Archives of Materials Science and Engineering
Volume 59
Issue 1
January 2013
Pages 40-45
International Scientific Journal published monthly by the World Academy of Materials and Manufacturing Engineering

Analysis of stress state of toothed ring of flexspline by means the BEM

P. Folęga*
Faculty of Transport, Silesian University of Technology,
ul. Krasińskiego 8, 40-019 Katowice, Poland
* Corresponding e-mail address: piotr.folega@polsl.pl

Received 30.11.2012; published in revised form 01.01.2013

ABSTRACT

Purpose: The paper presents an analysis of the influence of design features of the flexspline in a harmonic drive, such as the relative radial deformation, the relative coating thickness and the design features of the basic rack tooth profile, on stress values in the bottom lands of a toothed ring.

Design/methodology/approach: In numerical calculations, the software developed at the Faculty of Transport of the Silesian University of Technology was used. The program automatically generates a profile of the flexspline and a boundary elements mesh on the grounds of the flexspline and working tool assigned parameters. In the analysis of the state of stress, the boundary element method (BEM) was applied.

Findings: A decreased value of the curve radius of the head of the basic rack tooth profile results in a change in the width of the tooth bottom land, shortening of the transition curve in the tooth base and a reduction of the tooth thickness at its base, which in turn leads to increased values of stress. Yet, the influence of the relative curve radius of the head of the basic rack tooth profile on the stress value is insignificant. The increase of the torque and relative coating thickness for different value of the curve radius of the head of the basic rack tooth profile cause an increase of stress in the bottom lands of the toothed ring.

Research limitations/implications: The paper presents strength calculations for the teeth in toothed ring of a flexspline of a double harmonic drive by means of boundary elements method (BEM). The results of numerical calculations correspond in terms of their quality to the results presented in the literature, which were calculated by finite element method (FEM).

Originality/value: In the analysis of the state of stress of toothed ring of flexspline, the boundary element method (BEM) was applied.

Keywords: Flexspline; Harmonic drive; Boundary elements methods

Reference to this paper should be given in the following way:

P. Folęga, Analysis of stress state of toothed ring of flexspline by means the BEM, Archives of Materials Science and Engineering 59/1 (2013) 40-45.

METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

An increase of working loads transferred by harmonic drives causes an increase of forces acting on the teeth of their toothed rings. An important issue concerning the strength calculation of harmonic drives is the evaluation of the influence of stress values in the bottom lands of the toothed ring on the strength of the flexspline. As the results of empirical tests show, the bottom lands of the toothed ring are the places of reduced strength [1].

© Copyright by International OCSCO World Press. All rights reserved. 2013

A common phenomenon preceding the destruction of a flexspline is its cracking in the place of a local increase of stress values increase in the toothed ring [1]. In order to make the strength calculations and the choice of design features of a harmonic drive more specific, it is justified to use precise methods of determining stresses, which will facilitate an analysis of the influence of geometric features of toothed wheels and the basic rack tooth profile parameters on the strength. Currently, in order to calculate the stress values in the tooth base, one of the two numerical methods is used: the finite element method (FEM) or the boundary element method (BEM). The latter, in comparison to FEM, requires a considerably smaller number of computing nodes located within the tooth profile and a respectively smaller number of linear equations necessary to solve the task. In the analysis of the state of stress presented in the paper, the boundary element method (BEM) was used to determine the stress values in the bottom lands of the toothed ring of a flexspline in a harmonic drive co-operating with a mechanical cam wave generator.

2. Main elements of harmonic drive

A harmonic drive [2-9, 11, 13-15] consists of a toothed mechanism, which composed of three main elements: a rigid circular spline, an elliptical wave generator and a flexible spline, which is called the flexspline. The wave generator has a ball bearing and it is elliptical shape. The circular spline has a rigid ring with an internal toothed ring. The flexspline it is the main component of a harmonic drive, which can generate a repeated vibration by the wave generator. The wave generator deflects the elastically deformable flexspline elliptically across the major axis. Due to that, the teeth of the flexspline engage simultaneously with the ring gears of circular spline in two zones at either end of the major elliptical axis. Across the minor axis of the elliptically deflected flexspline there is no engagement. When the wave generator rotates, the meshing zones of the flexspline rotate with the generator. A difference in the number of teeth between the flexspline and the circular spline the latter has two teeth more results in a relative movement between these gear wheels. After a complete rotation of the wave generator, the flexspline moves relative to the circular spline by an angle equivalent to two teeth. Depending on the number of deformation waves, one may speak of single- or double-wave harmonic drivers.

