ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1987

Seria: ENERGETYKA z. 98 Nr kol. 1022

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NUMERICAL SOLUTION OF TRANSONIC FLOW THROUGH THE BLADE CASCADE

Summary. The application of a time marching method to determine the parameters of a transonic flow through blade cascades is presented. The unsteady Euler equations of motion are approximated by the discrete and conservative differential system using the Godunov scheme. An equipotential grid is used which has been obtained from the solution of the Laplace equation for geometry and boundary conditions under consideration. On the basis of detailed calculations of the flow in a turbine channel, the influence of some parameters on the analytical process and its results are discussed.

Notation

A - surface

a - speed of sound

c - absolute velocity

e - internal energy

h - static enthalpy

Ma - Mach number

n - normal vector

p - pressure

T - temperature

V - volume

ρ - density

τ - time

Subscripts and superscripts

n - normal component

t - tangent component

o - initial, total conditions

1 - inlet

2 - outlet

1. Introduction

The characteristic feature of steam turbines, gas turbines and compressors built nowadays is the constant increase of load on their particular stages. This is the consequence of aiming at a compact construction and the result of economizing on materials and energy in the process of producing these machines. In highly loaded stages of turbomachines, a gas velocity will usually exceed a speed of sound changing within the wide bounds its value in one channel.

The description of processes of energy conversion in this type of blade-to-blade channels is made up by equations of continuity, momentum (the Navier - Stokes equations), energy equations and relations describing the physical properties of mediums along with adequate initial and boundary conditions. Despite considerable development in numerical methods of fluid mechanics, their full solution is yet hardly probable for the geometry of turbomachines. Hence, there arises the necessity to assume physical simplifications which consequently result in simple mathematical descriptions, in the study of transonic flows in blade rows, main simplifications lead to omitting viscous elements in the equations of momentum and energy. The solution of equations simplified in such a way is now possible without additional simplifications in the geometry of flow systems.

Further on we shall consider the plane flow of a blade cascade, assuming for its determination, the time marching method using the GODUNOV scheme [3, 6, 8].

2. Presentation of the problem

As it has already been stated, in the blade-to-blade channels of turbomachines, a gas velocity may in a general case change its value from subsonic to supersonic. For the steady flow, it means that within the range of one channel we deal with the transition from eliptical to hyperbolic boundary problem (or vice versa). The difficulty in solving such a mixed problem causes frequent formulations of an auxiliary initial-boundary problem which within the range of the whole channel, disregarding the value of speed, is of hyperbolic type.

An auxiliary initial-boundary problem is established by the system of unsteady equations of mass, momentum and energy conservation, equation of state, an adequate initial condition, and boundary conditions in conformity with the steady state. The equation of conservation for an inviscid an non-conducting heat fluid can be expressed by

$$\frac{\partial}{\partial t}\iiint F\, dV + \iint G\, dS = 0 \qquad (2.1)$$

where:

$$F = \begin{bmatrix} \rho \\ \rho c \\ \rho(e + \frac{1}{2} cc) \end{bmatrix} \qquad G = \begin{bmatrix} \rho(cn) \\ \rho(cn)c + pn \\ \rho(h + \frac{1}{2} cc)(cn) \end{bmatrix}$$

This way of notating equations is advantageous mainly because of its independence from the coordinate system. It considerably simplifies the notation of differential analogues of the system (2.1).
Equations (2.1) along with the equation of state

$$p = p(\rho, T) \tag{2.2}$$

enable the calculation $\rho = \rho(\vec{\tau})$, $c = c(\tau)$, $E = e + \frac{1}{2} cc = E(\tau)$ inside an arbitrary element whose volume is V on the basis of the values p, ρ, c which have been determined for $\tau = \tau_0$ on the surface S that bounds volume V. In the two-dimensional flow: $dV = \Delta h\ dA$, $dS = \Delta h\ dl$ (Δh - constant depth of the considered element, dl - element of L_A contour that bounds dS, Fig. 1). The equation system referring to this case is expressed by

$$\frac{\partial}{\partial \tau} \int F\ dA + \int_{L_A} G\ dl = 0 \tag{2.3}$$

The system of equations (2.2) and (2.3) has been further on adopted in order to analyse the transonic flow through blade cascades in turbines (Fig. 1). A final closing of the initial - boundary problem requires the determination of initial and boundary conditions. In the particular case of the flow region shown in Fig. 1, the boundary velues on the segments AB, CD, EF, GH, HA as well as along the BC and FG contours of profiles should be determined.

