CEB
27 th PLENARY SESSION
PARIS SEPTEMBER 1980

COMITÉ EURO-INTERNATIONAL DU BÉTON
CONTRIBUTION OF THE POLISH NATIONAL GROUP

Zdzisław SULIMOWSKI [1)]

VECTOR ANALYSIS OF A REINFORCED CONCRETE CROSS-SECTION

1. General relationships

Let us consider a cross-section of a reinforced concrete member. The area limited by the contour of the cross-section will be regarded as the area of the concrete cross-section and denoted by A_c. In the centre of gravity O of this area we assume the beginning of the local ortho-Cartesian dextrorotatory system of coordinates u, v, w (Fig 1). The corresponding versors of the axis Ou. Ov, Ow are denoted by i, j, k. The cross-section considered belongs to the plane v,w.

The member is reinforced with flexible steel bars whose number is k. The area of the cross-section of the successive reinforcing bar is denoted by A_{si} and its distance from the axis Ow is determined by the vector $v_{si}=v_{si}j$. The cross-section of the concrete and reinforcement is symmetrical to Ov.

The cross-section load is a conjugated couple of internal forces: axial force N, and bending moment M. An equivalent load is the longitudinal force N on the arm e = ej, such that N×e = M (vector product). The denotations have been assumed acc. to the convention used in mechanics: axial force N>0 refers to tension in the cross-section, M>0 causes tension in the bottom [2] fibres of the cross section. In accordance with this, the normal stresses $\sigma = \sigma i$ and the tensile ones are assumed to be positive, and the compressive stresses - negative. The height of the cross-section h, width b_v, the area A_c and the area A_{si}, as well as the design compression strength of concrete f_{cd}, and of the compression and tensile strength of steel f_{yd} will be regarded as positive scalars. Thus, the situation when

1) Assoc. Professor, Silesian Technical University,
ul.Pstrowskiego 5, 44-100 Gliwice, Poland

(in a definite area of the cross-section) the stress in the concrete has been reached the design compression strength, will be written as $\sigma_{c} = -f_{cd}$.

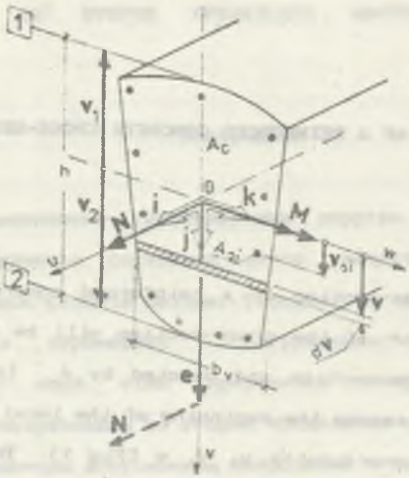

Fig. 1. Cross-section of a reinforced concrete member

Let an arbitrary, conjugated couple of external forces (N_S, M_S) caused by the load, operate on the cross-section. This will cause in the cross-section a certain state of normal stresses σ. If we make sufficient and consistent analytical assumptions defining all the possible distributions and values of normal stresses in concrete σ_{cu} and steel σ_{su}, determining the carrying capacity of the cross-section in the state considered, then we thus create a set of conjugated couples of internal forces

$$\{N_R,\ M_R\} \tag{1}$$

at which a realisation of the carrying capacity of the cross-section takes place. A graphic presentation of the set (1) is the interaction diagram.

We may easily find that

$$N_R = N_{cu} + N_{su} \equiv \int_{v1}^{v2} \sigma_{cu} b_v \, dv + \sum_{1}^{k} \sigma_{sui} A_{si} \tag{2}$$

$$M_R = M_{cu} + M_{su} \equiv \int_{v1}^{v2} \sigma_{cu} \times v b_v \, dv + \sum_{1}^{k} \sigma_{sui} \times v_{si} A_{si} \tag{3}$$

In the present paper we shall create the vector products only from the vectors which are perpendicular to each other, collinear with the assumed coordinate axes. The forces and stresses are collinear with the axis Ou; $N = [N^u,0,0]$, $\sigma = [\sigma^u,0,0]$, the arms of the forces are collinear with the axis Ov; $e = [0,e^v,0]$, $v = [0,v^v,0]$, with the vector of the bending moment being parallel to Ow; $M = [0,0,M^w]$.

Thus, e.g.

$$N \times e = \begin{vmatrix} i & j & k \\ N^u & 0 & 0 \\ 0 & e^v & 0 \end{vmatrix} = kN^u e^v = kM^w = M \qquad (4)$$

In view of this, the algebraic equation $N^u e^v = M^w$ permits the determination of an arbitrary chosen coordinate provided that remaining two are known.