Harmonic drivers are known of numerous advantages, but they also have certain disadvantages as compared with classical toothed transmission gears. The basic advantages include high torque on relatively small weight and compact structure, coaxiality of the drive shaft and the driven shaft, smooth operation and high kinematic accuracy, whereas the disadvantages of harmonic drivers include high elasticity and minimum gear ratio value as well as nonlinear rigidity and damping.

Flexsplines of harmonic drivers are used in various spheres of life in a more and more extensive scope of applications. They are currently used for a growing number of purposes in the automotive industry, aviation, medicine, automatic control and robotics [2].

In harmonic drivers, the external moment is transferred by means of the flexspline cyclic deformation with a wave generator causing a complex state of stress to emerge in this harmonic driver subunit. Therefore, the most heavily loaded, the weakest and the main element of toothed harmonic drivers is actually the flexspline. It is the design of this component and the choice of its material that determine the fundamental properties of the harmonic driver.

3. BEM model of flexspline

Creating parametric models of flexspline we must specify boundary conditions. This involves the problem of determining the load distribution in toothed ring gear of the flexspline and in contact zone the wave generator with the flexspline, depending by the torque. In his experiments Ivanov [3] observed that under the loading the flexspline tends to detach itself from the wave generator in two regions at some angle after the major deformation axis due to circumferential buckling under the loading. The approximate loading conditions of the flexspline and the analytical procedure for determination of the stress-deformation state of the flexspline is based on the modified theory of shells, described by Ivanov [3]. Due to its extensiveness is not given here.

Recently few researchers have successfully used the FEM to analyze the flexspline. Suvalov and Gorelov [4-5] conducted series of 2D FEM calculations of the flexspline cross-section in the region of two loaded teeth. Toropcjin [6] chose the axial cross-section for its 2D FEM calculations aiming at the optimum design of the flexspline, which is fixed on one end to the housing. However, this analysis failed to address the problem, because the flexspline is the buckling by the torque [4-7]. Kayabasi and Erzincanli [7] studied the stresses on flexspline teeth and find optimum shape of teeth to maximize fatigue life.

In this paper, the author proposed a numerical analysis of the flexspline divided into two stages. The first most important step involves a nonlinear finite element analysis of the contact between the flexspline and the wave generator [8-9]. The radial cross-section of the flexspline is analyzed in respect to the maximum displacements caused by the wave generator. Since the loading of the flexspline is not symmetrical the whole flexspline is divided into elements. The wave generator construction used in the model is opposite positioned discs. The contact between the flexspline and each of the discs is along the angle ±30º measured from the major symmetrical axis. The wave generators discs were modeled with a single line of finite elements along the discs boundaries in range ±90º from the major symmetrical axis. Inner nodes of the flexspline mesh and the outer nodes of the discs meshes were in contact along the angle ±30º, measured from the major symmetrical axis. The nodes on the potentially contacting surfaces of the flexspline and of the wave generator discs were connected with gap elements. MSC Patran/Nastran was used in calculations with FEM. The calculated radial (w) and tangential (v) displacements (Fig. 1) from the above nonlinear contact analysis were used in two-dimensional numerical models of the flexsplines with BEM. The values shown in Fig. 1 correspond qualitatively and quantitatively to the results presented in [7].

