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COMMENTS ON THE COMMAND „CONVERT" OF THE PROGRAM CC AND RELATED NOTICES *

Summary. In connection with the options 7,8,9, of the command „convert" of the program CC causal and noncausal discrete-time (DT) transfer functions (TF) for the system with ideal sampler (IS), zero order hold (ZOH) or first order hold (FOH) are distinguished and discussed. In the case of ZOH or FOH the discussed notions are essential for the DT system with a continuous-time (CT) plant having an uninertial channel, while for in the case of IS-for the rational TF having the order of numerator less by one from that of the denominator. It is noticed that the CC command „convert" in the mentioned options calculates only the noncausal DT TF while usually the causal DT TF-s represent the bahaviour of the real DT systems. It is also noticed that assumed in the program CC the manner of sampling of ideal impulses appearing at time t = 0 in time responses of some models can't be justified and accepted. The models related to these casses are not real and could be neglected in the program.

UWAGI O ROZKAZIE „CONVERT" PROGRAMU CC I POJĘCIA Z TYM ZWIĄZANE

Streszczenie. W związku z rozkazem „convert" programu CC wprowadza się pojęcia przyczynowych i nieprzyczynowych transmitancji dyskretnych dla układów z idealnym impulsatorem (II), ekstrapolatorem zerowego rzędu (E0) i ekstrapolatorem pierwszego rzędu (E1). W przypadku układów z E0 i E1 wprowadzone pojęcia mają znaczenie dla układów dyskretnych, dla których transmitancja obiektu ciągłego ma stopień licznika równy stopniowi mianownika, a dla układów z II - przy stopniu licznika mniejszym od jeden od stopnia mianownika. Zauważa się, że rozkaz „convert" programu CC dla omawianych układów wyznacza tylko transmitancje nieprzyczynowe, podczas gdy do opisu układów zamkniętych mają zazwyczaj zastosowanie transmitancje przyczynowe. Zauważa się także, że zastosowany w programie CC sposób próbkowania idealnych impulsów występujących w pewnych odpowiedziach czasowych nie ma żadnego uzasadnienia. Otrzymane dla tego przypadku modele są nierealne i nie powinny w ogóle występować.

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Introduction

The discussed command „convert" of the program CC [7] calculates the discrete-time (DT) transfer function (TF) corresponding to the given continuous-time one. The command has ten options, however our remarks concern the options 7,8,9. The option 8 concerns the DT system commonly considered, composed of sampler, zero order hold (ZOH) and CT plant G (s), in series (fig.1a), where G(s) denotes the TF of the plant.

The options 7 and 9 concern the same DT system in which the ZOH is replaced with the ideal sampler (IS) or with the first order hold (FOH), respectively. In the case of the systems with ZOH or FOH (options 8 or 9) the discussed question appears if the step response y_1 (t) of the CT plant G(s) is discontinuous at time t = 0, i.e. $y_1(0^+) \neq 0$, where $y_1(t) = \mathcal{L}^{-1}[(1/s)G(s)]$, $\mathcal{L}^{-1}$ is the symbol of the inverse. Laplace transform and $y_1(0^+)$ denotes the right--hand side limit at t = 0.

In the case of the system with IS (option 7) the question appears if the impulse response of CT plant G(s) is discontinuous at time t = 0 i.e. if $g(0^+) \neq 0$, where $g(t) = \mathcal{L}^{-1}[G(s)]$.

If discontinuity of the appropriate time response appears in some sampling instant there arises the question which kind of the limit right - or left-hand side is accounted by the output sampler 2. In connection with this it is reasonable to propose the distinction between the causal and noncausal DT TF-s, respectively. The distinction is not exactly seen in the literature, however in some items the problem is noticed e.g. in [1] only the causal TF-s are discussed, while in [5] the modified TF-s are used to obtain the same effect.

Thus, the causal DT TH H (z) of the system with ZOH (fig.1a) can be calculated from the formula

$$H(z) = (1 - z^{-1})\, \mathcal{Z}\, [y^*_1(nh)], \tag{1}$$

where $y^*_1(0h) = 0$, $y^*_1(nh) = y_1(nh)$ for n > 0, h is the sampling period, $\mathcal{Z}$ is the symbol of the Z-transform and $(1 - z^{-1})$ is the inverse of the Z-transform of the DT step function. The calculation of the noncausal DT TF $\bar{H}$ (z) differs only in this that $y^*_1(0h) = y_1(0^+)$. The causal and noncausal DT TF- s for the systems with IS or FOH can be calculated, similarly.

The program CC [7] in the discussed case, for all the mentioned options 7,8,9 (corresponding to IS, ZOH, and FOH, respectively) calculates only the noncausal DT TF without any comment. In accordance with my knowledge also in MATLAB [6] the problem of the causal DT TF-s is not noticed.

