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SOME THEORETICAL MODELS OF TECHNICAL WOVEN FABRIC AND THEIR USE IN FEM

Summary. The paper discusses the fundamental equations describing behaviour of membrane shells used for suspension roofs. An estimation of isotropic model and dense textile net model in the analysis of technical woven fabric is carried out. The results obtained from own finite element method (FEM) code and from commercial MARC software by MSC Company are compared. Good correlation of the results between both approaches has been obtained.

PEWNE MODELE TEORETYCZNE TKANINY TECHNICZNEJ I ICH ZASTOSOWANIE W MES

Streszczenie. Celem niniejszej pracy jest: przedstawienie niektórych typów równań konstytutywnych opisujących zachowanie powłok membranowych używanych do konstrukcji przekryć wiszących. Oceniono zastosowanie modelu izotropowego i sieci gęstej w analizie tkanin technicznych. Dokonano porównania wyników uzyskanych własnymi algorytmami metody elementów skończonych z powszechnie dostępnym na rynku programem komercyjnym MARC firmy MSC. Uzyskano dobrą zgodność wyników dla obu podejść.

1. Introduction

This paper concerns physical properties of PVC – coated fabrics which belong to modern group of materials applied for both seasonal and permanent constructions. The relations used for description of constitutive laws descend from long–standing research – see for example [6], [7], [3], [4]. An interesting approach to various aspects of the constitutive modelling for viscoelastic material is presented in [1].

The mathematical description of material behaviour and its constitutive characteristics enables the full utilisation of its properties in designing structural members.

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2. Material models of technical woven fabric

2.1. Elasticity matrix for isotropic materials

The isotropic material is frequently used for analysis of suspension structures made of plastic film (for example PVC), rubber-like materials, etc. In the analysis of membrane roofs we often use the stiffness parameter defined as:

$$F = Et \tag{1}$$

where:

E – Young's modulus

t – thickness of roof

In the analysis of roofs using FEM we assume plane stress condition. Therefore, the elasticity matrix **D** for an isotropic material is as follows:

$$\mathbf{D} = \frac{F}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \tag{2}$$

2.2. Elasticity matrix for dense net model

The dense net model provides more realistic working description than the isotropic model. The advantage of this model is that it enables taking into account the change of angle between families of threads during deformation. This approach has, however, a drawback i.e. forces in given family of threads depend on strains in the same direction only. Otherwise, the influence of PVC coat is neglected. The derivation of stiffness matrix is based on the assumption that one family of threads θ_1 is parallel to x axis of the local coordinate system and the second family θ_2 makes angle α with reference to the first family (Fig. 1) [3], [5]. The longitudinal stiffnesses F_1, F_2 of thread bundles are determined in the experimental way, on the basis of the tension tests in weft and warp direction. Therefore, the elasticity matrix has the following form:

$$\mathbf{D}=\begin{bmatrix} F_1+F_2\cos^4\alpha & F_2\sin^2\alpha\cos^2\alpha & F_2\sin\alpha\cos^3\alpha \\ F_2\sin^2\alpha\cos^2\alpha & F_2\sin^4\alpha & F_2\sin^3\alpha\cos\alpha \\ F_2\sin\alpha\cos^3\alpha & F_2\sin^3\alpha\cos\alpha & F_2\sin^2\alpha\cos^2\alpha \end{bmatrix} \tag{3}$$

The angle between threads families α, changes during deformation and is calculated on the basis of the current values of components σ_y and τ_{xy} in the plane stress state (Fig. 1).

$$\alpha=\operatorname{arctg}\frac{\sigma_y}{\tau_{xy}} \tag{4}$$

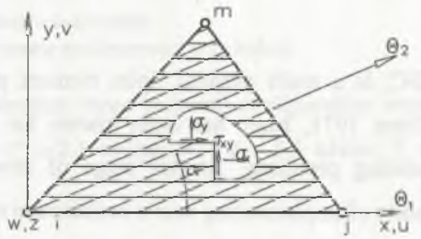

Fig. 1. Dense net model
Rys. 1. Model sieci gęstej

3. Membrane structures in FEM analysis

In the FEM analysis three-node triangular elements and isoparametric four-node of Lagrange type elements.[2] [8] were used. The local coordinate system for each element was assumed as shown in Fig. 2.