For the homogenous flow conditions along HA and DE, the analysis of the one-dimensional unsteady flow results in the following coefficients of angular characteristics $\lambda = \frac{dn}{d\tau}$ [8]

$$\lambda_1 = C_n, \quad \lambda_2 = C_n - a, \quad \lambda_3 = C_n + a, \quad \lambda_4 = C_n \tag{2.4}$$

Equations of movement along the particular characteristics are expressed as

$$\begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 \\ 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & D_4 \end{bmatrix} = \begin{bmatrix} C \\ J_- \\ J_+ \\ p-a^2 \end{bmatrix} \tag{2.5}$$

where

$D_i = \left[\frac{\partial}{\partial t} + \lambda_i \frac{\partial}{\partial m}\right]$ - differential operators along i of these characteristics.

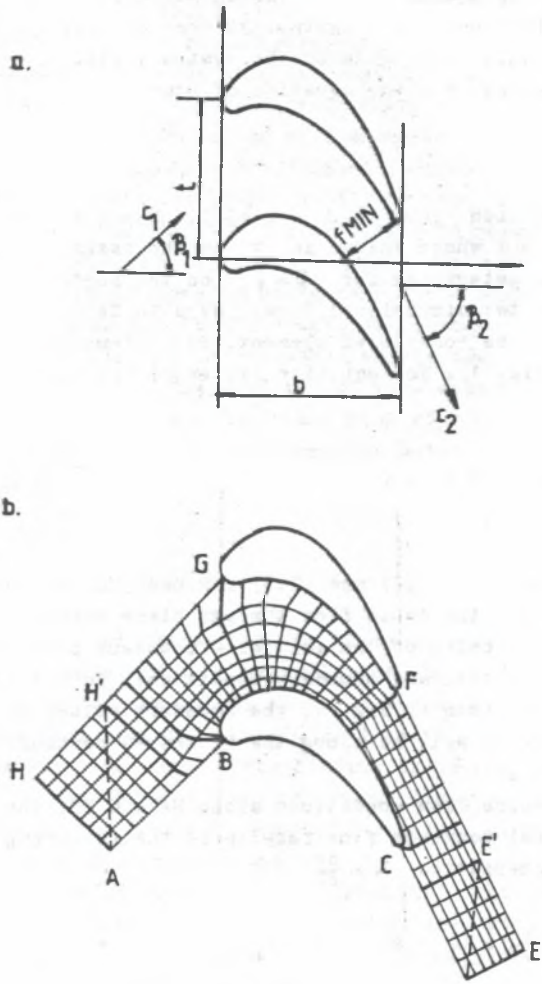

Fig. 1.
a) blade cascade used for calculations, b) computational grid
Rys. 1
a) palisada przyjęta do obliczeń, b) siatka obliczeniowa