Analogically, the coordinate v_c of the vector v_c of eccentricity of the resultant force N_{cu} in the concrete may be calculated as the quotient

$$v_c = \frac{\int_{v1}^{v2} \sigma_{cu} v b_v dv}{\int_{v1}^{v2} \sigma_{cu} b_v dv} \qquad ((5)$$

hence,

$$v_c = jv_c \qquad ((6)$$

whereas,

$$M_{cu} = N_{cu} x v_c = kN_{cu} v_c \qquad ((7)$$

Let us define the reduced vectors with dimensionless coordinates:

$$n_R = \frac{1}{f_{cu} A_c} N_R \qquad ((8)$$

$$m_R = \frac{1}{f_{cu} A_c h\psi} M_R \qquad (9)$$

where ψ is normalizing coefficient.

From formulae (2) and (3), taking into consideration the identity equalities, we shall obtain respectively:

$$n_R = n_{cu} + n_{su} \equiv n_{cu} + \sum_{i}^{k} n_{si} \qquad (10)$$

$$m_R = m_{cu} + m_{su} \equiv m_{cu} + \sum_{i}^{k} m_{si} \qquad (11)$$

thanks to which set (1) may be presented in the form

$$(n_R, m_R) \qquad (12)$$

Let us consider the axes Ou and Ow. Multiplaying their measures N, M respectively by $1/(f_{cu}A_c)$ and $1/(f_{cu}A_ch\psi)$, we shall obtain the transformed dimensionless system of coordinates which will be denoted on On, Om. In the plane Onm we may present graphically the reduced carrying capacity of the cross-sectin in the form of the relatioships (10), (11), (12); (Fig. 2).

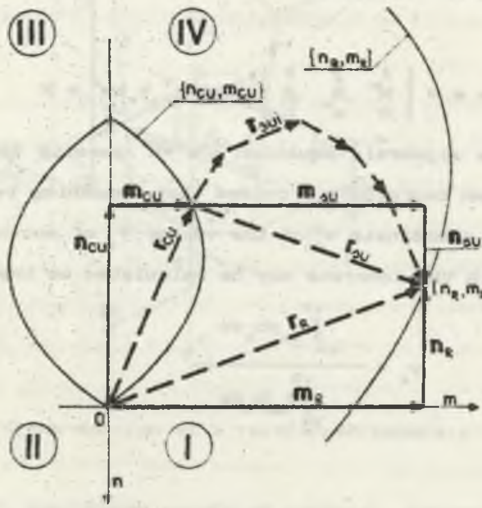

Fig. 2. Graphical representation of the reduced load carrying capacity of a reinforced concrete cross-section according to formulae (10), (11), (12)

Especially the vector

$$r_{cu} = n_{cu} + m_{cu} \tag{13}$$

may be interpreted as the reduced carrying capacity of the concrete cross-section on normal stresses. The set (n_{cu}, m_{cu}) of the coordinates of the end of this vector determines the line of interaction for the concrete cross-section, hence r_{cu} is the radius-vector of this line. On the cross-section of a definite shape, the coordinates of the line of interaction depend on the analytical assumptions made, defining the stresses σ_c.

In a similar way, the vector

$$r_{su} = n_{su} + m_{su} \tag{14}$$

represent the reduced carrying capacity of the total reinforcement of the cross-section. It is dependent on the quantity and position of this reinforcement and on the analitycal assumptions reffering to the stresses σ_s. The carrying capacities of the particuliar reinforcing bars are denoted by the successive vectors $r_{sui} = n_{sui} + m_{sui}$, whose superposition makes the vector r_{su}.

The total vector

$$r_R = r_{cu} + r_{su} \tag{15}$$

with the end coordinates (n_R, m_R) may be understood as radius-vector of the reduced line of interaction of whole reinforced concrete cross-section (n_R, m_R).

The procedure outlined above may be used to determine a set of coordinates of the reduced interaction diagram for the computational assumptions, provided the geometry and strength characteristics of the cross-section are known.

Since the carrying capacity of deformable element is analyzed here, the resultant N_{ui} of the normal stresses from the considered region of cross-section area must be regarded as attached vector. This may be, for example, the resultant of the stresses from the compressive zone of the concrete, or the resultant of the stresses in the reinforcing bar, or finally - the resultant of the stresses from a group of reinforcing bars in close proximity.