In the case of DT systems with ZOH or FOH the causal DT TF-s have meaning e.g. in the case of the rational TF-s G(s) with the order of numerator equal to that of the denominator, i.e. for the plants having an noninertial channel. In the case of DT systems with IS the problem appears when the rational TF-s G(s) have the order of numerator less by one from that of the denominator. Is should be stressed that the program CC for these models calculates only the noncausal DT TF-s without any warning. The cause that many researchers don't notice the problem results from the fact that the discussed models are rather seldom applied in practise. However in the case of relatively large sampling periods the models G(s) with uninertial channel can result from a justified neglection of the transients resulling from relatively small time constans. The important argument which should be taken into account is that for description of the closed loop (CL) systems usually the causal DT TF-s must by applied. Thus, the lack of the causal DT TF-s in so complete program as the program CC must be seen as an oversight.

From the other hand, applied in this program the manner of sampling of the ideal impulses can't be justified. The question is that the command „convert" of this program calculates the DT TF-s for the system with ZOH or FOH also for the rational TF-s G(s) with the order of numerator higher by one from that of the denominator, and for the systems with IS also for the rational TF-s G(s) with the order of the numerator equal to that of the denominator. In these cases, in the appropriate time responses the ideal impulses (in the form of the Dirac functions) at some sampling instants appear. In the program CC it as assumed that the ideal impulse $k\tilde{U}(t)$ (where $\tilde{U}(t)$ denotes the Dirac function and k is a real number) sampled at time $t = 0$ gives the discrete impulse $k\tilde{U}(n)$, where $\tilde{U}(0) = 1$ and $\tilde{U}(n) = 0$ for $n \neq 0$. It is incorrect assumption which can't be justified. It ssems that the cases of discussed here TF-s G(s) could be neglected since these kind of models are not applied.

Further on, the simple method of calculation of the causal DT TF-s in general case of DT systems with ZOH, as well as with IS and FOH is described. It is also shown that usually the causal DT TF-s are used in applications. The case when the noncausal DT TF-s could be used is also shown.

Calculation of Causal and Noncausal DT TF-s

Consider the DT system with ZOH and CT plant (fig.1a) described by a rational TF

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_m}{a_0 s^m + a_1 s^{m-1} + \ldots + a_m}, \tag{2}$$

where $b_0 \neq 0$ and $a_0 \neq 0$. The step response of the CT plant is shown in fig.1b and 1c, where additionally the DT transient responses y^{*}_{1} (nh) corresponding to notions of causal and noncausal DT TF-s (when $\Delta \to 0$) are shown.

Dividing the polynomial of the numerator by that of the denominator of (2) we obtain

$$G(s) = c + G_1(s), \tag{3}$$

where $c = b_0/a_0$ is the gain of the noninertial channel, $G_1(s)$ takes the form

$$G_1(s) = \frac{b'_0 s^{m-1} + b'_1 s^{m-2} + \ldots + b'_{m-1}}{a_0 s^m + a_2 s^{m-1} + \ldots + a_m} \tag{4}$$

and the numerator of (4) is the polynomial rest resulting from the division. Since the TF $G_1(s)$ is rational, strictly proper, the problem of causability does not concern it. Let us denote

$$H_1(z) = (1 - z^{-1}) \mathcal{Z}\{\mathcal{L}^{-1}[\frac{1}{s} G_1(s)]_{|t=nh}\}. \tag{5}$$

The causal DT TF H(z) corresponding to the TF G(s) can be calculated from the formula

$$H(z) = \frac{c}{z} + H_1(z), \tag{6}$$

where the first term on the right-hand side of (6) determines the causal DT TF of the noninertial channel of G (s).

The transformation of G (s) to the causal DT TF H(z) determined by the formulas (3) - (6) is denoted by

$$H(z) = D_c[G(s)]. \tag{7}$$

Of course, it is valid for the system with ZOH.

The noncausal DT TF $\bar{H}(z)$ corresponding to the TF G (s) can be calculated from

$$\bar{H}(z) = c + H_1(z), \tag{8}$$

where the first term on the right-hand side of (8) determines the noncausal DT TF of the noninertial channel of G (s).

From the comparison of the formulas (6) and (8) it results that in the causal DT TF (6) the additional mode z appears which usually increases by one the order of the TF H (z) in relation to that of G(s); (and $\bar{H}$ (z)).

The transformation of G(s) to the noncausal DT TF $\bar{H}(z)$, determined by the formulas (3) - (5) and (8), valid for the system with ZOH is denoted by

$$\bar{H}(z) = D_{nc}[G(s)]. \tag{9}$$

Of course, in the case of continuous step responses at t = 0 (i.e. for the strictly proper TF G(s)) the application of both the transformations D_c or D_{nc} gives the same result, since then the problem of causability does not appear.