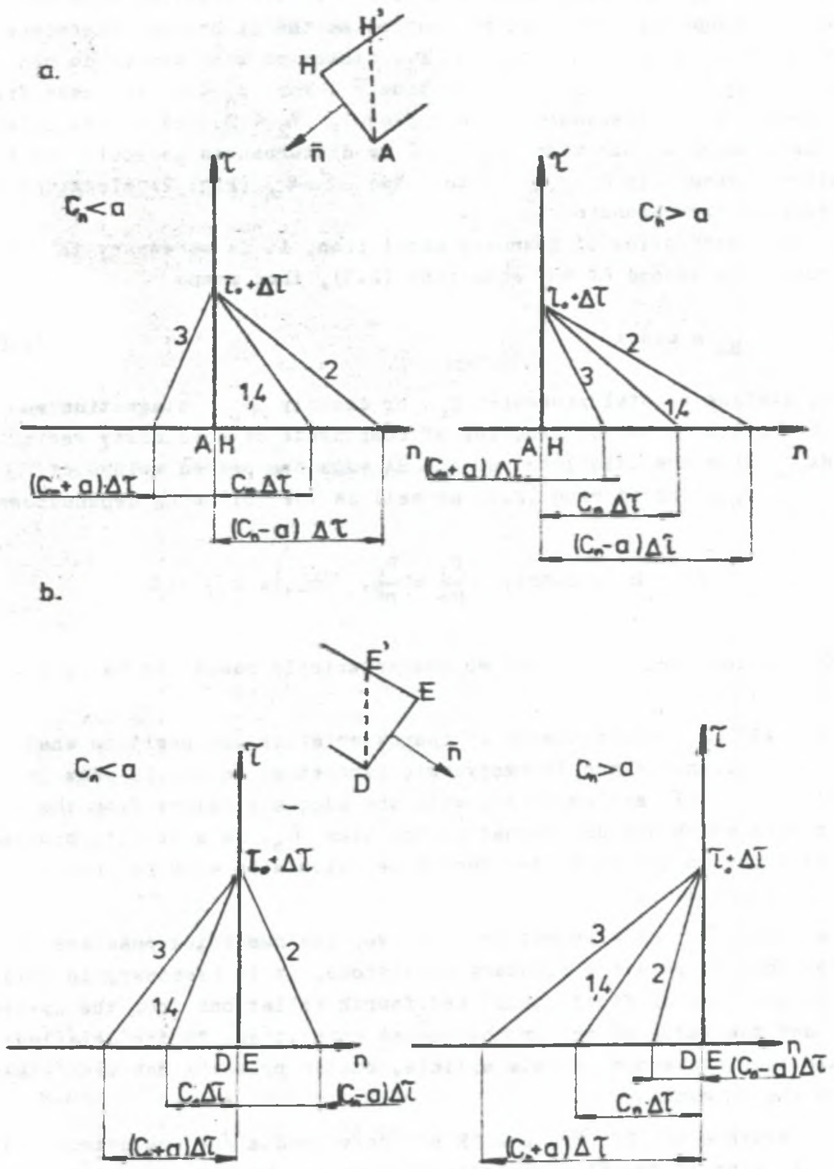

Fig. 2. Characteristics in the (n,τ) planes
Rys. 2. Charakterystyki w płaszczyźnie (n,τ)

We infer from (2.4) that for $c_n > a$ in HA section, coefficients λ_i are positive. It means that disturbances occuring in the examined flow region cannot change the values of parametres on the HA border. Therefore, in this case the values c_{n1}, c_{t1}, p_1, ρ_1, stated on this border do not undergo any change with the change of time τ. For $c_n < a$ (a case frequently occuring for transonic cascade flows), $\lambda_2 < 0$ and the HA inlet section is reached in the time $\tau_0 + \Delta\tau$ by disturbances generated in the computational range $(n > n_{HA})$ in the time $\tau = \tau_0$ (Fig. 2) along the characteristic corresponding to λ_2.
Then, in the description of boundary conditions, it is necessary to take into account the second of the equations (2.5), that means

$$J = (J_-)_{HA} = \text{const} \tag{2.6}$$

Moreover, stating a total pressure p_o or density ρ_o, stagnation enthalpy h_o and a linear combination of components of a velocity vector in the HA section are calculated on the HA edge the needed values of p_1, ρ_1, c_{n1}, c_{t1}, considering (2.6) as well as the following dependences:

$$\frac{k}{k-1}\frac{p_1}{\rho} + \frac{1}{2}cc = h_o = \text{const}, \quad \frac{p_1}{\rho^k} = \frac{p_o}{\rho^k}, \quad c_{t1} = c_{n1}\,tg\beta_1$$

In the DE section, the following two characteristic cases can be specified:

a. $c_n > a$. All λ_i coefficients of charakteristics are positive what means that kinematic and thermodynamic parametres on the DE edge in the time $\tau_0 + \Delta\tau$ are connected with the adequate values from the flow region which are determined in the time τ_0. As a result, boundary conditions on the DE border should be calculated with respect to all relations (2.5).

b. $c_n < a$. The λ_2 coefficient is negative, the remaining ones are positive. When formulating boundary conditions, it is necessary in this case to make use of first, third and fourth rellations from the system (2.5) and the value of one or the needed quantities. In the detailed calculations presented in this article, static pressure has been stated in the DE section.

Along the segments AB, CD, EF, and GH boundary conditions are determined owing to the fact of the flow periodicity through the infinite blade cascade. On the BC and FG contours, the normal of components velocity equals zero.

An arbitrary system of kinematic and thermodynamic parametres in the examined flow region can be assumed as an initial condition for the considered method. In order to shorten the computational time, the results obtained from the application of the water analogy have been assumed in this paper as the initial state.

