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DEADBEAT OBSERVERS FOR DISCRETE DESCRIPTOR SYSTEMS*

Summary. In this paper a deadbeat observer for discrete descriptor system is considered. The suggested procedure is based on the transformation of the system into a singular value decomposition co-ordinate form. Also, the concept of generalized matrix inverses is applied. The method does not presuppose the observer structure.

DOKŁADNY OBSERWATOR DLA DYSKRETNYCH UKŁADÓW SINGULARNYCH

Streszczenie. W pracy rozpatrzono dokładny obserwator dla dyskretnego układu singularnego. Procedura projektowania bazuje na transformacji układu z wykorzystaniem uogólnionych macierzy oraz wartości singularnych. Metoda ta nie wymaga zakładania z góry struktury obserwatora.

ТОЧНЫЙ НАБЛЮДАТЕЛЬ ДЛЯ ДИСКРЕТНЫХ СИНГУЛЯРНЫХ СИСТЕМ

Резюме. В работе расматривается точный наблюдатель для дискретной сингулярной системы. Процедура проектирования основана на преобразовании системы с использованием обобщенных матриц и сингулярных значений. Метод не требует априорной предпосылки о структуре наблюдателя.

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1. INTRODUCTION

Consider the discrete-time descriptor system described by

$$Ex(k+1) = Ax(k) + Bu(k) \tag{1a}$$

$$y(k) = Cx(k) \tag{1b}$$

where $x(k) \in R^n$ is the descriptor vector, $u(k) \in R^l$ is the input vector, and $y(k) \in R^m$ is the output vector. The matrices A,B and C are respectively dimensional and E is square and singular matrix and rank (E) = p.

Descriptor (singular or generalized state space) systems have recently received considerable effort [see, e.g., 6, 7, 8, 10,11, 16].

The observer desing problem for system (1) has been studied in [2, 4, 12]. For continuous time descriptor system, there are several approaches to solve this problem [see, e.g., 3, 5, 13, 14, 15].

In this paper a simple method to design a reduced-order deadbeat observer for discrete descriptor system is given. The suggested procedure does not presuppose the observer structure and is based on the singular value decomposition and the generalized inverses of matrices.

2. OBSERVER CONSTRUCTION

Let us perform a singular value decomposition (SVD) [9] of E

$$P^T E Q = \begin{bmatrix} \Sigma p & 0 \\ 0 & 0 \end{bmatrix}$$

where $\Sigma_p = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_p\}$ and σ_i $(i = 1,2,\ldots p)$ are the non-zero singular values of E, P and Q are orthogonal matrices.

Let us define

$$P^T A Q = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad P^T B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad CQ = [C_1 \quad C_2], \quad Q^T x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

with $x_1(k)$ being px1 and $x_2(k)$ being (n-p)x1. Then system (1) can be rewritten as

$$x_1(k+1) = \Sigma_p^{-1} A_1 x_1(k) + \Sigma_p^{-1} A_2 x_2(k) + \Sigma_p^{-1} B_1 u(k) \tag{2}$$

$$0 = A_3 x_1(k) + A_4 x_2(k) + B_2 u(k) \tag{3}$$

$$y(k) = C_1 x_1(k) + C_2 x_2(k) \tag{4}$$

It is evident that if A_4 is square and nonsingular matrix, then $x_2(k)$ can be evaluated from (3), and substituted in (2) and (4). Hence, a regular state-space system is found. Then an observer is constructed by using any of the well known approaches. However, if A_4 is singular matrix, then the following transformation is applied.
Equations (3) and (4) can be compressed into the equation

$$\begin{bmatrix} A_4 \\ C_2 \end{bmatrix} x_2(k) + \begin{bmatrix} A_3 \\ C_1 \end{bmatrix} x_1(k) + \begin{bmatrix} B_2 u(k) \\ -y(k) \end{bmatrix} = 0 \tag{5}$$

Clearly, (5) can be uniquely solved for $x_2(k)$ if and only if the matrix $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ has full column rank (n-p). It should be noted that if system (1) is strongly observable, i.e.: using a feasible transformation to bring the modal observability matrix

