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APPLICATION OF MULTISTAGE PROGRAMMING METHOD TO SCHEDULING PROBLEMS AT COMPLEX SYSTEMS WITH TECHNOLOGICAL MULTI-ROUTE

ZASTOSOWANIE METODY PROGRAMOWANIA WIELOETAPOWEGO DO HARMONOGRAMOWANIA KOMPLEKSOWYCH SYSTEMÓW Z ALTERNATYWNYMI MARSZRUTAMI TECHNOLOGICZNYMI

ПРИМЕМЕНИЕ МЕТОДА МНОГОЭТАПНОГО ПРОГРАММИРОВАНИЯ ДЛЯ КАЛЕНДАРНОГО ПЛАНИРОВАНИЯ КОМПЛЕКСНЫХ СИСТЕМ С АЛТЕРНАТИВНЫМИ ТЕХНОЛОГИЧЕСКИМИ МАРШРУТАМИ

Streszczenie: W referacie przedstawiony jest problem harmonogramowania w systemach z alternatywnymi marszrutami technologicznymi. Do rozwiązania problemu wykorzystano metodę programowania wieloetapowego.

Summary. In this paper, the scheduling problem at complex systems with technological multi-route is formulated. The problems with different operations sequencing were discussed. The problems have been solved by multistage programming method.

Резюме: В работе представлена проблема календарного планирования для систем с альтернативными технологическими маршрутами. Для решения проблемы использован метод многоэтапного программирования.

1. Introduction

The scheduling problems rise from practical manufacturing process in industry. It is very important in controlling of industrial processes. Many scholars have done a lot of work about it. In this paper the scheduling problem at complex systems will be discussed. The complex system consists of machines and objects. Every machine is with a input-store and a output-store. Every object has a lot of operations serviced at different machines. Each object can be put into system from any input-store and taken out system from any output-store. After one operation finished, the object can be transferred from the output-store to one input-store. The problem is to find a optimum schedule. It is very difficult problem. In this paper, the problems are consided under different relations among operations. The problems have been solved with aid of multistage programming method.

The multistage programming method is proposed by professor Franciszek Marecki, Silesian Technical University, Gliwice, Poland. He succeeded in applying the method to solve the scheduling problems at the typical systems and many other problems. The idea of this method was introduced in detail in his doctoral dissertation.

2. The multistage programming method

The multistage programming method is one of ways to solve combinatorial problems. It includes four basic concepts:

1) state of decision process,
2) value of state,
3) state generation procedure,
4) unprespective state elimination.

A simple introduction of the concepts will be given in following.

2.1. State of decision process

For any multistage process, there are some feasible decisions can be made at each stage. So there will be a lot of decision ways. The state is a concept describing the decision process. From one state we can get the information that which stage the process reaches and what decisions have been made. After one decision has been made, the state will be changed. So from initial state we will get many state sequences. A state sequence is called a trajectory. The final state of each trajectory gives out a feasible solution of the problem.

In the process we discuss in this paper the stages are denoted by

$$0,1,2,\ldots,e-1,e,e+1,\ldots,E.$$

At e-th stage, propose there are L_e states, and we denote these states with

$$p^{e,1},p^{e,2},\ldots,p^{e,l},\ldots,p^{e,L_e}$$

In general, the state is defined as a vector or matrix according to the problem that will be solved.

2.2. The value of state

The value of state is a function of state $p^{e,l}$, and corresponds to the optimum criterion. We denote the value of state with $v^{e,l}$.

$$V^{e,l}=Z(p^{e,l})$$

where: $Z(.)$ - a function dependents on the optimizations criterion.

If the problem is with only one optimum criterion, the value of state can be defined as a scalar. If the problem is with the multi-criterion, the value of state can be defined as a vector. Through comparing two state values, we can determine which state is better than another one.

2.3. The state generation procedure

To get a feasible solution of the problem, a feasible trajectory

$$p^{0,1},\ldots,p^{e-1,k},p^{e,l},\ldots,p^{E,1}$$

should be generated stage by stage. A state generation procedure is a mathematical formula that generates a new state $p^{e,l}$ from last state $p^{e-1,k}$.