In the actual cross-section, the distance of the considered vector N_{ui} from the axis Ow is determined by means of v_i. On the basis of (4),(8),(9), we conclude that there exists a dimensionless collinear with Ow, eccentric vector ν_i so that

$$n_{ui} \times \nu_i = m_{ui} \tag{16}$$

Taking into account the orthogonality of the vectors n_{ui} and m_{ui}, we are able to calculate the coordinate

$$\nu_i = \frac{m_{ui}}{n_{ui}} \tag{17}$$

Since we have the relationships defined by formula (7) , it may clearly be shown that:

$$\nu_i = \frac{1}{h\psi} v_i \tag{18}$$

and

$$\nu_i = j\nu_i \quad (19)$$

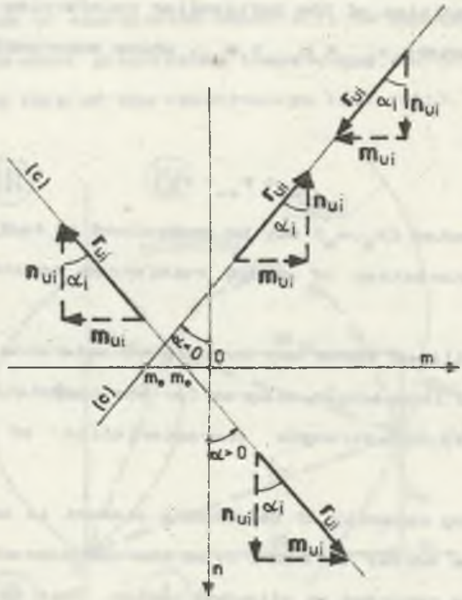

Fig. 3. Method of constructing of the vector inclination

The coordinate ν_i may be interpreted as the directorial coefficient of the straight line (c), given by the equation $m=\nu_i n+m_o$, collinear with the vector $r_{ui}=n_{ui}+m_{ui}$. Determining $\sphericalangle(On, r_{ui})=\alpha_i$ with $\alpha_i \in (-\pi/2, \pi/2)$, in compliance with Fig.3, we shall obtain

$$\nu_i = tg\ \alpha_i \quad (20)$$

hence, after substituting (18)

$$\alpha_i = arc\ tg\ (v_i/(h\psi)) \quad (21)$$

Making use of the above relationships we may solve an important problem consisting in the selection of the required reinforcement for the assigned couple of internal forces (N_S, M_S). Basing calculation on the ultimate state of carrying capacity, let us assume that

$$(N_S = N_R, M_S = M_R) \quad (22)$$

and applying the previous transformations also to the internal forces, we find the reduced vector of the internal forces

$$r_s = n_s + m_s \quad (23)$$

in view of which the condition of realization of the state being considered may be writtem in vector form:

$$r_s \in \{r_m\} \quad (24)$$

To solve the problem are required the directional angles α_{si} of the vectors r_{si} which - as arises from formula (21) - may be obtained by assuming the position of the reinforcing bars. The problem is very simple if reinforcement can be reduced to two areas A_{s1} and A_{s2} whose centres of gravity are distant respectively, by d_1 and d_2 from the upper [1] and lower [2] edge of cross-section.

In a general case of dimensioning of the cross-section, the number of the parameters necessary for the determination is greater than the number of the conditions of equilibrium, since apart from the required and most frequently given in the assumptions, design strengths of the concrete f_{cd} and steel f_{yd}, the following parameters must be determined: dimensions of the concrete cross-section (two for rectangular, and six for I-bar one), the areas of the reinforcement cross-sectios A_{s1}, A_{s2}, the distaances d_1, d_2 and the height of the compressed zone of the cross-section x dependent on the actual values (N_s, M_s); thus, in the simplest case - 7 unknown parameters, compared witch two conditions of equilibrum (22).

In the search for the solution, two kinds of approach are used:

(1) Assuming the dimensions proportion of the concrete cross-section, the reinforcement ratios ($\rho_1=A_{s1}/A_c$, $\rho_2=A_{s2}/A_c$), and the ratios d_1/h, d_2/h, it is possible to determine the dimensions of concrete cross-sections, as well as the area and position of reinforcing steel. Such procedure leads to the diagrams given, i.a. in [1], obtained in analytical way though they may be easily justified by means of vector relatioships.

(2) Assuming the dimensions of the concrete cross-section and the distances d_1, d_2, we search for A_{s1}, A_{s2}, and x. For an explicit solution we need an ancillary condition; in some cases it may be defined by the code rules, and where it is missing, we may rely on the optimizing condition, e.g.

$$A_{s1} + A_{s2} = \min(A_{s1} + A_{s2}) \quad (25)$$

Approach (2) is often used in engineering practice, and is well suited to vector analysis. It has been presented in paper [2] for the assumptions of the Polish Code [3].