$\begin{bmatrix} sE - A \\ C \end{bmatrix}$ to the form $\begin{bmatrix} sE_1 - A_1' \\ A_2' \end{bmatrix}$ with E_1 of full row rank

the following conditions will be satisfied [16]

$$\text{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n, \ \forall \text{ finite values of s, and} \tag{6a}$$

$$\text{rank} \begin{bmatrix} E_1 \\ A_2' \\ C \end{bmatrix} = n \tag{6b}$$

Then $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ has rank (n-p). This can be proved by noticing that

$$E_1 = \begin{bmatrix} \Sigma_p & 0 \end{bmatrix} Q^{-1}; \quad A_2' = \begin{bmatrix} A_3 & A_4 \end{bmatrix} Q^{-1}; \quad C = \begin{bmatrix} C_1 & C_2 \end{bmatrix} Q^{-1}.$$

Substituting into (6b), yields

$$n = \operatorname{rank} \begin{bmatrix} E_1 \\ A_2' \\ C \end{bmatrix} = \operatorname{rank} \begin{bmatrix} \Sigma_p & 0 \\ A_3 & A_4 \\ C_1 & C_2 \end{bmatrix} Q^{-1} = p + \operatorname{rank} \begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$$

Then

$$\operatorname{rank} \begin{bmatrix} A_4 \\ C_2 \end{bmatrix} = n - p$$

If equation (5) is consistence, it has unique solution for $x_2(k)$. So, the generalized matrix inverse can be used to solve the equation (5). Substituting $x_2(k)$ into (2) and then using the resulting equation with the consistency condition for relation (5), an observer may be constructed to estimate $x_1(k)$, once $x_1(k)$ obtained, $x_2(k)$ can be found. However, in order to work with matrices of smaller dimensions which reduce and simplify the computational effort, the following manipulation is applied.

Since the matrix $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ has full column rank, there exists an (n-p+m) x (n-p+m) nonsingular matrix T such that $T \begin{bmatrix} A_4 \\ C_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ 0 \end{bmatrix}$, where T_1 is an (n-p) x (n-p) nonsingular matrix. Premultiplying (5) by T and letting

$$T \begin{bmatrix} A_3 \\ C_1 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}; \quad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix};$$

$$T \begin{bmatrix} B_2 u(k) \\ -y(k) \end{bmatrix} = \begin{bmatrix} T_{11} B_2 u(k) - T_{12} y(k) \\ T_{21} B_2 u(k) - T_{22} y(k) \end{bmatrix}$$

we get

$$T_1 x_2(k) = -M_1 x_1(k) - T_{11} B_2 u(k) + T_{12} y(k) \tag{7}$$

$$M_2 x_1(k) = -T_{21} B_2 u(k) + T_{22} y(k) \tag{8}$$

Substituting (7) into (2), yields

$$x_1(k+1) = \overline{A}_1 x_1(k) + \overline{B}_1 u(k) + \overline{C}_1 y(k) \tag{9}$$

where

$$\overline{A}_1 = \Sigma_p^{-1} A_1 - \Sigma_p^{-1} A_2 T_1^{-1} M_1$$

$$\overline{B}_1 = \Sigma_p^{-1} B_1 - \Sigma_p^{-1} A_2 T_1^{-1} T_{11} B_2$$

$$\overline{C}_1 = \Sigma_p^{-1} A_2 T_1^{-1} T_{12} \quad .$$

Now, the following two different-order observers may be constructed for system (8) and (9).

I. Full-order observer:

This observer can be directly constructed for system (8) and (9) to estimate $x_1(k)$ as follows

$$\hat{x}_1(k+1) = \left(\overline{A}_1 - \overline{K} M_2\right)\hat{x}_1(k) + \left(\overline{B}_1 - \overline{K} T_{21} B_2\right)u(k) + \left(\overline{C}_1 + \overline{K} T_{22}\right)y(k) \tag{10}$$

where $\overline{K}$ is a p x m arbitrary matrix which must be chosen such that the matrix $\left(\overline{A}_1 - \overline{K} M_2\right)$ has arbitrarily specified eigenvalues. Obviously, this can be done if and only if $\left(M_2, \overline{A}_1\right)$ is observable pair of matrices [17].

Theorem 1: If system (1) satisfies the observability condition (6a) then $\left(M_2, \overline{A}_1\right)$ is observable pair of matrices.