2.4. The unperspective states elimination

The state p is unperspective one if the calculation of the optimum solution from it is impossible. For elimination of the unperspective states, the following three rules are often used:

1) the exhausting rule,
2) the domination rule,
3) the fathoming rule.

The exhausting rules eliminate the states which have not possibility to generate any feasible solution from it. The domination rules eliminate the states from which the optimum final state can not be generated. The fathoming rules eliminate the states which do not allow to find the final state better than current best one.

3. The scheduling problem at complex systems

Consider M machines, and each machine is with a input-store and a output-store. N objects should be serviced at machines. Each object has a lot of operations should be done. After one operation of a object has been finished, the object can be taken out from the output-store and be put into another input-store that next operation should be done at its machine. The system is shown on the figure 1, where:

B_m^I - the input-store of machine A_m

B_m^O - the output-store of machine A_m

A_m - the m-th machine

M - the number of machine

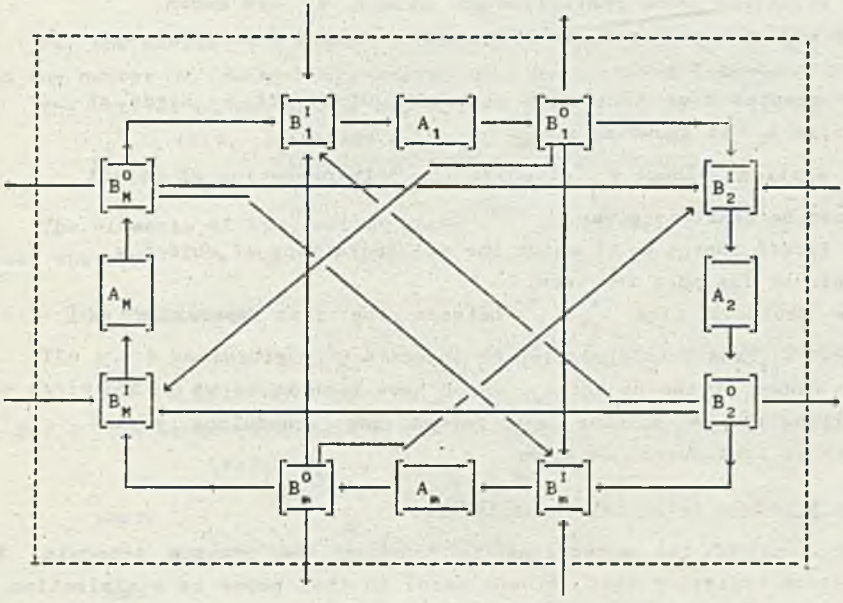

Fig.1. The system with complex structure
Rys.1. Struktura kompleksowego systemu

Any object can be put into system at any input-store and be taken out of system from any output-store. So the system is a complex system. In this paper the problem is discussed depending on following assumption:

3.1. Assumptions

- the system with the unlimited stores and independent objects,
- the M machines A_m, m=1,2,...,M,
- the N objects w_n,n=1,2,...,N,
- at any moment in each machine, only one object can be serviced,
- every object can be serviced in each machine only one time,
- all operations, which should be done to each object w_n are

known, and can be denoted by $w_{n_1}, w_{n_2}, \ldots, w_{n_{k_n}}$,

k_n-is the number of the operations of object w_n,

- all the operations of every object can be separated,
- the feasible technological routes, that each object w_n passes through some machines, are known and can be described by the matrix

$$U_n=[u^n_{k,m}]_{k_n \times M}$$

$$n=1,2,3,\ldots,N$$
$$k=1,2,3,\ldots,K_n$$
$$m=1,2,3,\ldots,M$$

where

$$u^n_{k,m}=\begin{cases} 1 \text{ if the operation } w_{n_k} \text{ can be serviced at } A_m \\ 0 \text{ otherwise} \end{cases}$$

- the relations among operations of object w_n are known,
- each object can be put into any input-store and be taken out from any output-store,
- the service time that i-th operation of object w_n needs at machine A_m is known as $v_{n_i,m}$,
- the earliest moment φ_{n_i} at which the i-th operation of object w_n can be start is given,
- the latest moment Ψ_n at which the all operations of object w_n should be finished is given,
- the shutdown time $\tau^m_{n1_i,n2_j}$ between the i-th operation of object w_{n1} and j-th operation of object w_{n2} is given,
- the number of the object α_m, which have been serviced as the last one in the machine A_m (before the scheduling perio which is considered) is given.