2. Solution for the ultimate limit state acc. to CEB-FIP Model Code 1978

Let us consider a rectangular cross-section with denotations as in Fig. 4 and accept the assumptions of [4, item 10.4.1.1], including the plane sections principle. To the denotations assumed we shall add the following

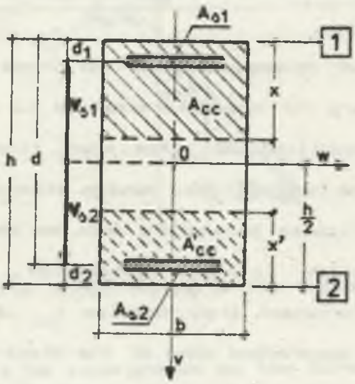

Fig. 4. Rectangular cross-section of a reiforced concrete member

assignations: Strains $\varepsilon = \varepsilon i$ of the particular fibres of the cross-section are regarded as the vectors taking positive values if they are elongations, and negative values - if they are contractions. The height x of the compressed zone of the section, distances d_1, d_2, and the moduli of elasticity E_c, E_s are interpreted as positive scalars.

On the basis of [4, item 10.4.3.1] we assume that the stress diagram in the compressed zone of concrete is parabolic-rectangular while ε_{cd}=-0.002, ε_{cu}=-0.0035; for the purpose of calculating we shall determine $f_{cd}=0.85 f_{ck}/\gamma_c$. Similary, on the basis of [4, item 6.4.2.3] we shall assume that steel behaves here like an ideally elasto-plastic material; $f_{yd}=f_{yk}/\gamma_s$, E_s=200000 MPa. In compliance with [4, item 6.4.2.3] it has been assumed that γ_c=1.50, γ_s=1.15. For reinforcing steel we find $\varepsilon_{sd}=f_{yd}/E_s$.

We shall introduce the ratios

$$\delta_1=\frac{d_1}{h}\ ,\quad \delta_2=\frac{d_2}{h}\ ,\quad \eta=\frac{\varepsilon}{\varepsilon_{cd}}\ ,\quad \eta_{sd}=\frac{-\varepsilon_{sd}}{\varepsilon_{cd}} \tag{26}$$

Now we shall write the assumed constitutive relationships between the stresses and strains for the carrying capacity of the cross-section.

At the strains ε in any fibre of the concrete

$$\sigma_c = \begin{cases} 0 & \text{for } \eta\le 0 \\ -f_{cd}\eta(2-\eta) & \text{for } 0<\eta\le 1 \\ -f_{cd} & \text{for } 1<\eta\le 1.75 \end{cases} \tag{27}$$

and respectively in steel

$$\sigma_s = \begin{cases} f_{yd} & \text{for } -5\le\eta\le-\eta_{sd} \\ -0.002\ E_s\eta & \text{for } -\eta_{sd}<\eta\le\eta_{sd} \\ -f_{yd} & \text{for } \eta_{sd}<\eta\le 1.75 \end{cases} \tag{28}$$

(for Polish steels with the successive values f_{yd}= 190, 310, 350 MPa we shall obtain respectively η_{sd}= 0.475, 0.775, 0.875).

By means of the reduced strain coordinates of the edge fibres η_1 and η_2, we can express the reduced strains of the reinforcing steel A_{s1} and A_{s2}.

$$\eta_{s1}=\eta_1(1-\delta_1)+\eta_2\delta_1 \tag{29}$$

$$\eta_{s2}=\eta_1\delta_2+\eta_2(1-\delta_2) \tag{30}$$

the height of the compressed zone of the section

$$x = \eta_1 h/(\eta_1-\eta_2)\ ;\qquad 0\le x\le h \tag{31}$$

and of this part of the height of the compressed zone in which $\sigma_c := -f_{cd}$

$$y = (\eta_1-1)h/(\eta_1-\eta_2)\ ;\qquad 0\le y\le 3h/7 \tag{32}$$

In Fig.5 are shown six (①....⑥) configurations of the cross-section strains which make possible the defining of the corresponding intervals of strain variations in concrete and steel. The limits of these intervals expressed by means of η_1 and η_2 are given in Table 1, whereas in Fig.6 is shown distribution of the stresses σ_c in the concrete which corresponds to them.

On the basis of relationships (2) and (3) in the terms reffering to concrete, and with due consideration to the reduction acc. to (8) and (9), for relation (27) the formulae for the coordinates n_{cu} and m_{cu} of the vector r_{cu} in relation to On and Om have been determined. Leaving out simple transfomations, the results are compiled in Table 1. Within the interval (5) - (6), the relationship between n_{cu} and m_{cu} is a linear one.

