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# Polar graphs and structural numbers in synthesis of active and passive mechanical systems

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**Analysis and modelling**

## ABSTRACT

**Purpose:** The main purpose of this work is the presentation analysis and synthesis of mechanical systems including passive and active elements reducing vibrations. In result of conducted synthesis was received structures and parameters of a discrete model meeting the defined requirements concerning the dynamic features of the system, in particular, the frequency spectrum. The aim of this paper is also comparison of these two methods of reduction of vibrations.

**Design/methodology/approach:** In this paper was used method of polar graphs and their relationship with algebra of structural numbers. This method is called a non-classical method. The use of such a method enables the analysis and synthesis of mechanical systems irrespective of the type and number of the elements of such a system.

**Findings:** The application of active elements to eliminate vibration enables overcoming limitations which occur if passive elements are used. Active elements give better results in case of reduction of low frequency vibrations. Presented approach simplifies the process of selecting the dynamical parameters of systems in view of their dynamical characteristics.

**Research limitations/implications:** The scope of discussion is analysis and synthesis of mechanical systems including passive and active elements reducing of vibrations, but for this type of systems, such approach is sufficient.

**Practical implications:** The practical realization of the reverse task of dynamics introduced in this work can find uses in designing of machines with active and passive elements with the required frequency spectrum.

**Originality/value:** Thank to the approach, introduced in this paper, can be conducted as early as during the designing of future functions of the system as well as during the construction of the system. Using method and obtained results can be value for designers of mechanical systems with elements reducing vibrations.

**Keywords:** Process systems design; Polar graphs; Structural numbers; Reduction of vibrations

## 1. Introduction

The use of passive systems is not satisfactory in case of broad-band frequency. The low-frequency character of vibration may result in the failure of passive vibroisolation to ensure efficient reduction of vibration or may even lead to the increase of vibration and that is why in such cases, the active reduction of vibration often replaces the passive one. The introduction of active elements to elimination of vibration enables the overcoming of limitations characteristic of passive methods such as low efficiency in the range of low frequency of input function and free vibration. A characteristic feature of the active vibration reduction is the fact, that vibration is compensated by interaction from additional sources. Active vibroisolation systems are controlled by input function.

Designer in conventional design methods of systems searches values of elements meeting specified requirements. If a given system does not meet the requirements, it needs further analysis and modification [1-9]. Such an approach can be called a method of successive trials. In case of complex systems, such an approach has been very time-consuming. By that reason it is possibility of apply non-classical design methods such as an inverse operation called ”synthesis” [1, 4, 5, 8, 10-14]. This method consists in searching for a system structure with such values of elements which meet required frequency characteristics.

## 2. Polar graphs and structural numbers in analysis and synthesis of mechanical systems

To solve the problem of reducing the vibration of mechanical system [1, 12, 14, 16-19], it is necessary to use passive or active elements reducing vibrations.

In case of use of passive elements synthesis of mechanical systems should was as following (Fig. 1).

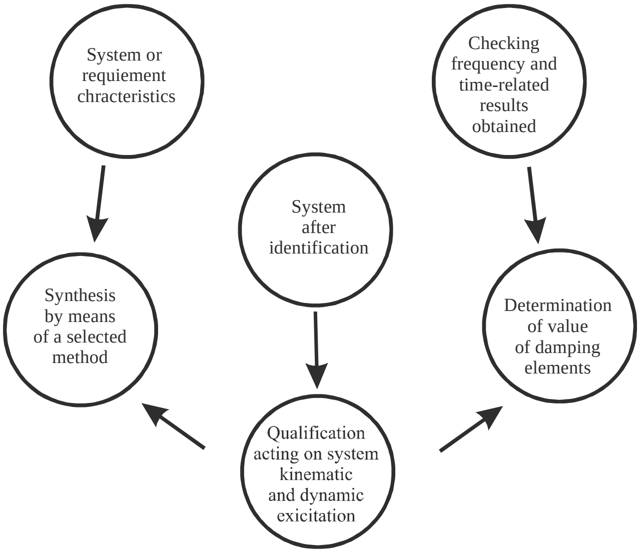



Fig. 1. Synthesis of mechanical systems with passive elements

In case of use of active elements synthesis of mechanical systems should was as following (Fig. 2).