3.2. The schedule optimization criterion

The goal of the scheduling is to find the optimum schedule. The optimization criterion that is considered in this paper is minimization of the maximum operation ending tardiness

$$Q=\max_{1\le n\le N}(t_{L,n}-\Psi_n)\to\min \qquad (1)$$

The $t_{L,n}$ is the moment at which the latest operation of n-th object has been finished.

4. Number-group-matrix and number-group-function

To define the state of this problem, we have to introduce two new concepts.

Def.1. If the elements of a matrix are not scalar, but are number-groups, we call this matrix as number-group-matrix.

Def.2. If (a,b) is a number-group, G(.) can be defined as a number-group-function as follow:

$$G(a,b)=b$$

and F(.) can be defined as a number-group-function as follow

$$F(a,b)=a$$

5. The algorithm

The algorithm depends on the multistage programming method.

5.1. The problem with independent operations

In this problem, there is not any relation among the operations. The operation sequence is not in order.

5.1.1. The state

Def.3 The state $p^{e,1}$ is a number-group-matrix

$$p^{e,1}=[p^{e,1}_{i,j}]_{N\times M} \qquad (2)$$

$$i=1,2,3,\ldots,N$$
$$j=1,2,3,\ldots,M$$

For the matrix, the number of rows corresponds the number of objects, and the number of columns corresponds the number of machines.

The entries of the matrix are determining in the following way

$$p^{e,1}_{i,j}=\begin{cases}(k,t_{i,j}) & \text{at stage } \eta,\eta\le e,\text{if operation } w_i \text{ is finished at moment } t_{i,j} \text{ at } A_j^k\\ (0,0) & \text{otherwise}\end{cases}$$

The elements of the initial state $p^{0,1}$ are equal to (0,0). From the final state $p^{E,1}$ the feasible schedule can be read out.

5.1.2. The state value

The state value function which is defined as following corresponds to the optimization criterion (1).

Def.4 The state value is the scalar, which can be find from the formula

$$V^{e,1}=\max_{n\in I^{e,1}}\ [\ \max_j(t_{n,j})-\psi_n] \qquad (3)$$

where

$$I^{e,1}=\{\ n\ :\ \sum_{j=1}^{M} R(p^{e,1}_{n,j})=K_n\} \qquad (4)$$

and

$$R(p^{e,1}_{i,j})=\begin{cases}1 & \text{if } p^{e,1}_{i,j}\neq(0,0)\\ 0 & \text{otherwise}\end{cases} \qquad (5)$$

The optimum global state is determined from the condition

$$\min_{1\le l\le L_E}\ (V^{E,l}=V^{E,l^0})\Rightarrow(p^{E,l^0}=p^0)$$

5.1.3. The state generation procedure

For the scheduling problem discussed in this paper, the state generation procedure has the form

$$\forall_{n\in I_k}\ \forall_m\left\{(p^{e-1,l}_{n,m}=0)\wedge(u^n_{k,m}=1)\wedge\left(\forall_j(F(p^{e-1,l}_{n,j})\neq k)\right)\right\}\Rightarrow$$

$$\Rightarrow(p^{e,l}=p^{e-1,l}+\delta p) \qquad (6)$$

The elements of the number-group-matrix $\delta P=[\delta p_{i,j}]_{N\times M}$ are defined as

following:

$$\delta p_{i,j}=\begin{cases}(k,t_{n,m}) & \text{if } (i=n)\wedge(j=m)\\ (0,0) & \text{otherwise}\end{cases} \tag{7}$$

The moment $t_{n,m}$ is defined by formula as following

$$t_{n,m}=\max(\varphi_{n_k,m}\ ,\ \tau^{m}_{h_m,n_k}+T^{e-1,l}_{m})+v_{n_k,m} \tag{8}$$

Where:

$T^{e-1,l}_{m}$ - the time at which the latest operation has been finished in state $p^{e-1,l}$ at machine A_m.

h - the number of the last operation serviced at machine A_m according to the state $p^{e-1,k}$.

