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DECOUPLING OF MULTIVARIABLE DISCRETE TIME SYSTEM USING OUTPUT FEEDBACK

Summary. A derivation of the output feedback decoupling for a time-invarinat multivariable discrete-time system is presented which is simple in concept and shows that decoupling produces a unity transmission system.

ODSPRZĘGANIE UKŁADÓW DYSKRETNYCH W CZASIE PRZEZ SPRZĘŻENIE OD WYJŚCIA

Streszczenie. Proste wyprowadzenie warunków odsprzęgania układów dyskretnych w czasie poprzez liniowe sprzężenie od wyjścia przedstawione zostało przy założeniu nieosobliwości odpowiednio skonstruowanej macierzy blokowej. Wykazano, że przedstawiona metoda odsprzęgania prowadzi do jednostkowej macierzy transmitancji.

РАЗВЯЗЫВАНИЕ СИСТЕМ ДИСКРЕТНЫХ ВО ВРЕМЕНИ ЧЕРЕЗ СОПРЯЖЕНИЕ ОТ ВЫХОДА

Резюме. В работе представлен простой вывод условий развязывания систем дискретных во времени через сопряжение от выхода. Отправной точкой являлась неособенность специально построенной блок-матрицы. Доказывается, что представляемый метод развязывания ведет к единочной матрице трансмиттанса.

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INTRODUCTION

Decoupling multivariable continuous-time system by output feedback has been discussed by several authors [1-6]. Falb and Wolovich [1] and Howze [2], developed output feedback decoupling in time domain and recently. Wang and Davison [3], employed a frequency domain approach based on a factorization of the transfer matrix of the given open-loop system. Wolovich [4], developed a necessary and sufficient conditions for output feedback decoupling and reassingning the closed-loop poles. Bayoumi and Duffiled [5], showed that the class of allowable decoupling matrices via linear output feedback can be extended to include the case where the feedback matrix is a function of s. Descusse and Malabre [6], solved the general decoupling problem of linear systems with constant (A, B, C, D) under the assumptions of regular output feedback. However, all these results are limited to continuous-time systems, no efforts have been made towards extending these results to discrete-time systems. Tan and Vandewalle [7], introduced the concept of complete decoupling of linear multivariable systems by means of linear static and differential state feedback. Kaczorek [8] extended the decoupling by state feedback problem to more general discrete-time systems. The aim of this paper is to present the conditions for decoupling linear multivariable discrete-time systems using linear output feedback.

STATEMENT OF THE PROBLEM AND MAIN RESULTS

Consider the linear-time invariant discrete-time system, which is assumed completely controllable and observable.

$$X(k+1)=\tilde{A}X(k)+\tilde{B}U(k) \qquad k=0,1,2,\ldots \tag{1-a}$$

$$Y(k)=CX(k) \qquad k=0,1,2,\ldots \tag{1-b}$$

where U(k) and Y(k) are m-input and m-output vectores, respectively, and X(k) are n-state vectors, and $\tilde{A}$, $\tilde{B}$ and C are nxn, nxm and mxn constant matrices respectively. K is any positive integer and the sampling time T=1 has been omitted for clarity.
The control law, has the from

$$U(k)=HY(k)+GV(k) \tag{2}$$

where H and G are mxm constant matrices.

Theorem 1: -

The necessary and sufficient condition for decoupling system (1) using output feedback control law (2) is that $\tilde{B}^*$ is nonsingular.