Mechanical systems can be described at using dynamic characteristics in form of dynamic slowness and mobility [1, 12, 14], about following figures:

$$U(s) = H\,\frac{d_l s^l + d_{l-1} s^{l-2} + \ldots + d_1 s}{c_k s^k + c_{k-1} s^{k-2} + \ldots + c_0} \tag{1}$$

$$V(s) = H\,\frac{c_k s^k + c_{k-1} s^{k-2} + \ldots + c_0}{d_l s^l + d_{l-1} s^{l-2} + \ldots + d_1 s} \tag{2}$$

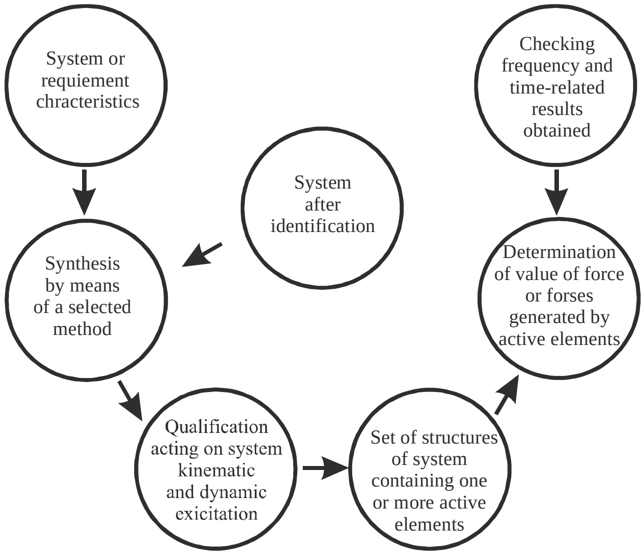



Fig. 2. Synthesis of mechanical systems with active elements

Polar graph of mechanical system of *n*-degree of freedom with kinematic, dynamic and active excitation was introduced in Fig. 3.

The above elements of polar graph (Fig. 3) are numbered according to the following standard:

$1,\ldots,n$ - edges of inertial elements,

$a_{w1},\cdots,a_{wn}$ - edges of dynamic excitations,

$a_{k1},\cdots,a_{kn}$ - edges of kinematic excitations,

$a_{a1},\cdots,a_{an}$ - edges of forces generated through active elements.

Kinematic and dynamic excitations acting on mechanical system cause dislocations and vibrating of inertial elements. Using active or passive elements it is possible to reducing this dislocations and vibrating.

Using the theory of polar graphs and their relation to structural numbers [1, 8-10, 12, 17, 20-22], it is possibility to determine the values of amplitudes of forces generated by active elements.

A general formula for amplitude value is as follows:

$$A_n = \frac{\left(\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[n]}\right)(F_1+F_{k1}+G_1)\right) + \left(\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[2]};\frac{\partial D(\omega)}{\partial[n]}\right)(F_2+F_{k2}+G_2)\right) + \ldots + \left(\left(\frac{\partial D(\omega)}{\partial[n]}\right)(F_w+F_{kl}+G_g)\right)}{D(\omega)} \tag{3}$$

where:

$D(\omega)$ - characteristic equation,

$\frac{\partial D(\omega)}{\partial[1]}$ - derivative of structural number the in relation to of edge [1],

$\underset{Z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[2]}\right)$ - function of simultaneousness of structural number,

$F_{k1},F_{k2},\dots,F_{kl}$ - kinematic excitation,

$F_1,F_2,\dots,F_w$ - dynamic excitation,

$G_1,G_2,\dots,G_g$ - forces generated through active elements.

Solving a system of equations (5) leads to the obtaining of values of individual amplitudes generated by active elements $G_1$, $G_2$, …, $G_g$.

To solve problem of reducing vibration of mechanical systems it is possible to use passive elements in form of dampers.

