ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2007
Seria: BUDOWNICTWO z. 112 Nr kol. 1763

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FROM THE FRACTURE-MECHANICAL EXPERIMENT TO THE STOCHASTIC MODEL OF QUASI-BRITTLE STRUCTURE/STRUCTURAL MEMBER

Summary. The paper contributes to the complex approach to the design and evaluation of quasi-brittle structures/structural members made of (advanced) cement based composites. It reviews recent results in the experimental fracture mechanics of quasi-brittle cement based composites obtained by the research group at Institute of Structural Mechanics (fracture parameters, incl. their statistics or influence of material degradation) as well as chosen aspects and results of nonlinear stochastic analysis of structural members made of these composites and inserts them into the presented complex approach.

OD EKSPERYMENTU DO MODELU STOCHASTYCZNEGO QUASI-KRUCHEGO ELEMENTU KONSTRUKCYJNEGO

Streszczenie: Referat przyczynia się do kompleksowego podejścia do problemu projektowania i oceny quasi-kruchych elementów konstrukcyjnych wykonanych z kompozytów cementowych. W pracy dokonano przeglądu dotychczasowych wyników doświadczeń z zakresu mechaniki pękania dla quasi-kruchych kompozytów cementowych prowadzonych przez grupę badawczą Instytutu Mechaniki Budowli. Dokonano także przeglądu wybranych aspektów i wyników nieliniowej analizy stochastycznej elementów konstrukcyjnych wykonanych ze wspomnianych kompozytów w kontekście kompleksowego podejścia do problematyki ich projektowania.

1. Introduction

The structural design which takes into account the nonlinear fracture mechanics principles as well as probability aspects is still not common in civil engineering. This situation is noticeable especially in the case of quasi-brittle structures/structural members made typically of cement based composites. The role of codes is still dominant in this field. The reasons why

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to use nonlinear fracture mechanical principles are reviewed e.g. in Bazant and Planas (1998). Two examples of the complex approach to the design and evaluation of structures/structural members made of (advanced) cement based composites are presented in the paper. Presented results can contribute to the optimal design of studied structural members. They can improve safety, reliability, durability or efficiency of the design/evaluation. The scheme of the complex approach is shown in Fig. 1.

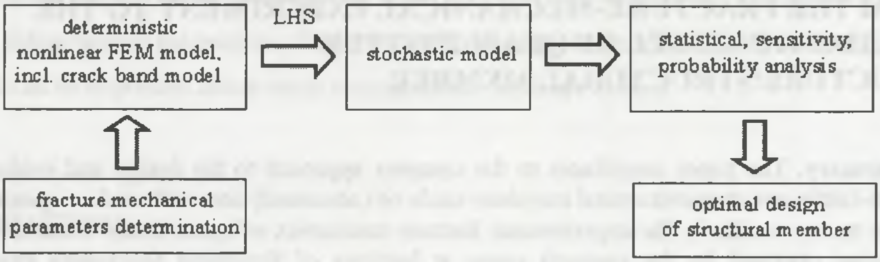

Fig. 1. Scheme of presented complex approach to the design and evaluation of structural members; LHS – Latin Hypercube Sampling Method

Rys. 1. Schemat prezentowanego, kompleksowego podejścia do projektowania oceny elementu konstrukcyjnego; metoda LHS

2. Fracture-mechanical experiments and their results and statistics

Fracture mechanical parameters play important role in the complex description/modelling of the cement based composite's response to the applied load. The knowledge of fracture energy is necessary for using cohesive crack models implemented into FEM codes (e.g. classical Bazant's crack band model [11] implemented also in software ATENA 2D/3D [2] which was used for the deterministic nonlinear numerical models presented in the paper). The fracture energy can be determined from the fracture mechanical experiments using work-of-fracture method [3,10,14] as well as sophisticated approach of inverse analysis based on coupling of nonlinear fracture mechanics FEM modelling, probabilistic stratified simulation for training neural network and artificial neural network [5].