In the second step with BEM, the flexspline is being analysed in the radial cross-section, where the circumferential stresses caused by the insertion of the wave generator into the flexspline are determined. In numerical calculations, the software developed at the Faculty of Transport of the Silesian University of Technology was used [8]. The way in which the flexspline was loaded and supported as well as the numerical BEM model employed in the calculations are presented in paper [8]. The computer program to calculate coordinates of the outline of the teeth used algorithm given in [10]. The calculation was considered isolated from the ring gear teeth, ignoring the impact of three remaining teeth. As shown in [4-5,11] is a simplification of the calculation is acceptable, because it causes only a few percent error of numerical calculations of stresses in the rate of tooth ring gear. The developed algorithm uses a computer program and a program for flat BEM calculation of the deformation (strain) in the published work [12]. Program adapted to operate on IBM PC type computer in a Windows environment, allowing using to calculate all the available main memory of your computer. This allows analyzing relatively large BEM systems, without collecting data on the disk. This considerably shortened and simplified the calculation time calculation algorithm. The calculation uses the elements of the three nodes and quadratic shape functions. Since the original program published in [12] only allowed the designation of displacement boundary points, it expanded the module calculates the stresses. Such a class of teeth has been considered which can be modeled as two-dimensional boundary value problems of linear theory of elasticity. In work [8] numerical algorithm of the method has been presented and possibilities of the developed computer program serving calculation of displacements and stresses in the teeth of flexspline have been characterized. The program automatically generates a profile of the flexspline (Fig. 2a) and a boundary elements mesh on the grounds of the flexspline and working tool assigned parameters (Fig. 2b).

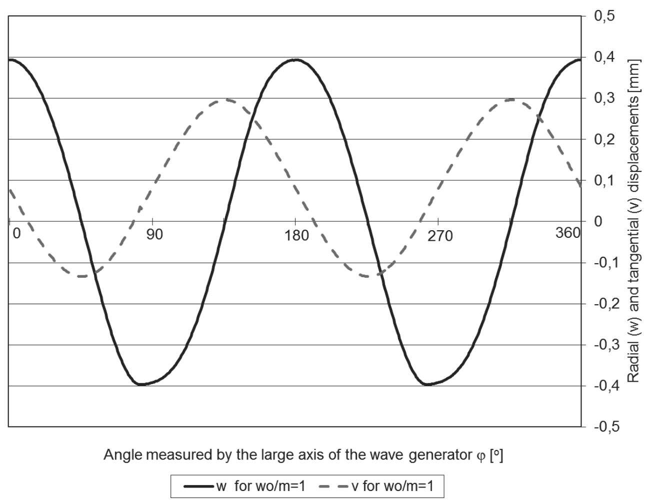


Fig. 1. Chart of radial (w) and tangential (v) displacements as a function of angle measured from the large axis of the wave generator

While choosing the flexspline material [15-18], one must consider the deformations and stresses occurring in the flexspline operating in the driver, both unloaded and loaded by the torsional moment. The heat treatment method to be applied to a spline must be determined entailing the criterion of ensuring its elastic properties as well as the service life assumed. Due to the fatigue strength and the permissible hardness necessary for the teeth to be able to cooperate and for the sake of their running-in, flexsplines are subject to the process of quenching and tempering. As a material flexspline adopted steel with the characteristics given in Table 1.

a)

b)

