathcal{Y})$ в устройствах. Зависимости эти выражаются в виде значений в лингвистических переменных расплывчатых множеств . Уравнение имеет вид

$$Y = X \circ R$$

где:

$X \in F(\mathcal{X}) \qquad Y \in F(\mathcal{X})$

Дано решение проблем определения матрицы зависимости R, а также рассмотрено вопрос решения этого уравнения даны условия существования решения и проанализировано алгебраические особенности класса решений .

TEORIA ZBIORÓW ROZMYTYCH JAKO PODSTAWA WSPOMAGANEJ KOMPUTEROWO DIAGNOSTYKI USZKODZEŃ

S t r e s z c z e n i e.

W pracy przedstawiono metodę równań relacyjnych pozwalających na formalne ujmowanie zależności pomiędzy objawami $X \in F(\mathcal{X})$ i uszkodzeniami $Y \in F(\mathcal{Y})$ w układach (urządzeniach), a wyrażonych w postaci wartości zmiennych lingwistycznych (zbiorów rozmytych).
Rozważane równanie ma postać:

$$Y = X \circ R,$$

gdzie:

$X \in F(\mathcal{X})$ i $Y \in F(\mathcal{Y})$.

Ponadto rozwiązanie problemów wyznaczania macierzy relacji R, jak i rozpatrzono zagadnienie rozwiązania powyższego równania (m.in. przedstawiono warunek istnienia rozwiązania i rozważano algebraiczne własności klas rozwiązań).

Przykład numeryczny stanowi ilustrację przedstawionych metod rozwiązywania równań relacyjnych.