The time $T^{e-1,l}_{m}$ is defined from the formula

$$T^{e-1,l}_{m}=\max_{1\le i\le N} G(p^{e-1,l}_{i,m}) \tag{9}$$

The number h_m is determined from the formula

$$h_m=\begin{cases} i_r & \text{if } (t_{i,m}=T^{e-1,l}_{m}>0)\wedge(F(p^{e-1,l}_{i,m})=r)\\ \alpha_m & \text{otherwise}\end{cases} \tag{10}$$

5.1.4. Unprespective state elimination

Exhausting rule:

Theorem 1. The state p is exhausted if condition is held

$$\exists_{n_k}\ (\forall_j F(p_{n,j})\neq k)\wedge(\forall_m\ (u^{n}_{k,m}=1)\wedge(p_{n,m}\neq 0)) \tag{11}$$

Domination rule:

Theorem 2. The state p is dominated by the state $p^{e,l}$ if the condition is held

$$\forall_{1\le n\le N}\ \forall_{1\le m\le M}\ \{(p^{e,l}_{n,m}=0)\vee(p_{n,m}=0)\}\wedge\{\forall_{1\le n_k\le K_n}\ (\exists_j F(p^{e,l}_{n,j})=k\vee\exists_i F(p_{n,i})=k)\}$$

$$\wedge\{T^{e,l}_{m}\le T_m\}\wedge\{v^{e,l}\le v\} \tag{12}$$

Fathoming rule:

Theorem 3. The state p is fathomed if condition is held

$$v^{c}\le v \tag{13}$$

where v^{c} is the value of the current best final state

5.2. The problem with sequencing relation

In this problem the relations among operations of object w_n are sequences, and it can be described by the matrix as following:

$$S_n=[s^{n}_{i,j}]_{K_n\times K_n}$$

$n=1,2,3,\ldots,N$

$i=1,2,3,\ldots,K_n$

$j=1,2,3,\ldots,K_n$

where

$$s_{i,j}=\begin{cases} 1 & \text{if operation } w_{n_i} \text{ must be finished before } w_{n_j} \\ 0 & \text{otherwise} \end{cases}$$

For this problem the state and the state value are defined as same as formal (2) and (3). The state generation procedure is defined as following:

$$\underset{n}{\forall}\underset{n_k}{\forall}\underset{m}{\forall}\left\{(p^{e-1,l}_{n,m}=0)\wedge(u^{n}_{k,m}=1)\wedge\left(\forall(s^{n}_{i,k}=1)\rightarrow\underset{1\le j\le M}{\exists}F(p^{e-1,l}_{n,j})=i\right)\right\}\rightarrow$$

$$\rightarrow(p^{e,l}=p^{e-1,l}+\delta p) \quad (14)$$

The elements of the number-group-matrix $\delta P=[\delta p_{i,j}]_{N\times M}$ are defined as:

$$\delta p_{i,j}=\begin{cases} (k,t_{n,m}) & \text{if } (i=n)\wedge(j=m) \\ (0,0) & \text{otherwise} \end{cases} \quad (15)$$

The moment $t_{n,m}$ and the other parameters are defined as same as in formula (6) and (7).

The elimination rules are as the same as that in 5.1.4.

5.3. The problem with excluding relation

The excluding relations among operations of object w_n can be described by the matrix

$$X_n=[x^{n}_{i,j}]_{K_n\times K_n}$$

$$n=1,2,3,\ldots,N$$
$$i=1,2,3,\ldots,K$$
$$j=1,2,3,\ldots,K^{II}_n$$

where:

$$x^{n}_{i,j}=\begin{cases} 1 & \text{if operation } w_{n_i} \text{ can not be serviced at same time as } w_{n_j} \\ 0 & \text{otherwise} \end{cases}$$

The state and state value are defined as same as that in 5.1.