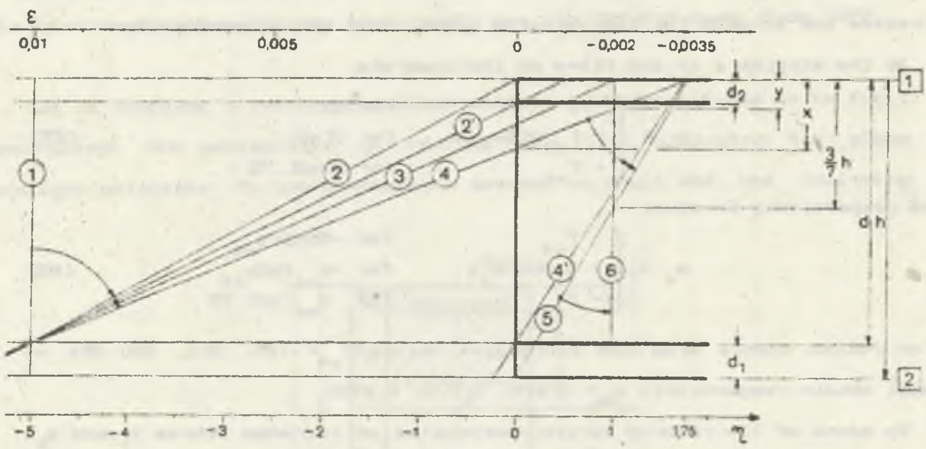

Fig.5 The configurations of the cross-section strains

For the same intervals, η_{s1} and η_{s2} may be calculated by means of formulas (29) and (30), and on the basis of (28) may be found the respective values of strsses σ_{s1} in the upper reinforcement and σ_{s2} in the bottom one.

Table 1

Scope	η_1	η_2	x	y	n_{cu}	m_{cu}
1	2	3	4	5	6	7
(1)-(2)	$-5 \le \eta_1 \le 0$		0	0	0	0
(2)-(3)	$0 < \eta_1 \le 1$	$-\frac{5+\eta_1\delta_2}{1-\delta_2}$	$0 < x \le \frac{1}{6}d$	0	$\frac{(\eta_1-3)\,\eta_1^2}{3(\eta_1-\eta_2)}$	$\frac{[\eta_1(2-\eta_1+2\eta_2)-6\eta_2]\eta_1^2}{12(\eta_1-\eta_2)^2\psi}$
(3)-(4)	$1 < \eta_1 \le 1{,}75$		$\frac{1}{6}d < x \le \frac{7}{27}d$	$0 < y \le \frac{3}{27}d$	$\frac{1-3\eta_1}{3(\eta_1-\eta_2)}$	$\frac{2(\eta_1+\eta_2)-1-6\eta_1\eta_2}{12(\eta_1-\eta_2)^2\psi}$
(4)-(5)	1,75	$-\frac{5+\eta_1\delta_2}{1-\delta_2} < \eta_2 \le 0$	$\frac{7}{27}d < x \le h$	$\frac{3}{27}d < y \le \frac{3}{7}h$		
(5)-(6)	$\frac{1}{4}(7-3\eta_2)$	$0 < \eta_2 \le 1$	h	$\frac{3}{7}h$	$\frac{4(1-\eta_2)^2}{21}-1$	$\frac{5(1+n_{cu})}{14\psi}$

In compliance with relationships (8) and (9) we shall now write the formulae for the coordinates of the vectors of reduced carrying capacity of reinforcement r_{s1u} and r_{s2u} to the axes On and Om.

$$n_{s1u} = \sigma_{s1} A_{s1} / (f_{cu} bh) \qquad (33)$$

$$m_{s1u} = \sigma_{s1} A_{s1} [-(0.5h-d_1)] / (f_{cu} bh^2 \psi) = -n_{s1u}(0.5-\delta_1)/\psi \qquad (34)$$

$$n_{s2u} = \sigma_{s2} A_{s2} / (f_{cu} bh) \qquad (35)$$

$$m_{s2u} = \sigma_{s2} A_{s2} (0.5h-d_2) / (f_{cu} bh^2 \psi) = n_{s2u}(0.5-\delta_2)/\psi \qquad (36)$$

In result from the above that for $\psi=0.5-\delta_1$ we obtain $m_{s1u}=-n_{s1u}$ and for $\psi=0.5-\delta_2$ we have $m_{s2u}=n_{s2u}$.

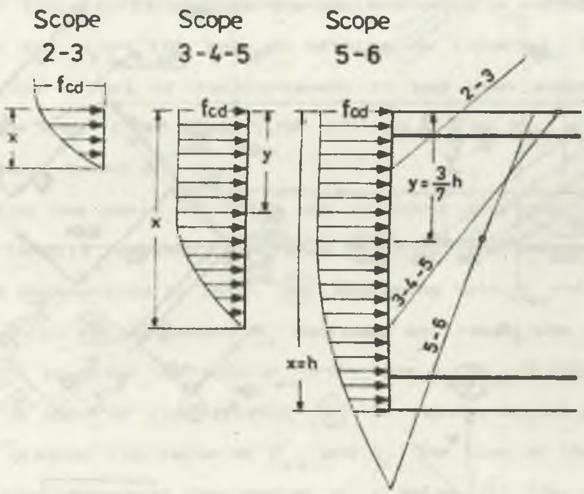

Fig.6. The distributions of the stresses in the concrete cross-section

3. Interaction diagram

The set $\{n_u, m_u\}$ of the coordinates of the end of the radius-vector r_u forms, in the plane Onm, a closed line called interaction diagram. We shall obtain the right branch of this diagram when the compressed zone of the section adheres to the upper fibre [1], and the left one - when it adheres to the bottom fibre [2] (Fig.4). As it is known, the interaction diagram is a complete representation of the carrying capacity of the cross-section