Proof: Using suitable matrix operations on (6a), yields

$$n = \text{rank}\begin{bmatrix} sE - A \\ C \end{bmatrix} = \text{rank}\begin{bmatrix} I_p & 0 \\ 0 & T \end{bmatrix}\begin{bmatrix} s\Sigma_p - A_1 & -A_2 \\ \begin{bmatrix} A_3 \\ C_1 \end{bmatrix} & \begin{bmatrix} A_4 \\ C_2 \end{bmatrix} \end{bmatrix} =$$

$$= \text{rank}\begin{bmatrix} \Sigma_p^{-1} & 0 & 0 \\ 0 & I_{n-p} & 0 \\ 0 & 0 & I_m \end{bmatrix}\begin{bmatrix} s\Sigma_p - A_1 & -A_2 \\ M_1 & T_1 \\ M_2 & 0 \end{bmatrix}\begin{bmatrix} I_p & 0 \\ -T_1^{-1}M_1 & I_{n-p} \end{bmatrix} =$$

$$= \text{rank}\begin{bmatrix} sI_p - (\Sigma_p^{-1}A_1 - \Sigma_p^{-1}A_2T_1^{-1}M_1) & -\Sigma_p^{-1}A_2 \\ 0 & T_1 \\ M_2 & 0 \end{bmatrix}$$

Consequetly,

$$\text{rank}\begin{bmatrix} sI_p - \overline{A}_1 \\ M_2 \end{bmatrix} = p \tag{11}$$

which completes the proof. ■

So, once $\hat{x}_1(k)$ is obtained, $\hat{x}_2(k)$ can be found by using (7), which is

$$\hat{x}_2(k) = -T_1^{-1}M_1\hat{x}_1(k) - T_1^{-1}T_{11}B_2u(k) + T_1^{-1}T_{12}y(k) \tag{12}$$

II. Reduced-order observer:

Let us put (8) in the following form

$$\overline{y}(k) = M_2x_1(k) \tag{13}$$

where $\overline{y}(k) = -T_{21}B_2u(k) + T_{22}y(k)$, and letting rank $M_2 = d$. Then an observer of order (p-d) is constructed as follows.

Using the generalized matrix inverses [1], the general solution of (13) is

$$x_1(k) = M_2^g \bar{y}(k) + \left(I_p - M_2^g M_2\right) f(k) \tag{14}$$

with consistency condition

$$\left(I_m - M_2 M_2^g\right)\bar{y}(k) = 0 \tag{15}$$

where M_2^g is a p x m generalized inveres of M_2 and f(k) is a p x 1 arbitrary vector. Let us take the (SVD) of M_2 which is

$$M_2 = U\begin{bmatrix} \Sigma_d & 0 \\ 0 & 0 \end{bmatrix} V^T; \text{ then } M_2^g = V\begin{bmatrix} \Sigma_d^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^T$$

where Σ_d is a dxd nonsingular matrix, and U and V are square orthogonal matrices of order m and p, respectively. Letting $V = [V_1, V_2]$, where V_2 is a p x (p-d) full colum rank matrix, (14) becomes

$$x_1(k) = M_2^g \bar{y}(k) + V_2 h(k) \tag{16}$$

where $h(k) = V_2^T f(k)$ is a (p-d) vector. Substituting (16) into (9), yields

$$V_2 h(k+1) = \bar{A}_1 V_2 h(k) + \bar{Q}_1 y(k) + \bar{L}_1 u(k) - M_2^g \bar{y}(k+1) \tag{17}$$

where $\bar{Q}_1 = \bar{A}_1 M_2^g T_{22} + \bar{C}_1$; $\bar{L}_1 = \bar{B}_1 - \bar{A}_1 M_2^g T_{21} B_2$.

Again, since V_2 has full column rank, there existe a p x p nonsingular matrix $\bar{P}$ such that $\bar{P}V_2 = \begin{bmatrix} \bar{V}_2 \\ 0 \end{bmatrix}$, where $\bar{V}_2$ is a (p-d) x (p-d) nonsingular matrix. Premultiplying (17) by $\bar{P}$, yields

$$\begin{bmatrix} \bar{V}_2 \\ 0 \end{bmatrix} h(k+1) = \begin{bmatrix} \bar{A}_{11} \\ \bar{A}_{21} \end{bmatrix} h(k) + \begin{bmatrix} \bar{Q}_{11} \\ \bar{Q}_{21} \end{bmatrix} y(k) + \begin{bmatrix} \bar{L}_{11} \\ \bar{L}_{21} \end{bmatrix} u(k) - \begin{bmatrix} \bar{D}_1 \\ \bar{D}_2 \end{bmatrix} \bar{y}(k+1) \tag{18}$$