A general formula for value of damping [12], when damping is proportional to elastic element, is as follows:

$$b_i = \lambda c_i \tag{4}$$

where:

$b_i$ - damping elements

$\lambda$ - modulus of proportionality $\left(0<\lambda<\frac{2}{\omega_n}\right)$

$\omega_n$ - the largest value of frequency

$c_i$ - elastic elements

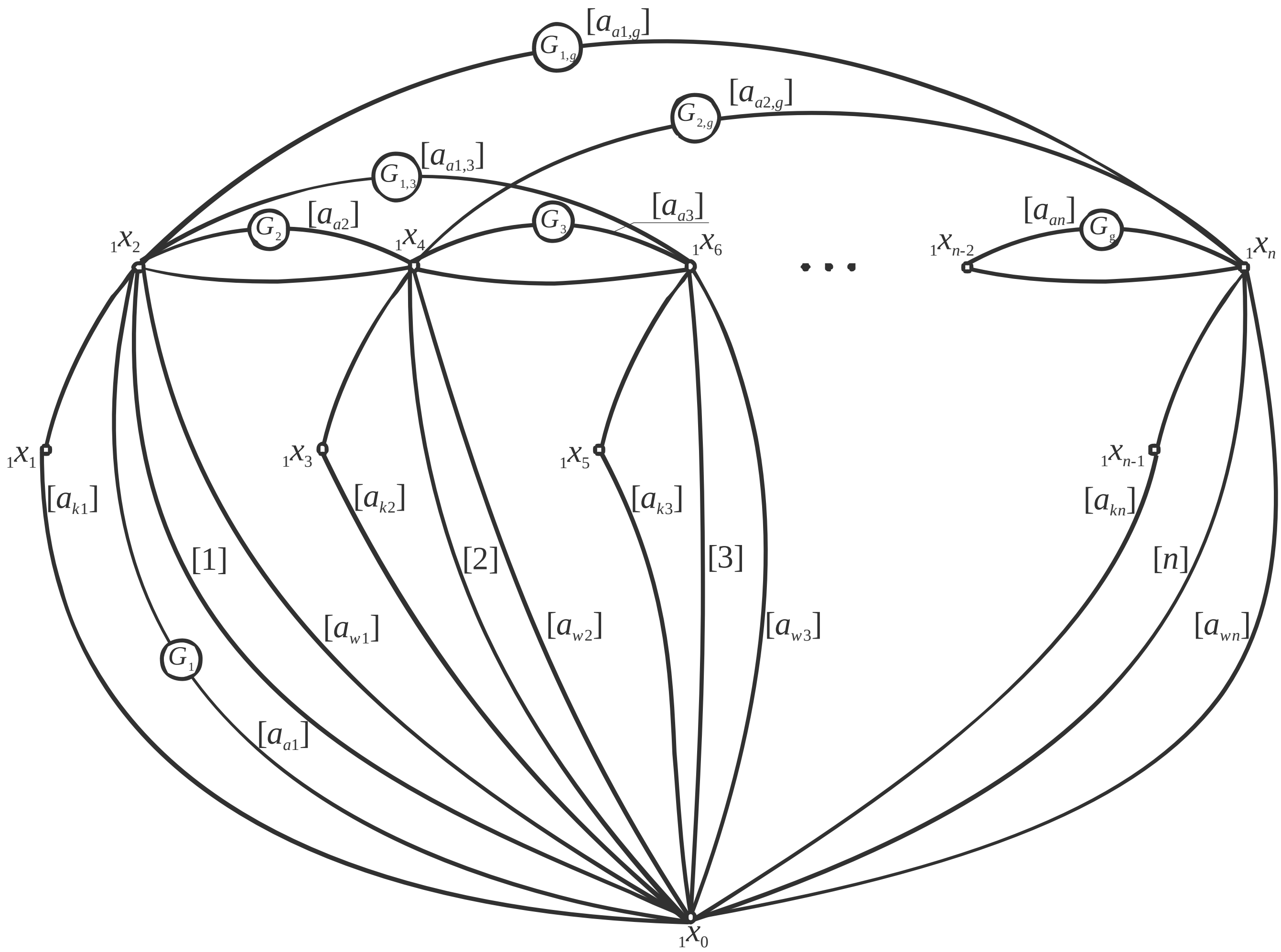



Fig. 3. Polar graph of mechanical system

$$\begin{bmatrix} \left(\frac{\frac{\partial D(\omega)}{\partial[1]}}{D(\omega)}\right) & \left(\frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[2]}\right)}{D(\omega)}\right) & \cdots & \left(\frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[n]}\right)}{D(\omega)}\right) \\ \left(\frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[2]}\right)}{D(\omega)}\right) & \left(\frac{\frac{\partial D(\omega)}{\partial[2]}}{D(\omega)}\right) & \cdots & \left(\frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[2]};\frac{\partial D(\omega)}{\partial[n]}\right)}{D(\omega)}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[1]};\frac{\partial D(\omega)}{\partial[n]}\right)}{D(\omega)}\right) & \left(\frac{\underset{z}{Sim}\left(\frac{\partial D(\omega)}{\partial[2]};\frac{\partial D(\omega)}{\partial[n]}\right)}{D(\omega)}\right) & \cdots & \left(\frac{\frac{\partial D(\omega)}{\partial[n]}}{D(\omega)}\right) \end{bmatrix} \cdot \begin{bmatrix} (F_{K1}+F_1+G_1) \\ (F_{K2}+F_2+G_2) \\ \vdots \\ (F_{Kl}+F_W+G_g) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{5}$$

## 3. Synthesis of mechanical systems

To of reduction of vibrations of mechanical systems in first step it is necessary to execute the passive synthesis or identification of system. In next step one should determine the structure of a system containing active or passive elements [2-7].