3. Differential scheme

Approximation of equations

After the integration of the system (2.3) in the interval $\Delta\tau = \tau^{(k+1)} - \tau^{(k)}$ we get [3, 6, 8]

$$\int F^{(k+1)}\, dA - \int_A F^{(k)}\, dA = - \int_{\tau^{(k)}}^{\tau(k+1)} (\oint G\, dl)\, d\tau$$

Proceeding to the discrete scheme, we find

$$\hat{F}_j^{(k+1)} = \hat{F}_j^{(k)} - \int_{\tau(k)}^{\tau(k+1)} \left[\frac{1}{\Delta A} \sum_{\alpha=1}^{M} (\hat{G}\,\Delta l_A)_{\alpha}\right]_j d\tau \tag{3.1}$$

Here, $\hat{F}$ is average values in the cell of ΔA surface; $\hat{G}$ is average values of streams flowing ghrough the segments Δl_{Ai} M is the number of elements bounding the cell of ΔA surface.

From the general scheme (3.1), we get some actual differential schemes which depend on the construction of a computational grid and the assumed way to determine the function

$$\chi(\hat{F},\tau) = \frac{1}{\Delta A} \sum_{\alpha=1}^{M} (\hat{G}\,\Delta l_A)_{\alpha}$$

The utilized differential scheme is based on the algorithm of GODUNOV and his collaborators [3, 6, 8]. In this case, when calculating the integral on the right we assume that the function $\chi(F,\tau)$ is constant in the $\Delta\tau$ interval, so instead of (3.1) we write for j - cell

$$\hat{F}^{(k+1)} = \hat{F}^{(k)} - \left[\left(\frac{\Delta\tau}{\Delta A}\right) \sum_{\alpha=1}^{M} (\hat{G}\,\Delta l_A)_{\alpha}\right]_j \tag{3.3}$$

For particular components of $\hat{F}$ from (3.3), we obtain (leaving out the j subscript):

$$\rho^{(k+1)} = \rho^{(k)} - \left\{ \frac{\Delta \tilde{\tau}}{\Delta A} \sum_{\alpha=1}^{m} \left[R \cdot (C.n) \, \Delta l_A \right]_{\alpha} \right\}^{(k)}$$

$$(\rho c)^{(k+1)} = (\rho c)^{(k)} - \left\{ \frac{\Delta \tilde{\tau}}{\Delta A} \sum_{\alpha=1}^{m} \left[P \, \Delta l_A n + RC(C.n) \, \Delta l_A \right]_{\alpha} \right\}^{(k)}$$

$$\left[\rho(e + \tfrac{1}{2} cc)\right]^{(k+1)} = \left[\rho(e + \tfrac{1}{2} cc)\right]^{(k)} - \left\{ \frac{\Delta \tilde{\tau}}{\Delta A} \sum_{\alpha=1}^{m} \left[R \cdot (E + \tfrac{1}{2} C \cdot C + P/R)(C.n) \, \Delta l_A \right]_{\alpha} \right\}^{(k)} \tag{3.4}$$

Averaged on the boundaries of the field A^k, the values of density R^k_α, velocity C^k_α, pressure P^k_α and internal energy E^k_α which determine the particular components of G are established for the assumed algorithm on the basis of the solution of the Riemann problem for the inviscid gas [6, 8].

Initial condition

In the investigations of the steady transonic flows by means of the time marching methods, it is possible to start calculations at hypothetical initial states. In the described algorithm, in order to render the time of calculations possibly the shortest, the results of measurements obtained from the application of the water analogy have been assumed as an initial state.

Boundary conditions

The numerical realization of boundary conditions presented in point 2 does not give rise to more serious troubles. It mainly concerns the conditions of symetry and the inlet into the blade-to blade channel.
The conditions at the trailing edge have been expressed with the use of the acoustic approximation of the Riemann problem with the stated value of static pressure in the section behind the cascade. The boundary condition on the surface of the profile ($c_n = 0$) has been realized with the assumption that the surface of tangent discontinuity is for good ascribed to the profile line which is at the same time a local axis of symmetry of the flow [6]. For such assumptions, the Riemann's solution has been applied to the points in the vicinity of the blade.

4. Results of calculations

Presented scheme of calculations of steady transonic flows can be applied to any geometry of a blade cascade, both for turbine and compressor stages. For the carried out calculations, a plane turbine blade cascade built form VKI profiles [2] has been engaged. For channels formed by these profiles, experimental researches [2] and results of numerical calculations [5] in the wide range of Mach number are available. In the calculations, as a boundary condition at the trailing edge: $M_{a2} = 121(\bar{p}_2 = \frac{p_2}{p} = 0{,}41)$ has been assumed. When testing the algorithm, the influence of some characteristic parametres on the results of calculations has been investigated. After analysing many applicable computational grids, decidedly the best results, with respect to time and calculations precision, have been achieved by engaging the equipotential grid drawn on the basis of the solution of the Laplace equation for velocity potencial. The global allowable step has been settled by examining local time steps (for each elementary region), which have been determined conforming to the conditions of stability in the GODUNOV scheme [1]. As a condition to achieve the steady state, the following has been assumed