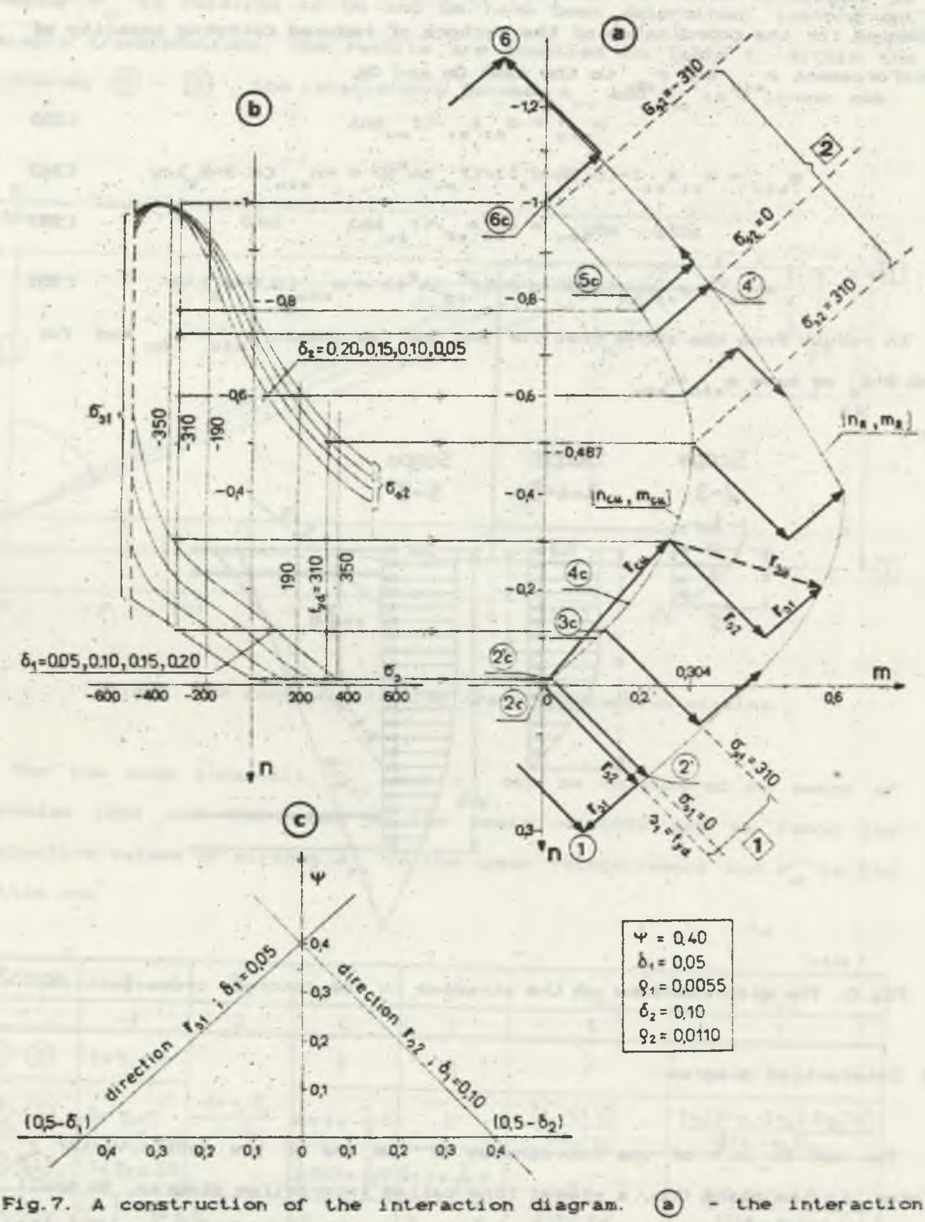

Fig.7. A construction of the interaction diagram. (a) - the interaction diagram; ⟨1⟩ - a band of inefficiency of the reinforcement A_{s1}. ⟨2⟩ - a band of inefficiency of the reinforcement A_{s2}. (b) - the values of the stresses in the reinforcement. (c) - the directions of the vectors r_{s1} and r_{s2}

subject to the bending moment and axial force.