In the MATLAB program [6] the toolboxes exist which make it possible to calculate the DT TF-s for the systems with ZOH. In the discretization toolbox „C2D" the matrix exponential function is utilized very skilfully. Adding insignificant modifications which realize the transformations D_c or D_{nc} the causal or noncausal DT TF-s can be also calculated. It can be noticed that the same transformations D_c or D_{nc} can be used to calculate the casual or noncasual DT TF-s for the systems with IS or FOH.

Let I (z) ($\bar{I}$ (z)) be the causal (noncausal) DT TF for the system with IS. It can be noticed that

$$I(z) = D_c\left[s\,G(s)\right]\frac{z}{z-1} \tag{10}$$

and $\bar{I}(z)$ results from (10) by replacing D_c with D_{nc}.

Similary, if F (z) ($\bar{F}(z)$) denotes the causal (noncausal) DT TF for the system with FOH then

$$F(z) = D_c\left[G(s)\left(1+\frac{1}{hs}\right)\right]\left(1-z^{-1}\right) \tag{11}$$

and $\bar{F}(z)$ results from (11) by replacing D_c with D_{nc}.

The formulas like (10) or (11) show that the MATLAB technique can be also utilized for calculations of the causal (noncausal) DT TF-s for the system with IS or FOH.

When to Apply the Causal and Noncausal DT TF-s

Let us focus our attention on the open-loop system with ZOH shown in fig.1a. The system can be described either by the causal or by the noncausal DT TF dependently upon the mutual shift in time of the sampling instants of the output y (sampler 2) and input u (sampler 1). If the sampling instants of the output y are close but somewhat delayed with respect to that of the input u (as in fig.1b) then the system is described approximately by the casual DT TF, while in the opposite case (as in fig.1c)-by the noncasual DT TF. If both the samplers 1 and 2 work precisely synchronously then we must apply the casual DT TF. It is the result of the causability principle in accordance with which, first appears sampling, then successively the ZOH response and G (s) response.

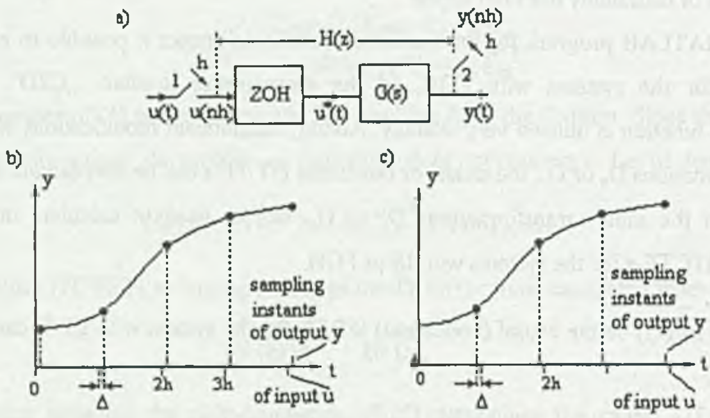

Fig.1 a) DT system with ZOH; b) and c) step responses sampling corresponding to the noncausal and causal DT TF, respectively

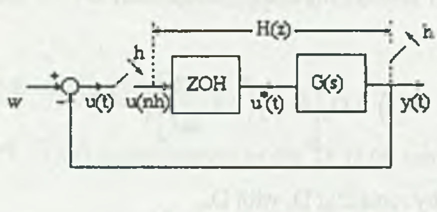

Fig.2. DT closed loop system

In order to describe the CL system shown in fig.2 the causal DT TF H (z) must be applied giving

$$Y(z) = \frac{H(z)}{1+H(z)} W(z). \tag{12}$$

Really, in the case of the CL system the sampling of the input e and the output y is realized physically by only one sampler and the sampling instants of the input e and the output y are synchroneous.

Now, there arises the question if there exist some CL systems for description of which the noncausal DT TF is used. Let us consider the system shown in fig.3. In this system C (z) determines the DT algorithm of the microprocessor controller in the form of the DT TF; the sampler 1 is related to DA converter (represented also by the ZOH); the sampler 2 corresponds to AD converter; G(s) is the TF of the CT plant. If the mutual shift of the sampling instants of the input u and the output y is such as in fig.1b then the noncausal DT TF $\bar{H}$ (z) describes the plant. However in this situation the DT TF C(z) of the controller must have the order of numerator less at least by one from that of the denominator. It means that the DT controller C (z) will then have one step delay in his output reaction to the input.