2.1. Railway sleeper's concrete

Fracture mechanical parameters of quasi-brittle materials can be obtained from fracture-mechanical experiment/test of notched specimen. Three point bending test of central edge notched specimen (in this case 480×80×80 mm, notch depth 25 mm, span 400 mm) presents the useful test configuration (Fig. 2 left, specimens made of sleeper's concrete, load-deflection diagrams from six tests are presented). Results obtained from both three point bending tests (modulus of elasticity, fracture energy) and "classical" tests of cubic specimens (compressive and tensile strength)/material model's inputs (inc. statistics of the variables important for stochastic model) are presented in tab. 1.

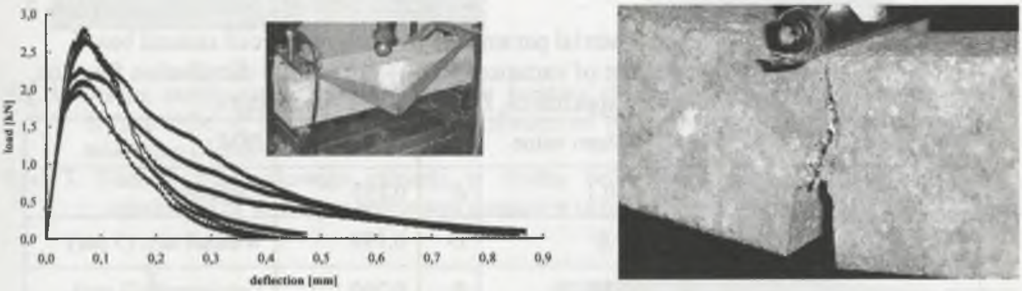

Fig. 2. Three point bending test of central notched specimen made of sleeper's concrete and load-deflection diagrams obtained from six tests (left); detail of three-point bending test of fibre-reinforced notched specimen (right)

Rys. 2. Próba trójpunktowego zginania środkowo naciętej próbki betonowej i wykres ugięcia pod wpływem obciążenia uzyskany z sześciu wykonanych prób (z lewej); detal próby trójpunktowego zginania naciętej próbki wykonanej z betonu zbrojonego włóknami (z prawej)

Table 1

Chosen results of sleeper's concrete tests inc. statistics of the variables [16]; *COV* – coefficient of variation, *PDF* – probability distribution function

Variable	Unit	Mean value	*COV* [–]	*PDF*
Modulus of elasticity	GPa	32.4	0.08	Normal
Compressive strength	MPa	75.0	0.12	Log-normal (2 par)
Tensile strength	MPa	4.0	0.12	Log-normal (2 par)
Fracture energy	J/m^2	180.0	0.254	Normal

2.2. Glass-fibre reinforced cement based composite, degradation of composite

Large set of experiments was performed to determine fracture mechanical parameters of advanced glass-fibre reinforced cement based composite. Also the influence of material degradation was taken into account. Altogether 80 specimens – notched beams made of the same batch – were tested (Fig. 2 right). Nominal geometry of specimens were 40×40×200 mm, notch depth 15 mm, span 180 mm. First 40 specimens were cured in normal conditions (reference specimens – *R*); the second half of the specimens was immersed into a special solution for 5 days, which is equal app. to degradation process of 20 years (degraded specimens – *D*). Already named approaches were used for the determination of fracture mechanical parameters. The synthesis of the results is summarized in table 2. It is obvious that the degradation evokes significant decrease of the material characteristics. Note also that the fracture energy of fibre reinforced composite is markedly higher than the value of sleeper's concrete.

Table 2

Basic random variables of material parameters of fibre-reinforced cement based composite [13]; *COV* – coefficient of variation, *PDF* – probability distribution function, *R* – reference specimens, *D* – degraded specimens

Variable	Unit	Mean value		*COV* [–]	*PDF*
Modulus of elasticity	GPa	10.1	*R*	0.195	Rayleigh
		7.8	*D*	0.199	Weibull min (3 par)
Compressive strength	MPa	53.5	*R*	0.250	Log-normal (2 par)
		31.5	*D*	0.250	Log-normal (2 par)
Tensile strength	MPa	6.50	*R*	0.250	Weibull min (2 par)
		3.81	*D*	0.250	Weibull min (2 par)
Fracture energy	J/m^2	816.2	*R*	0.383	Weibull max (3 par)
		195.8	*D*	0.418	Log-normal (2 par)