Let us denote

$$w(k)=\overline{V}_2 h(k)+\overline{D}_1\overline{y}(k) \tag{19}$$

Then (18) can be splitted into the following two equations

$$\begin{aligned} w(k+1)=&\overline{A}_{11}\overline{V}_2^{-1}w(k)+\left(\overline{Q}_{11}-\overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{22}\right)y(k)+\\ &+\left(\overline{L}_{11}+\overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2\right)u(k) \end{aligned} \tag{20}$$

$$\begin{aligned} \overline{A}_{21}\overline{V}_2^{-1}w(k)=&\overline{D}_2\overline{y}(k+1)+\left(\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_2T_{22}-\overline{Q}_{21}\right)y(k)-\\ &-\left(\overline{L}_{21}+\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2\right)u(k) \end{aligned} \tag{21}$$

Equations (20) and (21) can be interpreted as a dynamical system, where w(k) is the state vector, $\begin{bmatrix} y(k) \\ u(k) \end{bmatrix}$ is the input vector and the right hand side of (21) is the output vector.

An observer of order (p-d) can be initially constructed for system (20) and (21) as follows

$$\begin{aligned} \overline{w}(k+1)=&\left(\overline{A}_{11}\overline{V}_2^{-1}-\tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\right)\overline{w}(k)+\\ &+\left(\overline{Q}_{11}-\overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{22}-\tilde{K}\overline{Q}_{21}+\tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1T_{22}\right)x\\ &x\ y(k)+\left(\overline{L}_{11}+\overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2-\tilde{K}\overline{L}_{21}-\tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2\right)\\ &u(k)+\tilde{K}\overline{D}_2\overline{y}(k+1) \end{aligned} \tag{22}$$

where $\tilde{K}$ is a (p-d) x d arbitrary matrix which must be chosen such that the matrix $(\overline{A}_{11}\overline{V}_2^{-1}-\tilde{K}\overline{A}_{21}\overline{V}_2^{-1})$ has arbitrarily specified eigenvalues. As before, this is satisfied of and only if the pair of matrices $(\overline{A}_{21}\overline{V}_2^{-1},\overline{A}_{11}\overline{V}_2^{-1})$ is observable.

Theorem 2: If condition (11) is satisfied, then $(\overline{A}_{21}\overline{V}_2^{-1}, \overline{A}_{11}\overline{V}_2^{-1})$ is an observable pair of matrices.

Proof: Using suitable matrix operation on (11), yields

$$p = \operatorname{rank}\begin{bmatrix} sI - \overline{A}_1 \\ M_2 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} I & O \\ O & U^T \end{bmatrix}\begin{bmatrix} sI - \overline{A}_1 \\ U\begin{bmatrix} \Sigma_d & O \\ 0 & 0 \end{bmatrix} V^T \end{bmatrix}\begin{bmatrix} V_1 & V_2 \end{bmatrix}$$

$$= \operatorname{rank}\begin{bmatrix} [sI - \overline{A}_1]V_1 & [sI - \overline{A}_1]V_2 \\ \Sigma_d & O \end{bmatrix}$$

Clearly, rank $(sI - \overline{A}_1)V_2 = p - d = \operatorname{rank}[\overline{P}](sI - \overline{A}_1)V_2 = \operatorname{rank}\begin{bmatrix} s\overline{V}_2 - \overline{A}_{11} \\ \overline{A}_{21} \end{bmatrix}$,

which completes the proof ■

Returning to (22), $\overline{y}(k+1)$ can be eliminated by defining another new variable as follows

$$z(k) = \overline{w}(k) - \tilde{K}\overline{D}_2\overline{y}(k) \tag{23}$$

and then the final form of (22) may be written as

$$z(k+1) = Fz(k) + Gy(k) + Su(k) \tag{24}$$

where

$$F = \overline{A}_{11}\overline{V}_2^{-1} - \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}$$

$$G = \overline{Q}_{11} - \overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{22} - \tilde{K}\overline{Q}_{21} + \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1T_{22} +$$
$$+ \overline{A}_{11}\overline{V}_2^{-1}\tilde{K}\overline{D}_2T_{22} - \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\tilde{K}\overline{D}_2T_{22}$$

$$S = \overline{L}_{11} + \overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2 - \tilde{K}\overline{L}_{21} - \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2 -$$
$$- \overline{A}_{11}\overline{V}_2^{-1}\tilde{K}\overline{D}_2T_{21}B_2 + \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\tilde{K}\overline{D}_2T_{21}B_2$$

