Distortion evaluation of wavelet multiband decomposition of EEG signals

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Abstract – In this paper a cascade multiband filtration method based on a wavelet filters is described. This method can be an alternative solution to the traditional band-pass filters. Exemplary decomposition of EEG signal with evaluation of the filtration distortions is also included.

I. INTRODUCTION

Analysis of brain EEG signals is a part of nowadays non-invasive medical diagnosis and therapy. Because of complexity of the EEG signals, many analysis methods have been developed as far. Some of them are based on advanced signal analysis techniques like wavelet transform [1] or independent component analysis [2]. In many cases it is purposeful to divide the signal to some characteristic bands: delta (0.3-3Hz), theta (4-7Hz), alpha (8-13Hz), beta(14-30Hz) [5] before the main analysis is done. These bands are connected with physiological brain activity. Sometimes it is also desirable to filter the signal at the preprocessing stage to limit the signal frequency range (for example: 0.3-30 Hz) [3]. The medical diagnosis in most cases is based on the analysis of the wave shape [2] (for example: location and magnitude of spikes, characteristic frequencies), so filtration have to be done carefully to not introduce any significant distortions to the shape of the signal. Classical IIR (Butterworth, elliptic) and linear phase FIR filters dedicated to achieve such a filtration were evaluated in [6]. In this paper a method of distortion evaluation have been also proposed. Here, a concept of multiband signal decomposition using a cascade of wavelet filters is presented. Distortions of selected wavelet filters were also evaluated.

II. WAVELET TRANSFORM AND SUBBAND CODING

Wavelet transform is one of the multiresolution signal analysis methods. Continuous wavelet transform can be defined as follows[4]:

$$CWT\{x(t)\} = \int_{-\infty}^{\infty} x(t)\Psi^*\left(\frac{t-\tau}{s}\right)dt, \tag{1}$$

where:
CWT{$x(t)$} - continuous wavelet transform of signal $x(t)$,
$\Psi\left(\frac{t-\tau}{s}\right)$ - transform kernel (wavelet basis),
s - scale, τ - position.

The transform kernel is generated from a single function called mother wavelet $\Psi(t)$ using the following formula:

$$\Psi\left(\frac{t-\tau}{s}\right) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right), \tag{2}$$

so the kernel functions are generated by shifting (position changes - τ) and stretching or compressing (scale changes - s) the mother wavelet.
Making a substitution:

$$s = s_0^j, \tag{3}$$

$$\tau = k \cdot s_0^j \cdot \tau_0, \tag{4}$$

where:
j - scale index, k - position index, $j,k \in N$,
and changing the integral in (1) into sum:

$$DWT_{j,k}\{x[i]\} = \sum_i x[i] \cdot \Psi_{j,k}[i], \qquad i,j,k \in N, \tag{5}$$

where:
$DWT_{j,k}$ - discrete wavelet transform of time series x[i],
x[i] - discrete time series,
$\Psi_{j,k}$ - discrete wavelet basis,
j - scale index, k - position index.

Equation (5) defines a discrete version of the wavelet transform. In practical applications very popular is transform with s_0=2, because this value of s_0 parameter eliminates transform redundancy. Transform based on parameter s_0=2 is called a transform on a dyadic grid. There is a efficient algorithm to compute DWT on a dyadic grid [1]. This algorithm was proposed by S. Mallat and is based on the subband coding scheme[1]. Fig. 1 presents the idea of the algorithm.

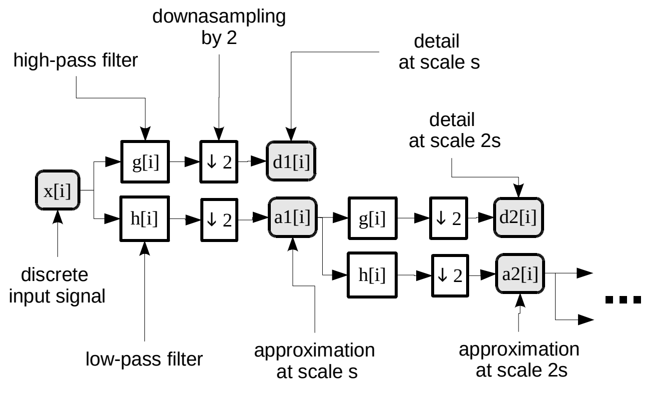

Fig. 1. Mallat algorithm of computing DWT

The Mallat algorithm uses two FIR filters: a high-pass filter g[i] and a low-pass filter h[i] to obtain wavelet coefficients at each decomposition level. The detail represents the upper half band of the signal and the approximation represents the lower half band of the signal. Filters h[i] and g[i] are directly connected with the mother wavelet [1]. This cascade filtration

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and downsampling scheme can be continued until the desired scale level is reached [1].