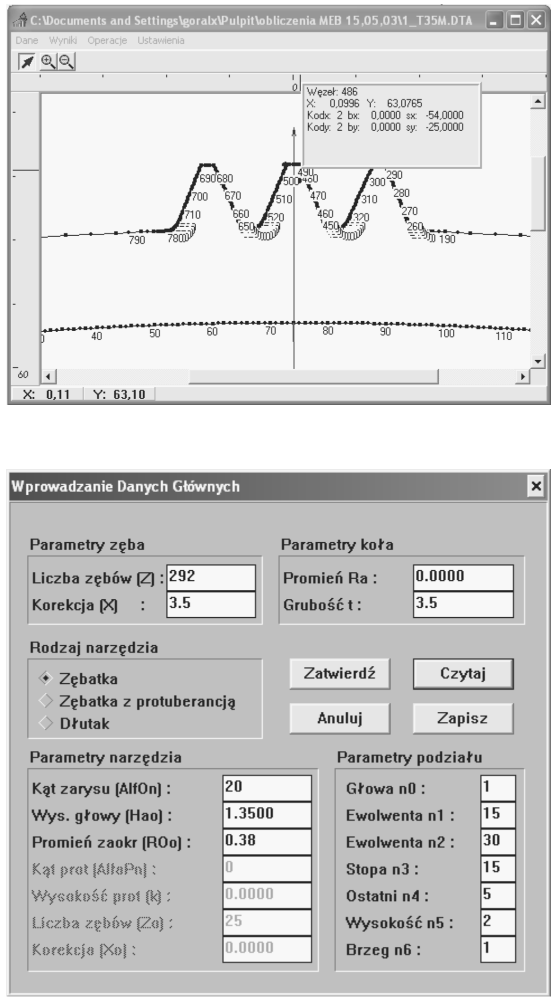


Fig. 2. The program window: a) a profile of the flexspline, b) sample data adopted in the calculations

4. Calculation results

In work [13], an analysis has been carried out regarding the effect of the adopted types of mechanical wave generators and design features of the flexspline, including the relative radial deformation w_o/m and the relative thickness g/d_f, on the distribution of stresses in the bottom land where: w_o - is the maximum radial deformation of the flexspline, m - the module, g- the flexspline thickness under the teeth, d_f - the inner diameter of the flexspline. Fig. 3 presents the maximum value of stress in the bottom land, depending on the adopted types of mechanical wave generators. With respect to the state of stress, it is advantageous to use cam or disc generators, which cause that the stress values in the bottom land are similar-respectively lower than while using a roll generator.

Table 1.
Properties of the steel 42CrMo4

Tensile modulus (GPa)	210
Shear modulus (GPa)	80
Poisson's ratio	0.3
Tensile strength (MPa)	1000
Density (kg/m^3)	7850

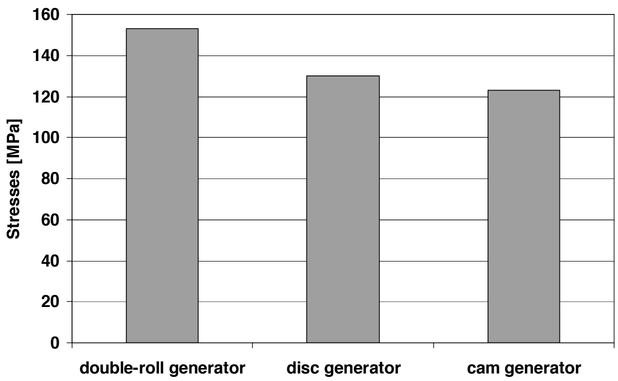


Fig. 3. The maximum value of stress in the bottom land of a tooth, depending on the type of wave generator with: g/d_f = 0.012; w_o/m = 1; M_{nom} = 0

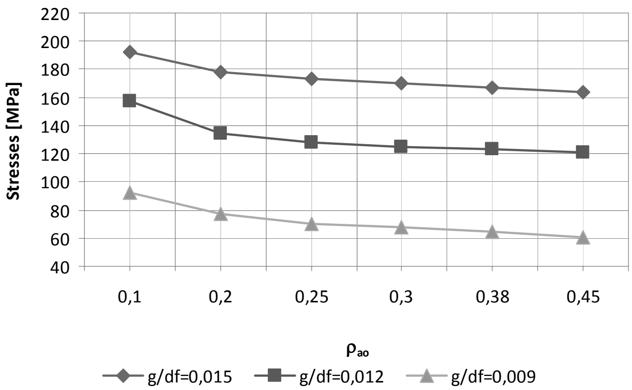


Fig. 4. The influence of the curve radius of the tool head ρ_{ao} and relative coating thickness g/d_f on value of stresses in the bottom land, with: w_0/m = 1; M_{nom}= 0