The generation procedure depends on following two conditions:

$$\text{(A)}.\ \underset{n}{\forall}\underset{n_K}{\forall}\left(\underset{n_j}{\exists}(x^{n}_{K,j}=1)\wedge\underset{m_j}{\exists}F(p^{e-1,l}_{n,m_j})=j\right) \quad (16)$$

$$\text{(B)}.\ \underset{m}{\exists}(p^{e-1,l}_{n,m_k}=(0,0)\wedge(u^{n}_{k,m_k}=1)\wedge$$

$$\wedge\{(t_{n,m_k}-v_{n,m_k}\ge t_{n,m_j})\vee(t_{n,m_k}\le t_{n,m_j}-v_{n,m_j})\} \quad (17)$$

m_j - the number of machine at which operation w_{n_k} was serviced.

m_k - the number of machine at which operation w_{n_k} can be serviced.

The conditions (A) means that there is excluding relation between operation w_{n_k} and w_{n_j}, and the operation w_{n_j} had been serviced at machine m_j.

The condition (B) means that the operation w_{n_k} can be serviced at machine m_k excluding from servicing time of operation w_{m_j}.

If condition (A) holds, but condition (B) does not held, in this case, the state $p^{e-1,l}$ is exhausting one.

If both condition (A) and (B) held, the generation procedure can be defined as following :

$$\forall_{n}\ \forall_{n_k}\ \forall_{n_j}\ \forall_{m}(p^{e-1,1}_{n,m}=(0,0))\wedge(u^{n}_{k,m}=1)\wedge((x^{n}_{k,j}=1)\wedge\exists_{m_j} F(p^{e-1,1}_{n,m_j})=j)\wedge$$

$$\wedge\{(t_{n,m}-v_{n,m}\geq t_{n,m_j})\vee(t_{n,m}\leq t_{n,m_j}-v_{n,m_j})\}\rightarrow$$

$$\rightarrow(p^{e,1}=p^{e-1,1}+\delta p) \qquad (18)$$

The elements of the number-group-matrix $\delta P=[\delta p_{i,j}]_{N\times M}$ are defined as the same as it in the formula (7).

If the condition (A) does not held, the generation procedure can be defined as following:

$$\forall_{n}\ \forall_{n_k}\ \forall_{m}\left\{(p^{e-1,1}_{n,m}=0)\wedge(u^{n}_{k,m}=1)\right\}\rightarrow(p^{e,1}=p^{e-1,1}+\delta p) \qquad (19)$$

The elements of the number-group-matrix $\delta P=[\delta p_{i,j}]_{N\times M}$ are defined as the same as it in formula (7).

For elimination of unprespective states, the three theorems in 5.1.4 can be used, and another exhausting rule is given as following:

Theorem 4. The state p is exhausted if condition is held

$$\forall_{n}\ \forall_{n_K}\ \forall_{n_j}\ (\exists_{m_j}(x^{n}_{k,j}=1)\wedge F(p^{e-1,1}_{n,m_j})=j)\wedge((p^{e-1,1}_{n,m}=(0,0)\wedge(u^{n}_{k,m}=1))\wedge$$

$$\wedge\{(t_{n,m}-v_{n,m}\leq t_{n,m_j})\wedge(t_{n,m}\geq t_{n,m_j}-v_{n,m_j})\} \qquad (20)$$

m_j - the number of machine at which operation w_{n_j} was serviced.

5.4. The problem with synchronization relation

The synchronization relations among operations of object w_n can be described by the matrix

$$Y_n=[y^{n}_{i,j}]_{K_n\times K_n}$$

$$n=1,2,3,\ldots,N$$
$$i=1,2,3,\ldots,K_n$$
$$j=1,2,3,\ldots,K_n$$

where:

$$y^{n}_{i,j}=\begin{cases}1 & \text{if operations } w_{n_i} \text{ and } w_{n_j} \text{ must be serviced in the same time}\\ 0 & \text{otherwise}\end{cases}$$

The state and state value are defined as same as that in 5.1.