III. MALLAT ALGORITHM IN EEG FILTRATION

Mallat algorithm described in previous paragraph can be, after a small modifications, adopted to perform a multiband decomposition of a signal. Adopted version of the algorithm dedicated to perform EEG signal division to: delta (0.3-3Hz), theta (4-7Hz), alpha (8-13Hz) and beta (14-30Hz) band is presented in fig. 2.

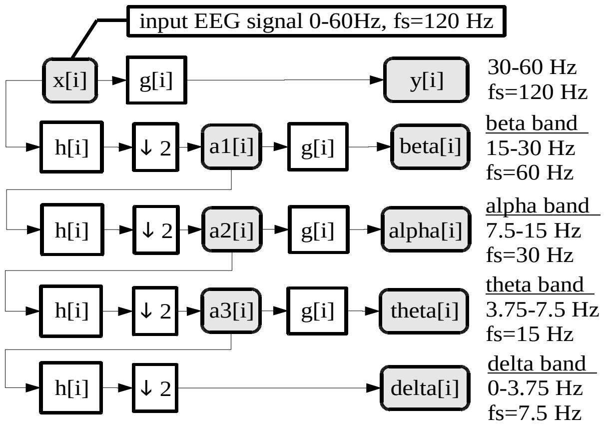

Fig. 2. Multiband EEG signal decomposition using wavelet filters and Mallat's algorithm. (fs - sampling frequency)

Comparing figures 1 and 2 can be seen, that the algorithm was changed only by removing the downsampling after the high-pass filter at each decomposition level. Analysing bands' edge frequencies can be noticed, that those from fig.2 do not match exactly the assumed ones. For example: delta band was defined as 0.3-3 Hz and in fig. 2 it has range 0-3.75Hz. The reason is that filters h[i] and g[i] are a half-band filters, so they can divide band only to two equal halves.

IV. EXEMPLARY DISTORTION EVALUATION OF WAVELET MULTIBAND DECOMPOSITION

To estimate possible distortions introduced by a wavelet filters, about one hundred wavelet filters from four wavelet families were evaluated. Distortion evaluation procedure with delay compensation described in [6] was used. This method is based on Sobolev's norm and direct comparison of signals filtered by the tested and the reference filter. In this paper the Sobolev's norm were taken up to the first derivative. Five different signals were used for testing. Signals denoted as per1 and per2 are synthetic periodic signals and signals x1, x2 and x3 are EEG signals of a healthy man. More detailed description of these signals can be found in [6]. Table 1 presents the minimal distortions obtained for each wavelet family calculated as an avarage for all five signals and both filters (low and high-pass). For Daubechies and symlet families, results when order is equal one were excluded from the table 1 (it is a Haar wavelet with linear phase filters, so D equals zero for low and high-pass filter).

Table I
MINIMAL DISTORTIONS FOR EVALUATED WAVELET FILTERS

wavelet family:	daubechies		symlet		coiflet		biorthogonal	
filter order:	5		20		5		all	
filter type:	low	high	low	high	low	high	low	high
avarage distortion measure D:	0,0376	0,310	0,0151	0,122	0,00182	0,0213	0	0

Results presented in table 1 shows, that minimal distortions for orthogonal families were reached for 5-th order coiflet wavelet. It is not a surprise, because coiflet family was especially designed to have maximal symmetry [4] (nearly linear phase). For biorthogonal wavelets, calculated distortion measure was equal zero for all wavelets. It can be explained by a phase linearity of all filters in these family. Fig. 3 presents exemplary distortion evaluation for high-pass filters.

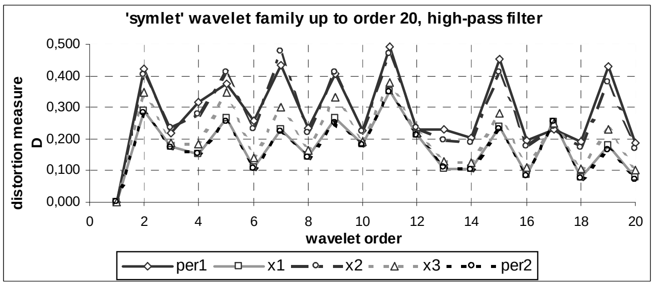

Fig. 3. Exemplary distortion evaluation results for high-pass filters from 'symlet' wavelet family.

As can be noticed in this case, measure D as a function of wavelet order is a multimodal function. Only for order 1 (Haar wavelet -linear phase filters) measure D is equal zero and besides this point the minimum is reached for order 20.

V. SUMMARY

By making just a slight modifications to the Mallat algorithm, it was possible to develop a cascade multiband filtration method, which can be alternative to the traditional band-pass filters described in [6].

Performed distortion analysis showed, that it is possible to asses and compare distortions introduced by different wavelet filters in the sense of measure D. For biorthogonal wavelet filters, because of phase linearity, distortions are equal zero. From tested orthogonal wavelet bases, minimal distortions were reached for coiflet order 5 wavelet, next are symlet wavelets and the worst results were obtained for Daubechies family.

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