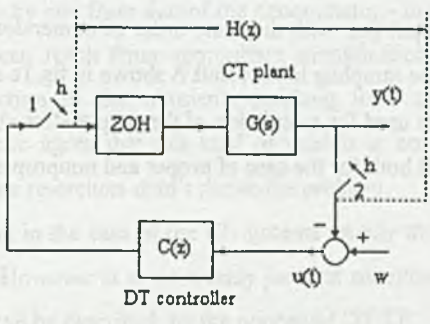

Fig.3. CL system with CT plant, DT controller and two samplers

It can been noticed that the casual DT TF H(z) has the order of numerator less by one from that of the denominator, while the noncausal DT TF $\bar{H}$ (z) has the same order of numerator and denominator. Thus, in the case shown in fig.1b the DT TF $\bar{H}$ (z) is noncausal but C (z) can be treated as the causal DT TF of the microprocessor algorithm. It can be noticed that in the case shown in fig.1c the system from fig.3 is described by the causal DT TF H(z), while the DT TF $\bar{C}(z)$ then can be noncausal i.e. $\bar{C}(z)$ can have the same order of numerator and denominator.

Till now, the considerations of the present section are valid if the mutual shift of the sampling instants of the output and of the input is very small in relation to the sampling period h. Of course the description of the DT system in the form od the causal or noncausal DT TF is then approximate.

If the shift Δ is such that if shuld be taken into account then for the case shown in fig.1c the precise DT TF H (z) results from usual formulas applied for the system shown in fig.1a in which G(s) exp(- sΔ) appears in the place of G(s). In the case shown in fig.1b the precise DT TF H(z) is determined by the so called modified DT TF H (z, ε), ($\varepsilon = \frac{D}{h}$) [2,4].

If the sampling instans of the input u and output y are precisely synchroneous, i.e. Δ = 0, then the causal DT TF-s H (z) and C (z) must be used. It denotes that then the controller with the TF C (z) having the same order of numerator and denominator can't be used since in each sampling instant no time appears for calculation of the output u after obtaining new value of the input e. It also means that if we would like to use the controller giving instantaneous output reaction to the input (i.e. with the same order of numerator and denominator of C (z) then we should design the sampling instant shift Δ shown in fig.1c and apply the causal DT TF H (z). The time Δ is then used for calculation of the output after obtaining new value of input. The latter remark is valid both for the case of proper and nonproper TF G (s).

Example

Let in the system shown in fig.1a h = 0.5 and the CT plant is described by

$$G(s) = \frac{2s+10}{s+1} = 2 + \frac{8}{s+1}. \tag{13}$$

The TF (13) can result e.g. from appropriate simplification of the primary TF in the form

$$G_p(s) = \frac{2s+10}{(T_s+1)(s+1)}. \tag{14}$$

If the time constant T is very small in relation to the sampling period h then the transients resulting from the mod (Ts + 1) can be neglected giving the simplified model (13).

The casual DT TF H (z) for h = 0.5 and G(s) determined by (13) takes the form

$$H(z) = 2\frac{1}{z} + 8\frac{1-D}{z-D} = \frac{5.1478z - 1.2131}{z(z-0.6065)}, \tag{15}$$

(where D = exp(-0.5), while the noncausal DT TF $\bar{H}(z)$ is

$$\bar{H}(z) = 2 + 8\frac{1-D}{z-D} = \frac{2z + 1.9347}{z - 0.6065}. \tag{16}$$

It is seen that the casual TF (15) has the additional causal mode z which increases its order by one in comparison to that of noncausal TF (16)

Final Conclusions

The program like CC [7] which calculates the noncausal DT TF without warning can cause in some situations an incorrect description of the system. This concerns the CT plants with an noninertial channel in the case of ZOH or FOH and the CT plants with a rational TF-s having the order of numerator less by one from that of the denominator - in the case of IS. This kind of the CT plant models can result from appropriate simplification of the primary models obtained from the neglection of the transients resulting from the relatively small time constants. However one can agree that this kind of models is not very frequently applied which could cause that some reserchers didn't notice the problem.

It should be stressed that in the case of the CL systems usually the causal DT TF-s should be used to describe them. However in some weakly justified situations described in the paper the part of the CL system can be described by the noncausal DT TF.

The property of the causal DT TF-s in the case of ZOH or FOH is that they have an additional causal mod z which increases the order of the system. It can be shown that the last statement does not concern the system with IS. Applied in the program CC the manner of sampling of ideal impulses appearing in some time responses at t = 0 has no justification and can't be accepted. The ideal impulse sampled at time of its occurence gives some indefinite result. In some justified situations as the result of sampling the left- or right-hand side limit of the appropriate time response at the sampling instant could give a definite value. However the related models are not real and can be neglected in considerations.

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