Also, the estimated state $\hat{x}_1(k)$ can be obtained by using (16), (19), and (23) as follows

$$\hat{x}_1(k) = V_2\,\overline{V}_2^{-1} z(k) + \overline{R}y(k) + \tilde{R}u(k) \tag{25}$$

where

$$\overline{R} = \left(M_2^g + V_2\overline{V}_2^{-1}\tilde{K}\overline{D}_2 - V_2\overline{V}_2^{-1}\overline{D}_1\right)T_{22}$$

$$\tilde{R} = \left(V_2\overline{V}_2^{-1}\overline{D}_1 - M_2^g - V_2\overline{V}_2^{-1}\tilde{K}D_2\right)T_{21}B_2$$

Deadbeat observer construction:
In the following the deadbeat observer construction is considered. Defining the state reconstruction error

$$e(k) = x_1(k) - \hat{x}_1(k) \tag{26}$$

an using (8), (9) and (10), yields

$$e(k+1) = \left(\overline{A}_1 - \overline{K}M_2\right)e(k) \tag{27}$$

So, at any instant q≥0

$$e(q) = \left(\overline{A}_1 - \overline{K}M_2\right)^q e(0) \tag{28}$$

It is required to choose the matrix $\overline{K}$ such that e(q) is reduced to zero in, at most, p steps. An observer with this property is known as a deadbeat observer, and it is evident that such an observer will produce a completely accurate reconstruction of the state of the pth-order system (8) and (9), after, at most, p steps.

Assuming m≤p and M_2 has full row rank. Then there exists a nonsingular matrix T which transforms the pair $\left(\overline{A}_1, M_2\right)$ to the following observable companon forms

$$
\hat{A}_1 = T^{-1}\overline{A}_1 T = \begin{array}{c}
\left[\begin{array}{ccccc|ccccc|ccccc}
0 & 0 & \dots & 0 & * & 0 & 0 & \dots & & * & 0 & 0 & \dots & & * \\
1 & 0 & \dots & 0 & * & 0 & 0 & \dots & & * & 0 & 0 & \dots & & * \\
\vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & & \vdots \\
0 & 0 & \dots & 1 & * & 0 & 0 & \dots & & * & 0 & 0 & \dots & & * \\
0 & 0 & \dots & & * & 0 & 0 & \dots & 0 & * & 0 & 0 & \dots & & * \\
0 & 0 & \dots & & * & 1 & 0 & \dots & 0 & * & 0 & 0 & \dots & & * \\
\vdots & \vdots & & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & & \vdots \\
0 & 0 & \dots & & * & 0 & 0 & \dots & 1 & * & 0 & 0 & \dots & & * \\
 & & \vdots & & & & & \vdots & & & & & \vdots & & \\
0 & 0 & \dots & & * & 0 & 0 & \dots & & * & 0 & 0 & \dots & 0 & * \\
0 & 0 & \dots & & * & 0 & 0 & \dots & & * & 1 & 0 & \dots & 0 & * \\
\vdots & \vdots & & & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & \dots & & * & 0 & 0 & \dots & & * & 0 & 0 & \dots & 1 & *
\end{array}\right] \\
\longleftarrow r_1 \longrightarrow \uparrow_{\sigma_1} \qquad \longleftarrow r_2 \longrightarrow \uparrow_{\sigma_2} \qquad \longleftarrow r_m \longrightarrow \uparrow_{\sigma_m}
\end{array}
$$

and

$$
\hat{A}_2 = M_2 T = \begin{array}{c}
\left[\begin{array}{ccccc|ccccc|ccccc}
0 & 0 & \dots & & 1 & 0 & 0 & \dots & & 0 & 0 & 0 & \dots & & 0 \\
0 & 0 & \dots & & * & 0 & 0 & \dots & & 1 & 0 & 0 & \dots & & 0 \\
\vdots & \vdots & & & \vdots & \vdots & \vdots & & & \vdots & \vdots & \vdots & & & \vdots \\
0 & 0 & \dots & & * & 0 & 0 & \dots & & * & 0 & 0 & \dots & & 1
\end{array}\right] \\
\longleftarrow r_1 \longrightarrow \uparrow_{\sigma_1} \qquad \longleftarrow r_2 \longrightarrow \uparrow_{\sigma_2} \qquad \longleftarrow r_m \longrightarrow \uparrow_{\sigma_m}
\end{array}
$$