3. Stochastic models of chosen structural members and their analysis

3.1. Railway sleepers

The pre-stressed sleepers build the crucial part of the railway building. It is necessary to check their quality during the production and to ensure the sufficient reliability during their using. Standardized three-point bending tests (for the profile in the middle of the sleeper (Fig. 3) and for the profile under the rail (Fig. 4)) on the hydraulic press were defined to ensure checking of the sleeper's quality. The amount of the manufactured sleepers as well as the large number of prescript check tests presents the high requirements on the testing laboratory. The suitable numerical model (stochastic level) can extend experimental dates and contributed to the ensuring of the pre-stressed sleeper's reliability. The numerical model also provides possibilities to make other parametric studies. The scheme of numerical model of railway sleeper tested in the profile under the rail (created in software ATENA 2D with SBeta material model) is presented in the Fig. 4. Parameters of material model were set up according to experimental results (tab. 1).

Fig. 3. Testing configuration of the three-point bending check test of the sleeper in the profile in the middle of the sleeper (left); installation of the deflectometers for the experimental measuring (right)

Rys. 3. Badanie trójpunktowego zginania w środku belki podwalinowej (z lewej); instalacja ugięciomierza w miejscu wykonania pomiarów (z prawej)

Fig. 4. Scheme of the testing configuration for the profile under the rail (left); illustration of corresponding FEM model (created in software ATENA 2D) with finite elements mesh, pre-stressing reinforcement and macro-elements (right) [15]

Rys. 4. Schemat badanego układu dla profilu pod szyną (z lewej); przedstawienie odpowiadającego mu modelu (stworzonego w programie ATENA 2D) z siatką elementów skończonych i wstępnym sprężeniem wzmocnienia (z prawej)

After the agreement between experiments and numerical models was verified the set of numerical parametric study dealing with influence of material properties as well as influence of pre-stress (e.g. Fig. 5 for the profile under the rail) on the sleeper's behaviour was performed (also Routil et al. 2006a, Stancl et al. 2006). Also stochastic models of three-point bending tests were created – the first took into account randomness of concrete parameters (Fig. 6 for the profile in the middle of the sleeper, Routil and Kersner 2004) while the second was focused to the randomness of pre-stress. Stochastic models of the described experiment combine deterministic nonlinear numerical model in ATENA 2D (used material model – SBeta(Cervenka, Pukl 2005)) and Latin Hypercube Sampling Method (*LHS*, Novak et al. 2006) used for generating of set of random input parameters. The complex software system SARA was developed for this approach (Pukl et al. 2003). The most important material input parameters (modulus of elasticity, fracture energy, tensile and compressive strength) according to sensitivity analysis are considered as random variables (tab. 1). Statistical correlation among random input variables is also considered. The use of crack band model allows determining of the crack width during the increasing load. The probabilistic analysis (besides statistical and sensitivity analysis) focused among others on probability of crack width achievement for different levels of load were performed (Fig. 6).

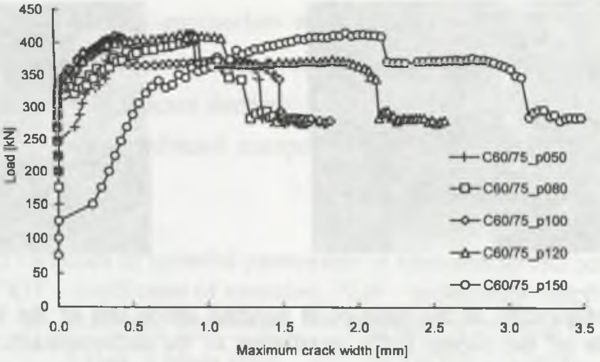

Fig. 5. Graphs of load vs. maximum crack width (for the profile under the rail) obtained from numerical parametric study: concrete class C60/75; 50% (*p050*), 80% (*p080*), 100% (*p100*), 120% (*p120*) and 150% (*p150*) of pre-stressing force [12]