The synthesis of mechanical systems to be applied be able through distribution of characteristic function into partial fraction, continued fraction expansion or mixed method [2, 7-13, 16-18].

The required frequency spectrum:

$$\begin{cases} \omega_1 = 10\frac{rad}{s}, \ \omega_3 = 30\frac{rad}{s}, \ \omega_5 = 50\frac{rad}{s}, \\ \omega_0 = 0\frac{rad}{s}, \ \omega_2 = 20\frac{rad}{s}, \ \omega_4 = 40\frac{rad}{s}. \end{cases}$$

The structures of systems after accomplishment the synthesis was introduced in Table 1.

## 4. The system of the research

System number 6 (from Table 1) was selected to more far considerations. This system was weighted dynamic excitation (Fig. 4). Polar graph of the system was introduced in Figure 5.

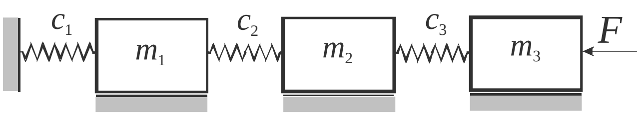



Fig. 4. Model of system with dynamic excitation

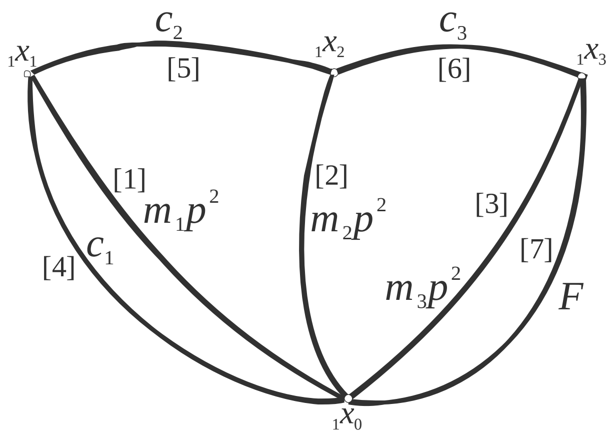



Fig. 5. Polar graph of the system with dynamic excitation

## 5. Reduction of vibration of mechanical system with passive and active elements

Systems with passive elements reducing vibrations they be introduced in Figure 6 (polar graph in Fig. 7):

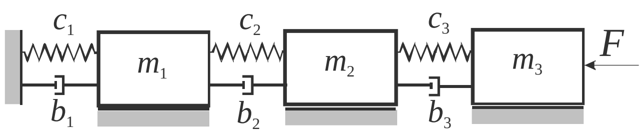
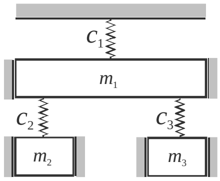
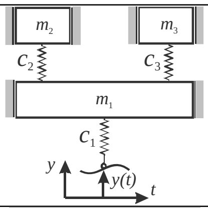
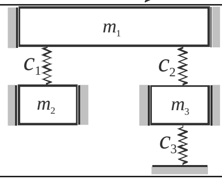
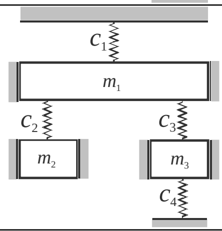
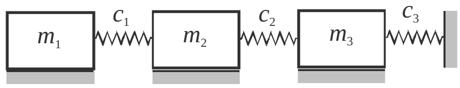
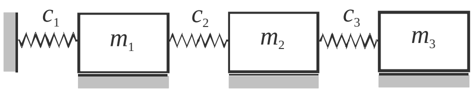
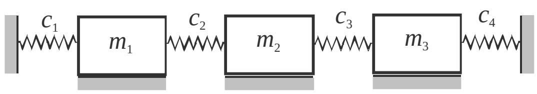
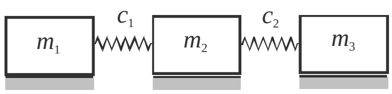
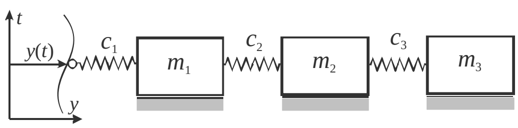