The generation procedure depends on following condition:

$$\text{(A)}.\ \forall_{n}\ \forall_{n_K}\ \forall_{n_j}\ (\exists_{m_j}(y^{n}_{K,j}=1)\wedge F(p^{e-1,1}_{n,m_j})=j) \qquad (21)$$

$$\text{(B)}.\ \exists_{m_k}(p^{e-1,1}_{n,m_k}=(0,0))\wedge(u^{n}_{k,m_k}=1)\wedge(t_{n,m_k}=t_{n,m_j}) \qquad (22)$$

m_j - the number of machine at which operation w_{n_j} was serviced,

m_k - the number of machine at which operation w_{n_k} will be servised.

The condition (A) means that there is synchronization relation between

operation w_{n_k} and w_{n_j} and the operation w_{n_j} had been serviced at machine m_j.

The condition (B) means that the operation w_{n_k} can be serviced at machine m_k in the same time as operation w_{m_j}.

If condition (A) holds, but condition (B) does not hold, in this case, the state $p^{e-1,l}$ is exhausting one.

If both condition (A) and (B) held, the generation procedure can be defined as following :

$$\forall_n \forall_{n_k} \forall_{n_j} \forall_m (p^{e-1,l}_{n,m}=(0,0)) \wedge (u^n_{k,m}=1) \wedge ((y^n_{k,j}=1) \wedge \exists_{m_j} F(p^{e-1,l}_{n,m_j})=j) \wedge$$

$$\wedge (t_{n,m}=t_{n,m_j}) \rightarrow (p^{e,l}=p^{e-1,l}+\delta p) \qquad (23)$$

The elements of the number-group-matrix $\delta P=[\delta p_{i,j}]_{N\times M}$ are defined as the following formula:

$$\delta p_{i,j}=\begin{cases} (k,t_{n,m}) & \text{if } (i=n)\wedge(j=m) \\ (0,0) & \text{otherwise} \end{cases}$$

where

$$t_{n,m}=t_{n,m_j}$$

If the first condition does not hold, the state generation procedure is as the same as formula (19) in 5.3.

For elimination of unperspective states, the three theorems in 5.1.4 can be used, and another exhausting rule is given as following:

Theorem 5. The state p is exhausted if condition is held

$$\forall_n \forall_{n_k} \forall_m \forall_{n_j} (\exists (y^n_{k,j}=1) \wedge \exists_{m_j} F(p^{e-1,l}_{n,m_j})=j) \wedge ((p^{e-1,l}_{n,m}=(0,0)) \wedge (u^n_{k,m}=1)) \wedge$$

$$\wedge (t_{n,m} \neq t_{n,m_j}) \qquad (24)$$

m_j- the number of machine at which operation w_{n_j} was serviced.

6. Final remarks

In the paper the multistage programming was applied to scheduling problem at complex systems. A feasible algorithm is given. If the system is with more complex structure, the problem will be more difficult than the one in this paper. The application of multistage programming method to the other problems are presented in others paper.

I would like to thank professor Franciszek Marecki, who as my professor gave me many good suggestions in my working.

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Streszczenie:

W referacie przedstawiony jest problem harmonogramowania w systemach z alternatywnymi marszrutami technologicznymi. System składa się z agregatów posiadających wejściowy i wyjściowy magazyn buforowy. Obiekty obsługiwane w agregatach systemu przesuwane są pomiędzy odpowiednimi magazynami buforowymi. Marszruty technologiczne obiektów (kolejne agregaty) są alternatywne. Problem polega na wyznaczeniu harmonogramu minimalizującego maksymalne opóźnienie obsługi obiektów.

Do rozwiązania tak sformułowanego problemu wykorzystano metodę programowania wieloetapowego. W tym celu zdefiniowano: stan procesu decyzyjnego, wartość stanu, procedurę generowania stanów oraz procedurę eliminacji stanów nieperspektywicznych.

Podano algorytmy harmonogramowania zadań zależnych oraz niezależnych.