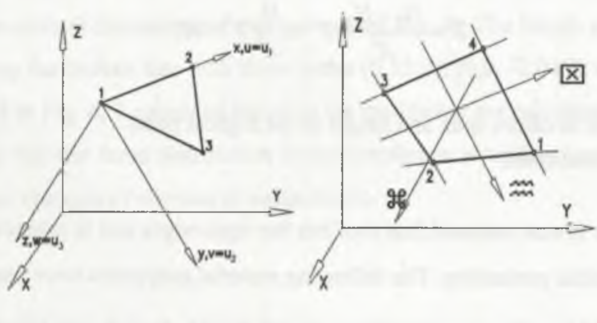

Fig. 2. Membrane finite elements: three-node triangular and isoparametric four-node
Rys. 2. Elementy membranowe: trójwęzłowy i czterowęzłowy izoparametryczny

4. Description of applied programs

4.1. FEM code (OFC)

The self-made FEM code for membrane analysis was used. The operational methodology, development and application limits of these codes are discussed in [5]. The main advantage of this approach is possibility of interference into all subroutines and adjusting them to a particular problem.

4.2. MARC

Program Marc, by MSC, is a multi purpose finite element program for an advanced engineering simulation. Since 1971, Marc has been known for its versatility in various industries to solve engineering problems. At this stage of investigation, only standard procedures, without possibility of implementing user's constitutive relations, were used.

5. Numerical example

5.1. Structure description

The geometrical non-linear calculations of a suspension roof of the hyperbolic paraboloid initial shape with square $2A \times 2A$ orthogonal projection (Fig. 3) were carried out. The vertical coordinates were computed from the following surface equation

$$Z = \frac{H_1 - H_2}{A^2} X^2 + \frac{H_1}{A^2} Y^2 - H_1 \qquad (5)$$

were:

H_1, H_2 – height in centre span and height in the highest point

$2A$ – diagonal span.

In the analysis it was assumed that roof has the rigid edges and is subjected to deadweight loading and an initial pretension. The following material properties were assumed: F_x=147.0 kN/m, F_y=83.4 kN/m (for dense textile net model) and $F = (F_x + F_y)/2$=115.2 kN/m (for isotropic model); ν=0.2; γ=0.10 kN/m^2 (deadweight of roof). The geometrical data are: A=15 m; H_1=2.045 m; H_2=3.546 m, initial angle between thread families $\alpha = 90^\circ$.

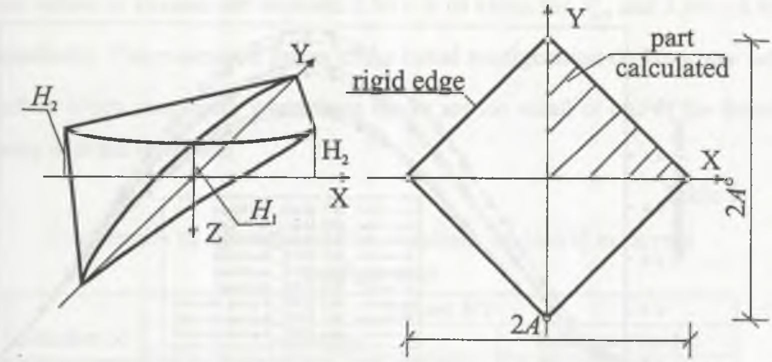

Fig. 3. Three-dimensional roof visualization
Rys. 3. Wizualizacja przestrzenna analizowanej konstrukcji

Three variants of pretension force in initial configuration were considered in this paper i.e.: *W1*: $T_x = 5.5$ kN/m, $T_y = 2.5$ kN/m; *W2*: $T_x = 5.5$ kN/m, $T_y = 4.0$ kN/m; *W3*: $T_x = 5.5$ kN/m, $T_y = 5.5$ kN/m.

5.2. Analysis of the results[1]

A sufficient agreement in vertical displacements and in stress distributions from OFC and Marc calculations was obtained. A 5% reduction in vertical displacements was obtained from dense net model in comparison with isotropic model. The resulting differences between these two models are caused by small change of the angle between threads families during deformation in the dense net model. It should be expected that these effects will grow when operational load will be applied. With increase of the initial force T_y in the undeformed configuration the vertical displacement also grows (see Fig. 4). The length parameter in Fig. 4 is calculated along the broken line with three nodes (0,15,0); (0,0, -2.045); (15,0, -3.546) (see Fig. 3). Presented in Fig. 6, 7 values of forces in the membrane are calculated for the *W2* case. It can be noticed that the force distribution in the membrane is uniform and the influence of the deadweight on changes of stresses is insignificant.

[1] Calculations were carried out at the Academic Computer Center in Gdańsk (TASK).