The rest of the calculation results, presented in Figs. 4 to 6, refer to the interaction of the flexspline with the cam wave generator. In addition, an analysis has been made of the influence of the selected design features of the basic rack tooth profile on the stress values in the bottom lands of the teeth in the toothed ring of a flexspline. The influence of the following relative values was also analysed: the height of the head of basic rack tooth profile h_{ao} and the curve radius of the head of basic rack tooth profile assuming the following values: $\rho_{ao} \in$ (0.1; 0.2; 0.25; 0.3; 0.38; 0.45). The teeth in toothed ring of the flexspline was made assuming the following parameters for the basic rack tooth profile: profile angle α_{on} = 20^o, relative curve radius of the head of the basic rack tooth profile ρ_{ao} = 0.1-0.45, relative height of the head of the basic rack tooth profile h_{ao} = 1.25; 1.35.

The influence on stresses of values of the curve radius of the tool head ρ_{ao}, assuming the following design features of the flexspline: the relative radial deformation w_o/m = 1 and the relative coating thickness $g/d_f \in$ (0.009; 0.012; 0,.015) is presented in Fig. 4. The influence of the torque and relative coating thickness g/d_f on value of stresses in the bottom land of a tooth, assuming the following design features of the flexspline: the relative radial deformation w_o/m = 1, the curve radius of the tool head ρ_{ao} = 0.25 and ρ_{ao} = 0.38 are presented in Figs. 5 and 6.

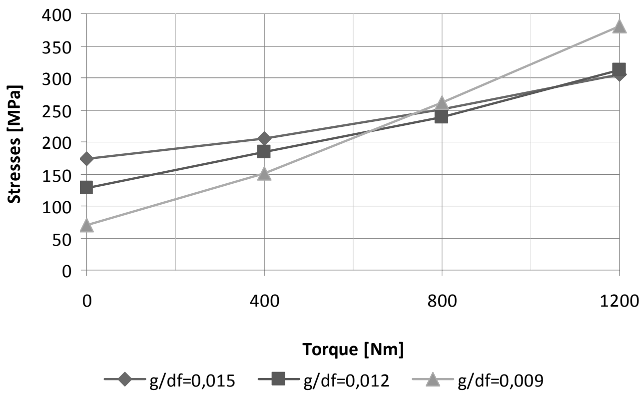


Fig. 5. The influence of the torque and relative coating thickness g/d_f on value of stresses in the bottom land of a tooth, with: ρ_{ao} = 0.25; w_0/m = 1

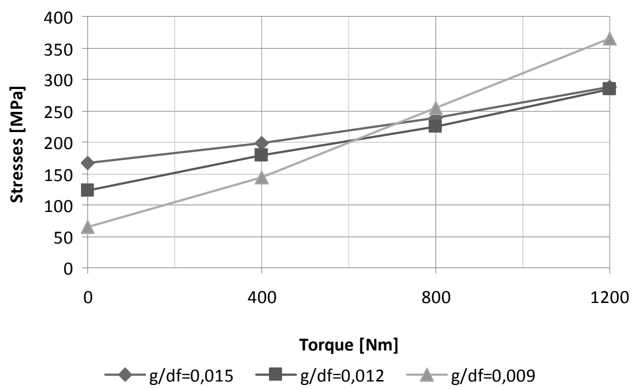
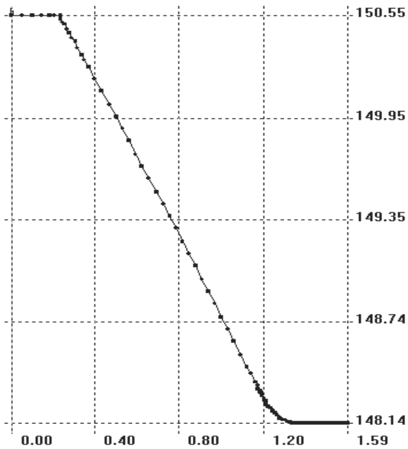
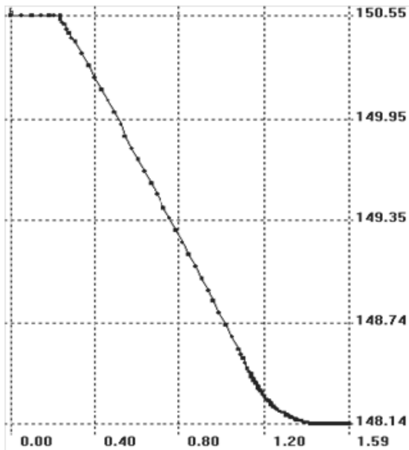