We shall construct an interaction diagram (Fig.7a) of a rectangular cross-section for the assumptions defined in item 2. It has been assumed that ψ=0.4 which means that for δ_2=0.1, the vector r_{s2u} is inclined towards On at an angle $\alpha=\pi/4$. The directions of the vectors r_{s1u} and r_{s2u} at arbitrary values of δ_1 and δ_2 may easily be plotted by means of the construction given in Fig.7c. The effective values of the stresses σ_s in the reinforcement, depending on $\delta_{1,2} \in$ (0.05, 0.10, 0.15, 0.20) and on f_{yd} (vertical lines) are presented in Fig 7b. The reduced diagram of interaction of concrete (n_{cu}, m_{cu}) is of universal character, independent of the dimensions of the cross-section; it has been calculated acc. to the formulae of Table 1. It reaches the maximum value m_{cu}=0.304 for n_{cu}=-0.487 at η_1=1.75 and η_2=-1.162 and so within the interval (4) - (5). To illustrate the effect of reinforcement it has been assumed that f_{yd}=310 MPa, f_{ck}=30 MPa, f_{cd}=0.85*30/1.5= 17 MPa, δ_1=0.05, $\rho_1=A_{s1}$/b/h=0.0055, δ_2=0.10, $\rho_2=A_{s2}$/b/h=0.011.

Since, (on the basis of (27)) the concrete does not cooperate in the bearing of tensile stresses, for range (1) - (2) the interaction diagram of concrete is degenerated to point (2c). Beginning with $\eta_{s1}=-f_{yd}/0.002/E_s$, the stresses σ_{s1} in reinforcement A_1 decrease and reach the value σ_{s1}=0 for position (2') in order to reach $\sigma_{s1}:=-f_{yd}$ for $\eta_{s1}=f_{yd}/0.002/E_s$. In this way is created a band of inefficiency ◇1 of reinforcement A_{s1}. This is the wider, the greater the value of f_{yd} and δ_1. The line of the equation σ_{s1}=0 separates the senses of the vector r_{s1}; below it, the component n_{s1} is directed in compliance with the axis On, and abowe it - in the opposite.

For the range (2) - (4), the interaction diagram of concrete depends on δ_2. The diagram in Fig.7a has been calculated at δ_2=0.1, and next the percentage relative error Δ has been determined for δ_2=0.05 and δ_2=0.20. Since it has been found that |Δ|<2%, it may be accepted that the error is comprised witin the limits of tolerance of the diagram.

In the ranges (4)-(5)-(6) there occurs a decrease of the elongations of the bottom reinforcement A_{s2}, and beginning with the value $\eta_{s2}=-f_{yd}/0.002/E_s$ this is accompanied by a drop of the stresses; in position (4') we obtain σ_{s2}=0 to reach, for $\eta_{s2}=f_{yd}/0.002/E_s$, the value

$\sigma_{a2}:=-f_{yd}$. In this way a band of inefficiency ◇2 of reinforcement A_{a2} is created. This is the wider, the greater is the value f_{yd} and the greater is δ_2. The line of equation $\sigma_{a2}=0$ separates the senses of vector r_{a2}: below it, the component n_{a2} is directed in compliace with the axis On, abowe - it is opposite.

A consequence of the separation of the plane Onm with half-lines $\sigma_{a1}=0$ and $\sigma_{a2}=0$ is that the beginning of the vector r_{au} must rest on the line of interaction of the concrete section in the same subdomain to which belongs the assigned coordinate (n,m) determining its end.

The left branch of the interaction diagram may be obtained when the system of the possible strains of the section, analyzed acc.to Fig.5, is reversed in relation to edge fibres. For the section with a horizontal axis of symmetry, the diagram $\langle n_{cu}, m_{cu}\rangle$ will be symmetrical to On.

All the remarks on the intervals will remain valid provided the upper reinforcement $(A_{a1}, \sigma_{a1}, \delta_1)$ is exchanged for the bottom one $(A_{a2}, \sigma_{a2}, \delta_2)$.

4. Final remarks

The analysis of the interaction diagram presented, makes it possible to explain in a graphic way, the role played by the particuar elements of the cross-section (concrete, upper and bottom reinforcements) in shaping of the carrying capacity of the cross-section. It may be useful for the verification and comparison of the anaytical assumptions. For example, the comparative calculations made for $\varepsilon_{au}=0.005$ (with unchanged remaining values assumed in the present paper) show that the interaction diagram of concrete A_{a2} differ insignificantly; somewhat greater differences occur in the estimation σ_{a1} of the upper reinforcement A_{a1}, but only in a relatively narrow interval ② - ④. In viev of this, a discussion on the subiect of the value of ε_{au} assumed for the calculation of the cross-section carrying capacity seems to be insignificant.