where $\tilde{B}^*$ is defined as:

$$\tilde{B}^* = \begin{bmatrix} C_1 & \tilde{A}^{d_1} & \tilde{B} \\ C_2 & \tilde{A}^{d_2} & \tilde{B} \\ & \vdots & \\ C_m & \tilde{A}^{d_m} & \tilde{B} \end{bmatrix} \qquad (3)$$

and

$$\left.\begin{aligned} d_i &= \{\min \ j \,|\, C_i \tilde{A}\tilde{B} \neq 0 && i = 1,2,\dots,m \\ & && j = 0,1,\dots,k-1 \\ d_i &= k-1 \ \text{if} \ C_i\tilde{A}\tilde{B} = 0 && \text{for all } j \end{aligned}\right\} \qquad (4)$$

where C_i is the i-th row of C

Proof I -

Substituting the control law (2) into eqn. (1) and Solving the resultant, with initial conditions X(0) = 0 leads to

$$Y(k) = C \sum_{j=0}^{k-1} (\tilde{A} + \tilde{B}HC)^j \, \tilde{B}GV(k-1-j) \qquad (5)$$

where v(k) is a reference m-input vector.

The pair H and G will decouple the system (1) if

$$Y_i(k) = e_i \, V(k-1-j) \qquad (6)$$

is satisfied, where $Y_i(k)$ is the i-th element of y(k) and e_i is an m-row vector with all elements zero except the i-th element.

Using eqn. (4) it can easily be shown that

$$C_i (\tilde{A}+\tilde{B}HC)^q \tilde{B}G = C_i \tilde{A}^q \tilde{B} = 0, \qquad q = 0,1,\ldots, d_i - 1 \tag{7}$$

and

$$C_i (\tilde{A}+\tilde{B}HC)^q \tilde{B}G = C_i \tilde{A}^{d_i} (\tilde{A}+\tilde{B}HC)^{q-d_i} \tilde{B}G, \qquad q = d_i, d_i+1, \ldots, k-1 \tag{8}$$

From eqn. (5) the i-th element of Y(k) can be written in the from

$$Y_i(k) = C_i[\tilde{B}GV(k-1) + (\tilde{A}+\tilde{B}HC)\tilde{B}GV(k-2) + \\ + \ldots + (\tilde{A}+\tilde{B}HC)^{d_i}\tilde{B}GV(k-d_i-1) + \ldots + (\tilde{A}+\tilde{B}HC)^{k-1}\tilde{B}GV(0)] \tag{9}$$

The object is to select H and G such that, by using eqns. (4), (7) and (8) each term in the series (of eqn. 9) is either zero or diagonal matrix.
By eqn. (7), Y (k) is reduced to

$$Y_i(k) = C_i[(\tilde{A}+\tilde{B}HC)^{d_i}\tilde{B}GV(k-d_i-1) + \\ + (\tilde{A}+\tilde{B}HC)^{d_i+1}\tilde{B}GV(k-d_i-2) + \ldots\ldots + (\tilde{A}+\tilde{B}HC)^{k-1}\tilde{B}GV(0)] \tag{10}$$

and by eqn. (8),

$$C_i(\tilde{A}+\tilde{B}HC)^{d_i+1} = C_i\tilde{A}^{d_i+1} + C_i\tilde{A}^{d_i}\tilde{B}HC \tag{11}$$

If H is chosen such that

$$C_i\tilde{A}^{d_i}\tilde{B}HC = -C_i\tilde{A}^{d_i+1} \tag{12}$$

eqn. (11) becomes

$$C_i(\tilde{A}+\tilde{B}HC)^{d_i+1} = 0 \tag{13}$$

Therefore, let

$$\tilde{A}^* = \begin{bmatrix} C_1 & \tilde{A}^{d_1+1} \\ C_2 & \tilde{A}^{d_2+1} \\ & \vdots \\ C_m & \tilde{A}^{d_m+1} \end{bmatrix} \tag{14}$$

then it is clear that if

$$HC = -\bar{B}^{*^{-1}} \bar{A}^{*} \qquad (15)$$

eqn. (12) becomes

$$C_i \tilde{A}^{d_i} \tilde{B}HC = -\tilde{B}_i^{*} \tilde{B}^{*^{-1}} \tilde{A}^{*} = -C_i \tilde{A}^{d_i+1} \qquad (16)$$