Fig. 6. The models of the system with passive elements

Table l.
The structures of systems after accomplishment the synthesis

| No | FUNCTION | STRUCTURE |
|---|---|---|
| 1 | $U(s)\frac{1}{H}=\frac{351}{s}+s+\frac{1}{\frac{s}{656}+\frac{1}{1{,}64s}}+\frac{1}{\frac{s}{492}+\frac{1}{0{,}31s}}$ |  |
| 2 | $V(s)\frac{1}{H}=\frac{s}{395}+\frac{1}{1s+\frac{1}{\frac{s}{1500}+\frac{1}{0{,}85s}}+\frac{1}{\frac{s}{1095}+\frac{1}{22{,}98s}}}$ |  |
| 3 | $U(s)\frac{1}{H}=s+\frac{1}{\frac{s}{656}+\frac{1}{1{,}64s}}+\frac{1}{\frac{s}{492}+\frac{1}{0{,}31s}+\frac{1}{\frac{s}{351}}}$ |  |
| 4 | $U(s)\frac{1}{H}=\frac{100}{s}+s+\frac{1}{\frac{s}{656}+\frac{1}{1{,}64s}}+\frac{1}{\frac{s}{492}+\frac{1}{0{,}31s}+\frac{1}{\frac{s}{251}}}$ |  |
| 5 | $U(s)=s+\frac{1}{\frac{s}{1500}+\frac{1}{2.14s+\frac{1}{\frac{s}{1285}+\frac{1}{2.86s}+\frac{1}{\frac{s}{714}}}}}$ |  |
| 6 | $U(s)=\frac{351}{s}+s+\frac{1}{\frac{s}{1149}+\frac{1}{1.26s+\frac{1}{\frac{s}{485}+\frac{1}{0.69s}}}}$ |  |
| 7 | $U(s)=\frac{175}{s}+s+\frac{1}{\frac{s}{1325}+\frac{1}{1.67s+\frac{1}{\frac{s}{848}+\frac{1}{1.6s}+\frac{1}{\frac{s}{268}}}}}$ |  |
| 8 | $U(s)=s+\frac{1}{\frac{s}{1000}+\frac{1}{2.22s+\frac{1}{\frac{s}{777}+\frac{1}{3.89s}}}}$ |  |
| 9 | $V(s)=\frac{s}{1500}+\frac{1}{s+\frac{1}{\frac{s}{700}+\frac{1}{1.17s+\frac{1}{\frac{s}{525}+\frac{1}{2.1s}}}}}$ |  |

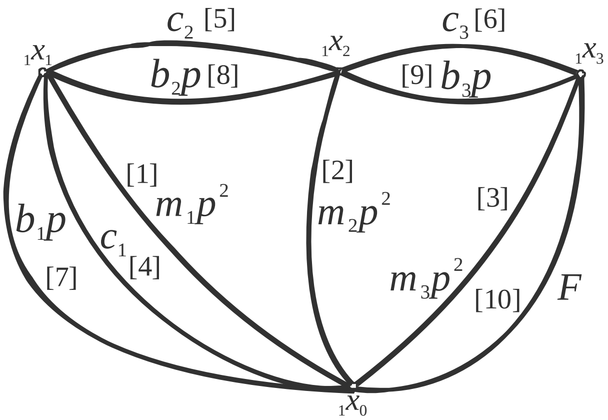



Fig. 7. Polar graph of the systems from Fig. 6

Using formula (4) it is possible to mark the values of damping elements:

$\lambda = 0.01$

$b_1 = 3.51\frac{Ns}{m}$ $\quad b_2 = 11.49\frac{Ns}{m}$ $\quad b_3 = 4.85\frac{Ns}{m}$

Systems with active elements reducing vibrations they be introduced in Figure 8 (polar graph in Fig. 9):

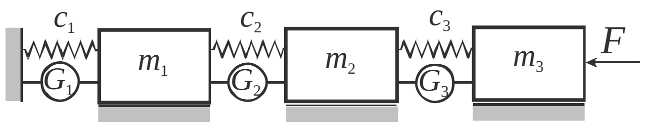



Fig. 8. The models of the system with active elements

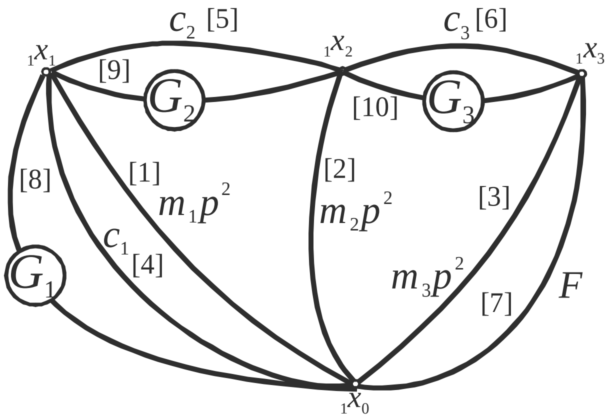



Fig. 9. Polar graph of the systems (Fig. 8)

The comparison of amplitudes of vibrations is introduced in Figs. 10-18. Symbols in Figs. 10-18:

A1(ω), A2(ω), A3(ω) – amplitudes of system without reduction,

Ap1(ω), Ap2(ω), Ap3(ω) – maximum displacements of system with passive reduction,

Aa1(ω), Aa2(ω), Aa3(ω) – amplitudes of system with active reduction.