Fig. 6. The influence of the torque and relative coating thickness g/d_f on value of stresses in the bottom land of a tooth, with: ρ_{ao} = 0.38; w_0/m = 1

a)



b)



c)

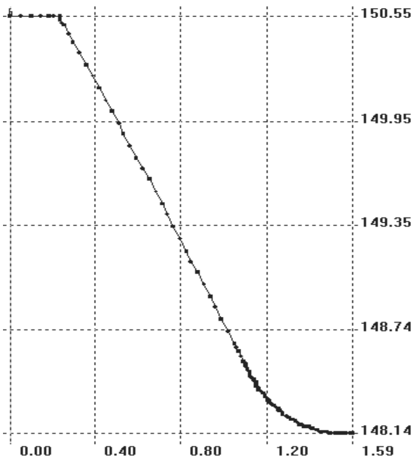
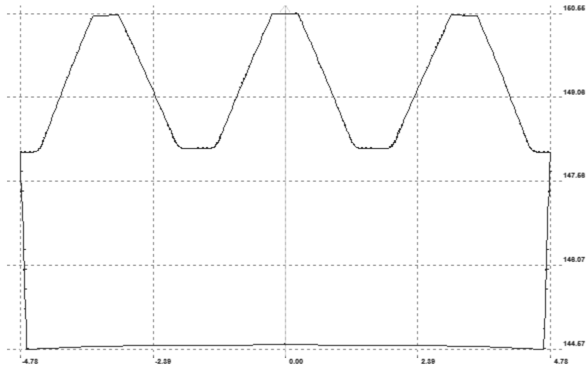
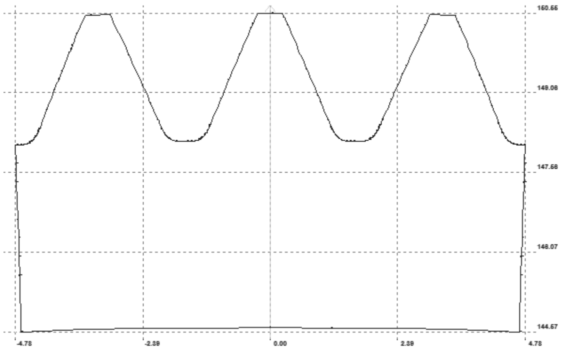


Fig. 7. The lateral contour lines of the tooth generated by tooth profile generator for: a) $\rho_{ao} = 0.1$; b) $\rho_{ao} = 0.25$; c) $\rho_{ao} = 0.38$

a)



b)



c)

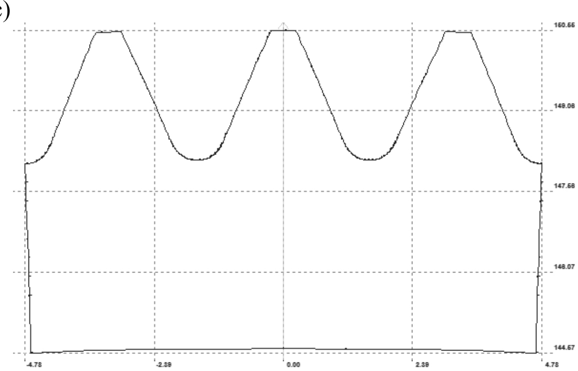


Fig. 8. The contours of the toothed ring generated by tooth profile generator for: a) $\rho_{ao} = 0.1$; b) $\rho_{ao} = 0.25$; c) $\rho_{ao} = 0.38$