Rys. 5. Wykres obciążeń dla maksymalnego rozwarcia szczeliny (dla profilu pod szyną) uzyskany z badań numerycznych: klasa betonu C60/75; 50% (*p050*), 80% (*p080*), 100% (*p100*), 120% (*p120*) i 150% (*p150*) siły wstępnie sprężającej [12]

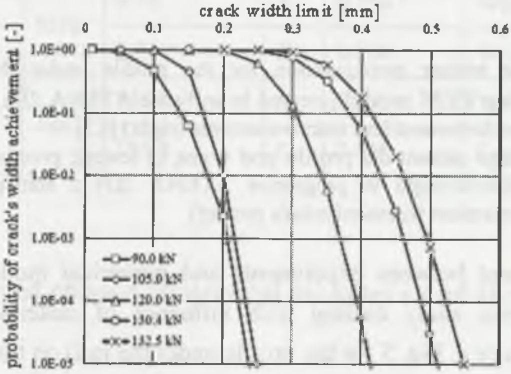

Fig. 6. Probability of crack width achievement for different levels of load for the profile in the middle of the sleeper [11]

Rys. 6. Prawdopodobieństwo osiągnięcia stopnia rozwarcia szczeliny dla różnych poziomów obciążenia dla profilu w środku belki podwalinowej [11]

3.2. Facade panels made of glass-fibre reinforced cement based composite

This stochastic model deals with experimental test of facade panels (2050×1050×13,5 mm, Fig. 7). The loading action caused by wind intake (area load) was simulated by vacuum-treated laboratory experiment [6]. Stochastic model of facade panel's test combines deterministic nonlinear numerical model in ATENA 3D (with material model 3D Nonlinear Cementitious [8] and Latin Hypercube Sampling Method (*LHS*) used for generating of 2×30 sets (reference and degraded) of random input parameters (SARA complex software system). The most important material input parameters according to sensitivity analysis (modulus of elasticity, fracture energy, tensile and compressive strength) are considered as random variables according to experimental results (tab. 2) and their decrease due to material degradation is taken into account. The stochastic model was evaluated using statistical, sensitivity and probability analysis. Note that substantial decrease of the mean value of ultimate load occurred due to degradation (more than 50%, from 13.23 kN/m^2 to 6.52 kN/m^2).

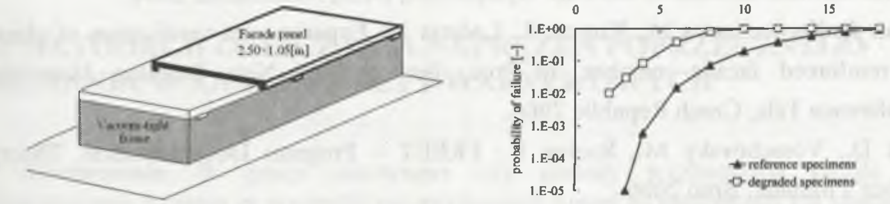

Fig. 7. Scheme of facade panel's testing (left); result of the stochastic model – theoretical failure probabilities for different levels of load (right) [4]

Rys. 7. Schemat badanych paneli elewacyjnych (z lewej); wyniki dla modelu stochastycznego – teoretyczne prawdopodobieństwo błędu dla różnych poziomów obciążenia (z prawej) [4]

The chosen result of the reliability study is shown in Fig. 7. The theoretical failure probability – the probability that the panel will not resist the load (wind intake) is essentially higher in the case of degraded panel (for more details see Kersner et al. 2007).

4. Conclusion

The paper presents two examples (railway sleeper and fibre reinforced facade panel) of the complex approach to the design and evaluation of structures/structural members made of (advanced) cement based composites. Chosen results from the stochastic analyses as well as recent works of the research group at Institute of Structural Mechanics dealing with fracture mechanical parameters determination (including their statistics or influence of degradation) are reviewed and inserted into the complex structure's design/evaluation approach. Presented approach can contribute to the optimal design of structural members made of cement based composites.

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Recenzent: Prof. dr hab. inż. Grzegorz Prokopski