Assuming that $\bar{B}^{*}$ is nonsingular, $\bar{B}_i^{*} \bar{B}^{*^{-1}}$ is an m-row vector e_i, with 1 in the i-th position and zero elsewhere.
From eqns. (3), (11), (14) and (15)

$$C_i (\bar{A} + \tilde{B}HC)^q \tilde{B}G = 0, \qquad q = d_i + 1, d_i + 2, \ldots, k-1 \qquad (17)$$

and hence eqn. (10) is further reduced to

$$Y_i(k) = C_i (\bar{A} + \bar{B}HC)^{d_i} \bar{B}GV(k - d_i - 1) \qquad (18)$$

Remark: -

In the decoupling discrete-time system there is a delay time in the input with (d_i+1) i.e., the delay dependent on the system description.
The use of eqns. (3), (4) and (8) gives

$$C_i (\bar{A} + \bar{B}HC)^{d_i} \bar{B}G = C_i \bar{A}^{d_i} \bar{B}G = \bar{B}_i^{*} G \qquad (19)$$

Then (18) will be

$$Y_i(k) = C_i (\bar{A} + \bar{B}HC)^{d_i} \bar{B}GV(k - d_i - 1) = \bar{B}_i^{*} GV(k - d_i - 1) \qquad (20)$$

and from (6)

$$Y_i(k) = e_i V(k - d_i - 1)$$

$$Y_i(k) = \bar{B}_i GV(k - d_i - 1) = e_i V(k - d_i - 1) \qquad (21)$$

$$\bar{B}_i^{*} G = e_i \qquad \text{i.e.,} \quad G = \bar{B}^{*^{-1}} \qquad (22)$$

therefore

$$HC = -\tilde{B}^{*^{-1}}\ \tilde{A}^{*} \quad \text{or}$$

$$H = -\tilde{B}^{*^{-1}}\ \tilde{A}^{*}C^{\dagger}$$

and

$$G = -\tilde{B}^{*^{-1}}$$

decouple the system (1) if and only if $\tilde{B}^{*}$ is nonsingular.

$C^{\dagger}$ is the pseudo-inverse of the matrix C.

CONCLUSIONS

A formulation of the discrete-time problem was presented. The output feedback of time-invariant linear multivariable discrete-time systems was explained. The structure of decoupled discrete-time systems is studied, a central therm was presented, giving a necessary nad sufficient condition for output feedback decoupling. Also eqn. (21) shows that the decoupling produces a unity transmission system.

REFERENCES

[1] Falb, P. L. and Wolovich, W. A.: Decoupling in the desing and synthesis of multivariable control systems, IEEE Trans. Automatic Control, Vol. Ac-12, No.6, pp. 651-659, Dec. 1967.

[2] Howze, J. W.: Necessary and sufficient conditions for decoupling using output feedback, IEEE Trans. Automatic Control (short papers), Vol. Ac-18, No. 1, pp. 44-46, Feb. 1973.

[3] Wang, S. H. and Davison E.J.: Desing of decoupling control systems: A frequency domain, INT. J. Control, Vol. 21, No. 4, pp. 529-536, Apr. 1975.

[4] Wolovich, W. A.: Output feedback decoupling, IEEE Trans. Automatic Control, (Tech. Notes and Corresp.), Vol. Ac-20, No.2, pp. 148-149, Feb. 1975.

[5] Bayoumi, M. M. and Duffield, T. L.: Output feedback decoupling and pole placement in linear time-invariant System, IEEE Trans, Automatic Control, Vol. Ac-22, No. 2, pp. 142-143, Feb. 1977.

[6] Descusse, J. and Malabre, M.: Solvability of the decoupling problem for linear constant (A,B,C,D) quadruples with regular feedback, IEEE Trans. Automatic Control, Vol. Ac-27, No. 2, pp. 456-458, Feb. 1982.

[7] Tan S. and Vandewalle J.: Complete decoupling of linear multivariable systems by means of linear static and differential state feedback, INT. J. Control, Vol. 46, No. 4, pp. 1261-1266, Apr. 1987.
[8] Kaczorek T.: Two Dimensional Linear System, Springer-Verlag, Berlin 1985, pp. 285.

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