Since for all the assumptions made it is always possible to construct one, generally valid reduced interaction diagram of the concrete cross-section $\langle n_{cu}, m_{cu}\rangle$, it is wortwile to carry out a comparative analysis while assuming various relationships $\sigma-\varepsilon$ for the concrete, e. g. rectangular, triangular-rectangular, parabolically-rectangular one, acc. to the curves CEB - FIP [4,formula (7.1)], or the later suggestions [5,formula (2.4.4)]. This may have a practical aspect, as with a developed interaction diagram of concrete cross-section, the selection of reinforcement takes place in the same way, independently of the assumed relationships $\sigma_c-\varepsilon_c$. There is no doubt that desirable here would be a compliance of the $\sigma-\varepsilon$ relationships assumed for the determination of the internal forces as the result of the loads, with those $\sigma-\varepsilon$ dependences which are used for the estimation of the carrying capacity of the cross-sections.

REFERENCES

1. E. Grasser, Bemessung für Biegung und Biegung mit Länggskraft, Beton-Kalender 1980, Teil I, 701-819, Verlag W. Ernst und Sohn., Berlin 1980
2. Z. Sulimowski, Wektorowe ujęcie nośności przekroju żelbetowego;; (Load carrying capacity of a reinforced concrete cross-section in vector approach), Archiwum Inżynierii Lądowej, 30, 2-3, 1984, 515-533
3. PN-84/B-03264 Konstrukcje betonowe, żelbetowe i sprężone. Obliiczenia statyczne i projektowanie; (Concrete, reinforced concrete and pprestressed concrete. Design rules), Wyd. ALFA, Warszawa 1984
4. CEB-FIP Model Code for Concrete structures, London 1978
5. CEB-FIP Model Code 1990, First Predraft 1988, CEB Bulletin No. 190, Luasanne 1988

VECTOR ANALYSIS OF A REINFORCED CONCRETE CROSS-SECTION

Summary

The resultant of the normal stresses in reiforced concrete cross-section has been interpreted as a conjugated couple of vectors (N_R, M_R) reduced to the centre of gravity of the section, composed of the vectors assigned to the particular areas of concrete and reinforcing bars. Transforming these vectors to dimensionless values it is possible to obtain their sum $r_R = n_R + m_R$ which is the radius-vector of the interaction diagram. Of practical importance is the opposite problem: at the assigned load (N_S, M_S), through the decomposition of r into components pertinent to concrete and steel, it is possible to determine the reinforcement area needed.

The general solution has been developed for the assumptions of the carrying capacity acc. to CEB-FIP Model Code 1978, assuming parabolically-rectangular diagram of the stresses in concrete. With the assumptions made, it has been found that the limit strains in steel ε_{su} do not show a significant efect on the carrying capacity of the whole section.

KEY WORDS

reinforced concrete,

ultimate limit state,

carrying capacity,

vector analysis,

interaction diagram.

WEKTOROWA ANALIZA PRZEKROJU ŻELBETOWEGO

Streszczenie

Wypadkową naprężeń normalnych w przekroju żelbetowym zinterpretowano jako sprowadzoną do środka ciężkości przekroju sprzężona parę wektorów (N_R, M_R), złożonych z wektorów przyporządkowanych określonym powierzchniom betonu i wkładek zbrojenia. Przekształcając te wektory do wartości bezwymiarowych, można utworzyć ich sumę $r_R = n_R + m_R$,która jest promieniem wodzącym wykresu interakcji. Praktycznie ważne jest zadanie odwrotne: przy znanym

obciążeniu (N_S, M_S), poprzez dekompozycję r na składowe przynależne do betonu i stali można wyznaczyć potrzebne pole zbrojenia.

Rozwiązanie ogólne rozwinięto dla założeń stanu granicznego nośności według CEB-FIP Model Code 1978, przyjmując paraboliczno-prostokątny wykres naprężeń w betonie. Przy tych założeniach okazało się, że odkształcenia graniczne w stali ε_{su} nie wykazują istotnego wpływu na nośność całego przekroju.

ВЕКТОРНЫЙ АНАЛИЗ ЖЕЛЕЗОБЕТОННОГО СЕЧЕНИЯ

Резюме

Равнодействушая нормальных напряжений в железобетонном сечении интерпретируется как приведенная к центру тяжести сечения связанная пара векторов (N_R, M_R), состоящих из векторов приуроченных к определенным поверхностям бетона и отдельным стержням арматуры. Преобразовывая векторы в безразмерные величины можно составить их сумму $r_R = n_R + m_R$, которая является радиусом графика интеракции. Практически важной является обратная задача: при известной нагрузке (N_s, M_S), путем декомпозиции r на составляющие, принадлежащие к бетону и стали можно определить необходимое поле арматуры.

Общее решение разработано для предельного состояния несущей способности по CEB-FIP Model Code 1978, учитывая параболическо-прямоугольный график напряжений в бетоне. При этих принципах оказалось, что предельная деформация в стали ε_{su} не оказывает существенного влияния на несущую способность всего сечения.