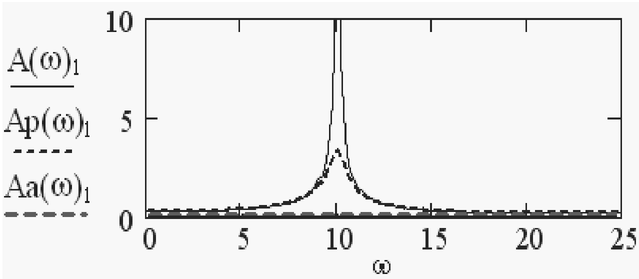



Fig. 10. Diagram of $A_1$ amplitude at $\omega=\omega_1$

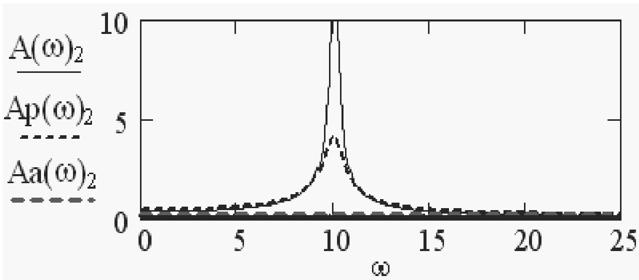



Fig. 11. Diagram of $A_2$ amplitude at $\omega=\omega_1$

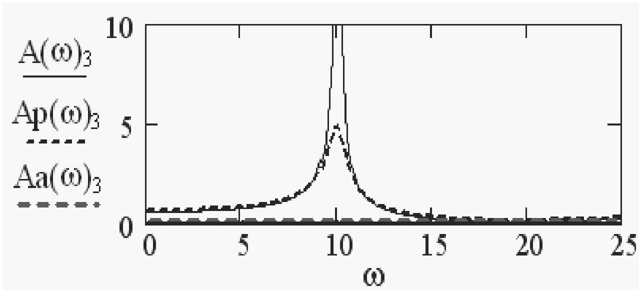



Fig. 12. Diagram of $A_3$ amplitude at $\omega=\omega_1$

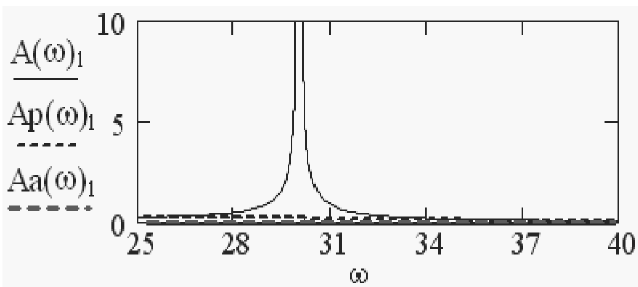



Fig. 13. Diagram of $A_1$ amplitude at $\omega=\omega_2$

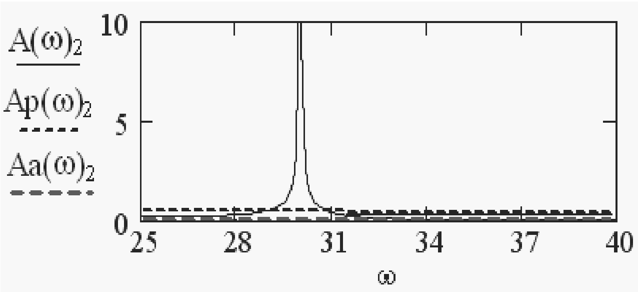



Fig. 14. Diagram of $A_2$ amplitude at $\omega=\omega_2$

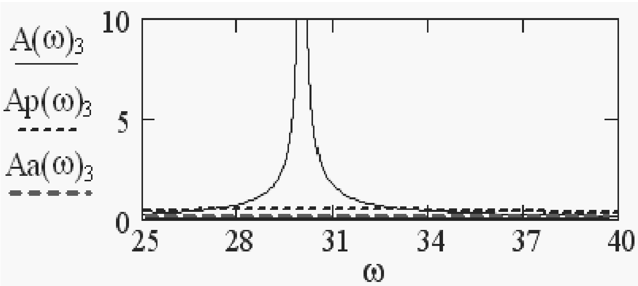



Fig. 15. Diagram of $A_3$ amplitude at $\omega=\omega_2$

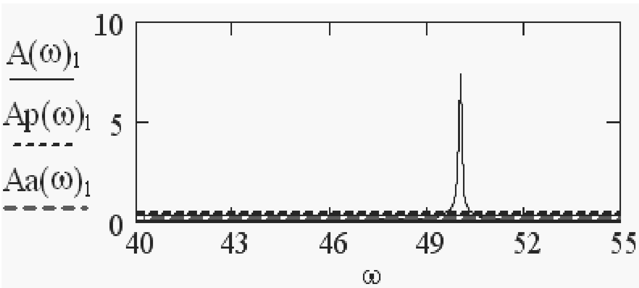



Fig. 16. Diagram of $A_1$ amplitude at $\omega=\omega_3$

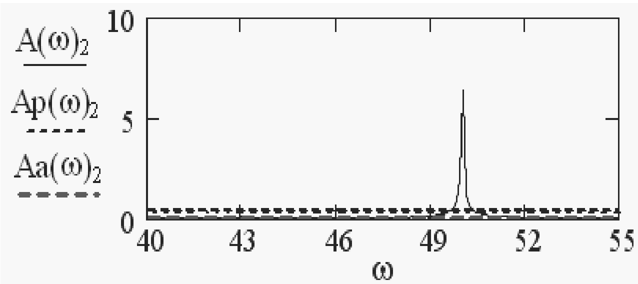



Fig. 17. Diagram of $A_2$ amplitude at $\omega=\omega_3$

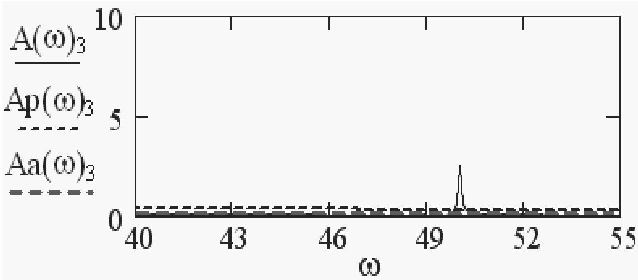



Fig. 18. Diagram of $A_3$ amplitude at $\omega=\omega_3$

## 6. Conclusions

Using non-classical design methods (reverse task) make possible to obtain system parameters and structure meeting previously adopted requirements in relation to dynamic properties. During designing a machine it is necessary to take into consideration various factors which may affect its operation. Proper modelling enables appropriate optimisation of machine construction as early as at the design stage.

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