where $\sum_{i=1}^{m} r_i = p$ and $\max_{1\le i\le m} r_i$ is the so called observability index of $(\overline{A}_1, M_2)$. The matrix $\hat{A}_1$ consist of m observable companion forms, each of dimension r_i, along the leading diagonal of which the non-trivial columns are denoted by σ_i. So, the square matrix $\hat{A}_1$ has m non-trivial columns which constitute p x m matrix $\bar{A}_1$, and the matrix $\hat{A}_2$ has m non-trivial columns which constitute m x m nonsingular matrix $\bar{A}_2$. Then the matrix $\overline{K}$ can be determined by

$$\overline{K} = T\bar{A}_1 \bar{A}_2^{-1} \tag{29}$$

This $\overline{K}$ makes the matrix $(\overline{A}_1 - KM_2)$ a nilpotent matrix with nilpotency index equal to $\max_{1 \le i \le m} r_i$, the dimension of the largest companion form of $\hat{A}_1$. So, the p eigenvalues of the observer are zeros. This complete the deadbeat observer construction for the full-order observer, and in the same manner, this may be done for the reduced-order observer.

3. CONCLUSIONS

A straightforward method to desing full-order and reduced-order deadbeat observers for the discrete descriptor system is presented. Firstly, the given descriptor system is transformed into singular value decomposition descriptor-space coordinates by performing a SVD of matrix E. And then, a full-order - rank E- deadbeat observer may be designed. However, in order to design a reduced-order observer the generalized matrix inverse and the SVD of matrices is used. The suggested procedure does not presuppose the observer structure.

REFERENCES

[1] S.Barnett, Matrices in Control Theory with Application to Linear Programming, Van Nostrand Reinhold, London 1971.

[2] L.Dai, Observers for discrete singular systems. IEEE Trans. on Automat. Control, 33 (1988) 187-191.

[3] M. El-Tohami, V. Lovass-Nagy and R. Mukundan, On the desing of observers for generalized state space systems using singular value decomposition, Internat. J. Control 38 (1983) 673-683.

[4] M. El-Tohami, V, Lovass-Nagy and R. Mukundan, Design of observers for discrete-time descriptor systems, Internat. J. Control, 46 (1987) 841-848.

[5] M.M. Fahmy and J. O'Reilly, Observers for descriptor systems, Internat. J. Control 49 (1989) 2013-2028.

[6] T. Kaczorek, Different forms of the description of generalized linear systems, Proceeding of 15-th Seminar on Fundamentals of Electrotechnics and Circuit Theory, Wisła, Poland, May (1992) 13-29.

[7] T. Kaczorek, Linear Control System, Vol. 1 entitled "Analysis of Multivariable Systems", John Wiley and Sons Inc., New York 1992.

[8] T. Kaczorek, Theory of Control and Systems, PWN, Warszawa 1993. (in Polish).

[9] V.G. Klema and A.J. Laub, The singular value decompsition: Its computation and some applications, IEEE Trans. Automat. Control 25 (1980) 164-176.

[10] F. L. Lewis, A survey of linear singular systems, Circutis Systems Signals Process, 5 (1986) 3-36.

[11] D.G. Luenberger, Dynamic equations in descriptor form, IEEE Trans. Automat. Control, 22 (1977) 312-321.

[12] A. - M. Mohamed, Observers for discrete descriptor systems. Proceeding of Second MEPCON-92, January 6-9, Assiut Egypt, (1992) 221-223.

[13] A.-M. Mohamed, Observer design for descriptor systems, Foundation of Computing and Decision Sciences, 15 (1991) 173-180.

[14] B. Shafai and R.L. Caroll, Desing of minimal order observers for singular systems, Internat. J. Control 45 (1987) 1075-1081.

[15] D.N. Shields, Observers for descriptor systems, Internat. J. Control 55 (1992) 249-256.

[16] G.C. Verghese, B.C. Levy and T. Kailath, A generalized state-space for singular systems, IEEE Trans. Autoamt. Control 26 (1981) 811-831.

[17] W.H. Wonham, On pole assignement in multi-input controllable linear systems, IEEE Trans. Automat. Control 12 (1967) 660-665.

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