5. Conclusions

The paper presents strength calculations for the teeth in toothed ring of a flexspline of a double harmonic drive by means of boundary elements method (BEM). In the numerical analysis conducted, the influence was investigated of the design features of a flexspline and of the basic rack tooth profile on the value of stresses in the bottom lands of the toothed ring. The results of numerical calculations correspond in terms of their quality to the results presented in the literature [1,5-6], which were calculated by finite element method (FEM). On the basis of the analysis of the results the following conclusions can be formulated:

1. With respect to the state of stress in the bottom lands of the teeth, it is beneficial to use cam or disc wave generators (Fig. 3).
2. A decreased value of the curve radius of the head of the basic rack tooth profile ρ_{ao} results in a change in the width of the tooth bottom land, shortening of the transition curve in the tooth base and a reduction of the tooth thickness at its base (Figs. 7 and 8), which in turn leads to increased values of stress (Fig. 4). Yet, the influence of the relative curve radius of the head of the basic rack tooth profile on the stress value in range $\rho_{ao} \in (0.25\text{-}0.38)$ is insignificant (Fig. 4).
3. The increase of the torque and relative coating thickness g/d_f for different ρ_{ao} (Figs. from 4 to 6) cause an increase of stress in the bottom lands of the toothed ring.

References

[1] A. Nalepa, Stress state analysis of harmonic drive flexspline, Overview of Mechanical Engineering 11 (1976) 376-378 (in Polish).

[2] General Catalogue Harmonic Drive AG, 05.2009.

[3] M. N. Ivanov, Harmonic gear drive, Moscow, 1981.

[4] W. N. Gorelow, Stress state of the toothed ring of harmonic drives flexible gear, Soviet Engineering Research 5 (1979) 7-10.

[5] S. A. Suvalov, V. N. Gorelov, Stress in the flexible gear by finite element method, Soviet Engineering Research 3 (1983) 9-11.

[6] V. A. Toropičjin, Stress-deformation analysis of the flexible gear of harmonic drive by finite element method, Soviet Engineering Research 2 (1988) 17-21.

[7] O. Kayabasi, F. Erzincanli, Shape optimization of tooth profile of a flexspline for a harmonic drive by finite element modelling, Materials and Design 28 (2005) 441-447.

[8] P. Folęga, A. Wilk, The selection construction feature of harmonic gear drive flexspline with FEM, Overview of Mechanical Engineering 10 (2002) 31-35 (in Polish).

[9] P. Folęga, A. Wilk, Numerical analysis of flexspline with gap contact elements, Scientific Books of the Silesian Technical University, Series of Transport 41 (2000) 35-42 (in Polish).

[10] L. Müller, Gears - design, WNT, 1996 (in Polish).

[11] P. Folęga, Stress analysis of gears by means boundary elements method, Maintenance Problems 62 (2006) 157-164 (in Polish).

[12] C.A. Brebbia, J. Dominguez, Boundary elements, An introductory course, CMP and McGraw-Hill, Southampton/New York, 1989.

[13] P. Folęga, Determining stresses in bottom lands of the toothed ring of a flexspline in a harmonic drive by BEM, Overview of Mechanical Engineering 7-8 (2009) 54-58 (in Polish).

[14] W. Ostapski, Harmonic drive, University of Technology Publishing House, Warsaw, 2011 (in Polish).

[15] M. Mijał, Synthesis of toothed harmonic drive, University of Technology Publishing House, Rzeszow, 1999 (in Polish).

[16] R. Przyłucki, S. Golak, B. Oleksiak, L. Blacha, Influence of the geometry of the arrangement inductor - crucible to the velocity of the transport of mass in the liquid metallic phase mixed inductive, Archives of Civil and Mechanical Engineering 11/1 (2011) 171-179.

[17] J. Łabaj, G. Siwiec, B. Oleksiak, Surface tension of expanded slag from steel manufacturing in electrical furnace, Metalurgija 50/3 (2011) 209-211.

[18] L. Blacha, Examinations of antimon displacement speed from blister copper in the process of vacuum refining, Archives of Metallurgy and Materials 50/4 (2005) 989-1002.