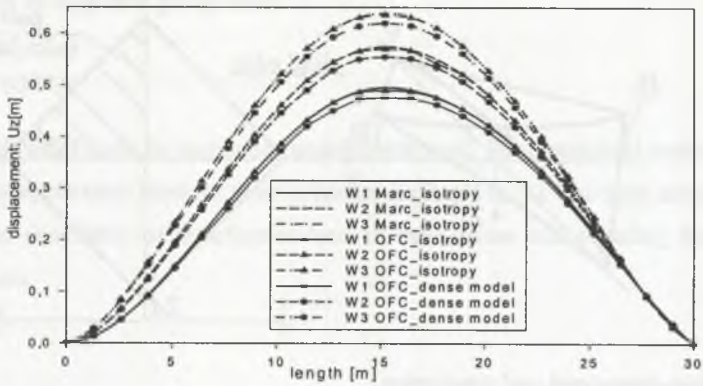

Fig. 4. Displacement's Uz profile
Rys. 4. Profil przemieszczenia Uz

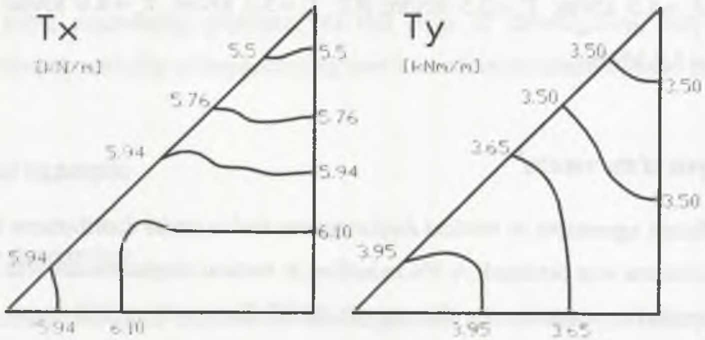

Fig. 5. The isolines of membrane forces (OFC)
Rys. 5. Izolinie sił błonowych (OFC)

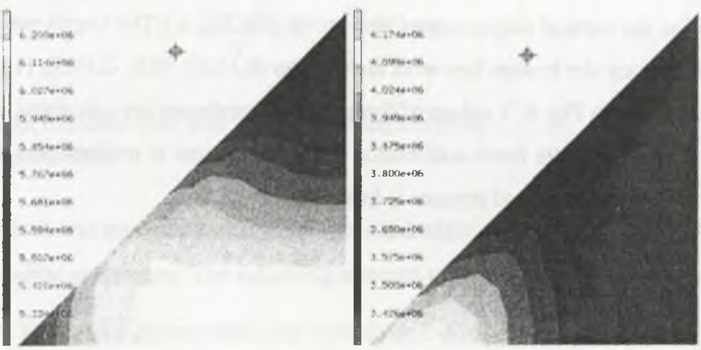

Fig. 6. The map of membrane forces (MARC)
Rys. 6. Mapy sił błonowych (MARC)

The final values of stresses are between 5.50 – 6.10 kN/m for T_x, and 3.50 – 3.95 kN/m for T_y, respectively. The pretension forces in the initial configuration do not cause folding of the roof surface which may occur when these forces are too small or one of the forces is too big comparing with the other one.

Table 1

Frequencies of symmetrical free vibrations ω_i [rad/s] in current configuration

Number of frequency	variant *W2*				
	isotropy			dense net model	
	MARC	OFC	{(2-3)/2} 100 %	OFC	{(5-3)/5} 100 %
1	*2*	*3*	*4*	*5*	*6*
1	5,10	5,05	*1,0*	5,13	*1,6*
2	9,20	8,99	*2,3*	9,09	*1,1*
3	10,60	10,40	*1,9*	10,40	*0,0*
4	13,40	12,70	*5,2*	12,80	*0,8*
5	15,40	14,50	*5,8*	14,60	*0,7*
6	17,10	16,10	*5,8*	16,20	*0,6*

Comparing values of the symmetrical frequencies in current configuration it can be noticed that obtained results differ by 1% – 5% only (Table 1). These differences can be caused by different type of finite elements or different material models.

6. Conclusion

The presented FEM approach and theoretical models of technical woven fabric seems to be useful in designing of the suspension structures. The proposed formulation is open and flexible. It has perspectives of many industrial applications.

6.1. Principal further research

The further research should concentrate on development of the dense net model, new constitutive relations with the emphasized influence of coating, rheological properties and temperature influences. Furthermore, the development of numerical methods, especially FEM techniques, and study on the original membrane shape and optimisation of membrane structures should be the subject of future research.

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Abstract

This paper deals with the analysis of theoretical woven fabric models used in membrane suspension structures. For the proposed problem the finite element method was used. The constitutive laws for both isotropic model and special model for description of woven fabric, named dense net model, were applied.