$$\left|\frac{E^{(k+1)} - E^{(k)}}{E^{(k)}}\right| < 10^{-6} \tag{4.1}$$

This has been determined after the examination of many actual geometries for the different numbers of iterations. In the investigation of the influence of the way to set initial conditions on the speed of calculations process, no major difference in attaining the solution from "top" or "bottom" has been found (Fig. 3). The linear distribution of the parametres between the trailing and loading edges assumed as an initial state was leading to a longer time of calculations than in the case of utilizing the data obtained from the method of water analogy. An important purpose of the research was to determine the influence of the manner of forming the grid within the trailing edge on the results of calculations. Some results of this research is contained in Figure 4. The modification of the trailing edge resulting form the various discrete means of the approximation of a profile shape has a crucial influence on velocity distributions not only in the region of the trailing edge but in the whole channel as well, particularly along the convex surface of the profile. The extension of the profile (Fig. 4, case 4) changes the value of a minimal section of the channel and shifts its position in relation to the real object. As a consequence, it intensifies the shock wave. In this case, the greatest departure from the experiment s results occured.
For the other cases of correction, the distribution lines of Mach number go up to greater values.

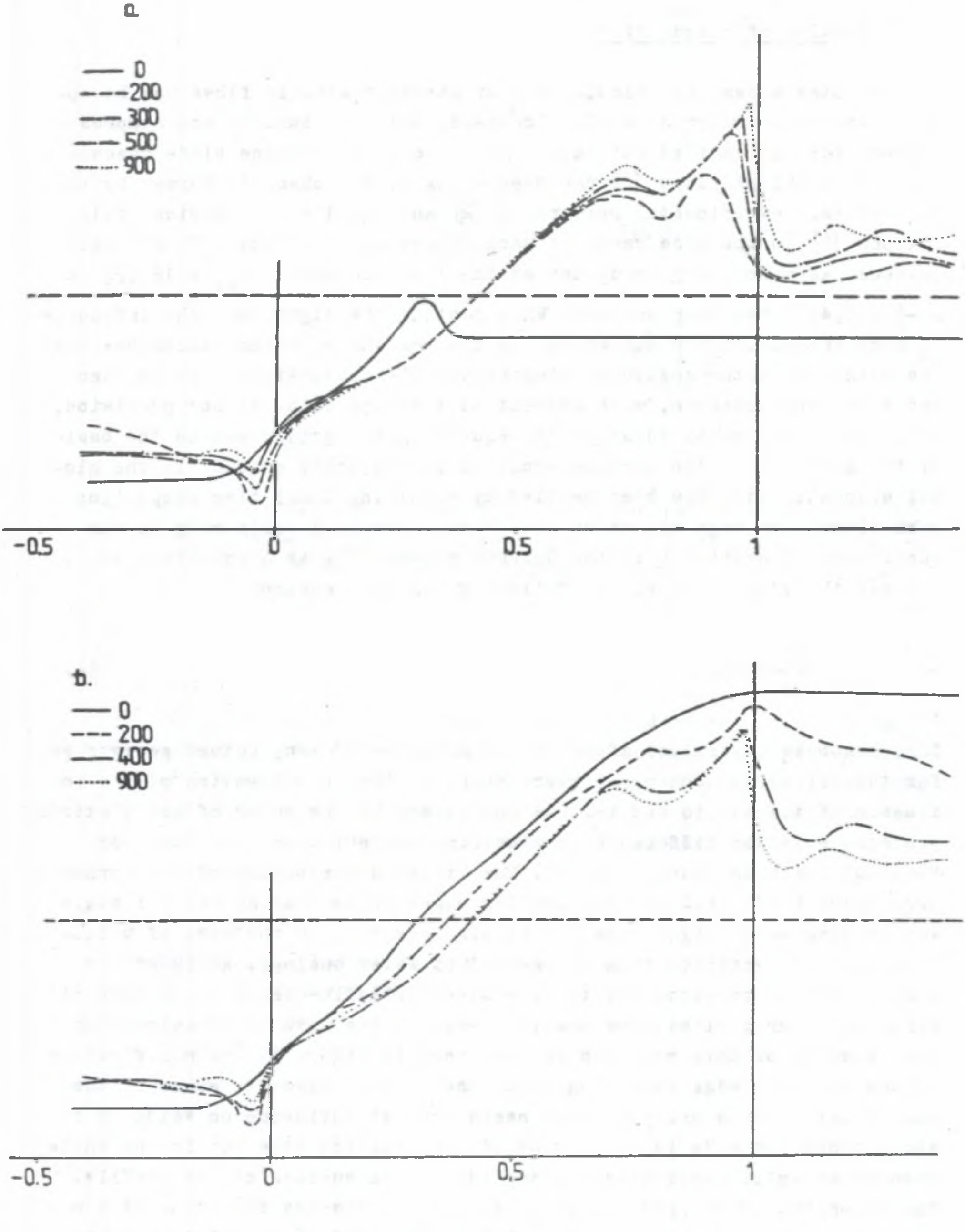

Fig. 3. The course of iteration process
a) for $Ma_{2(\tau=0)} > 1$, b) for $Ma_{2(\tau=0)} < 1$
Rys. 1. Przebieg procesu iteracyjnego
a) dla $Ma_{2(\tau=0)} > 1$, b) dla $Ma_{2(\tau=0)} < 1$

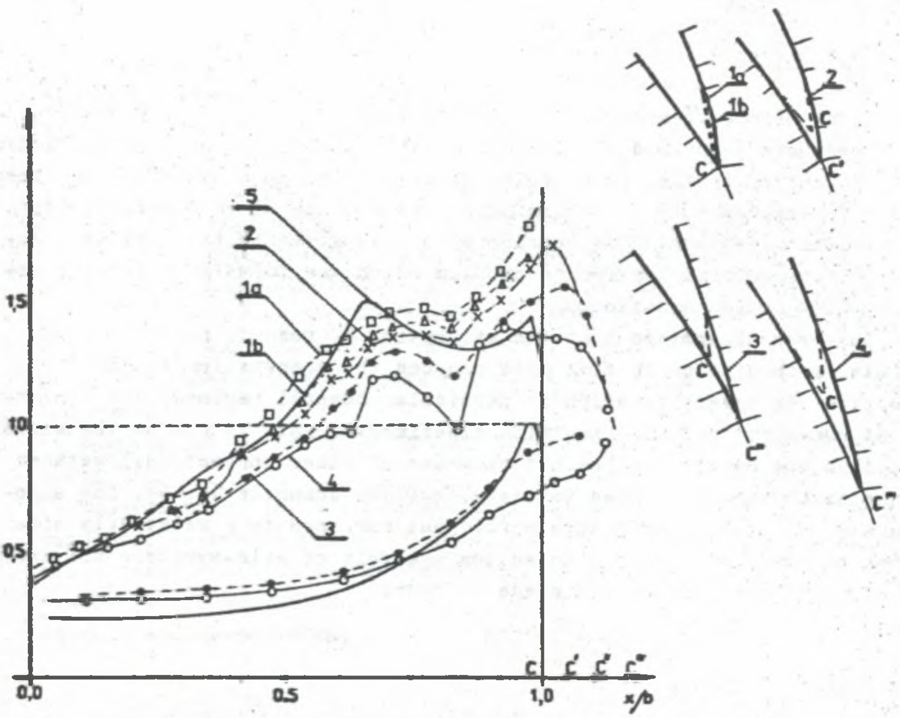

Fig. 4. Calculation results (5 - result of the testings in the air tunnel)
Rys. 4. Rezultaty obliczeń (5 - rezultaty pomiarów w tunelu powietrznym)

In Figure 4, the course of Mach number for 350 and 650 cells of division has been shown. The solution for 650 cells refers to the first version of the correction of the trailing edge. Slightly different value of the angle of the outlet stream in CF section corresponds with each considered geometrical shape of the trailing edge. These differences do not exceed $30' - 1^{o}$. The angle β_2 computed for the version is closest to the experimental value and amounts to $\beta_2 = 62^{o}$ (experimental value $\beta_{2e} = 63^{o}$).

The algorithm and calculation programme have been organized and optimized in such a way to achieve an effective solution for the number of the cells $j = 350$ by the use of the microcomputer MERA 60.
The time of performing one iteration was 29s, and the number of iterations necessary to fulfill the condition (4.1) amounted to 895. For j=850 the calculations were done on the digital computer ODRA 1305. In thic case, the time of performing one iteration was 8 s and the number of necessary steps was 1490.

5. Conclusion

A direct method of successive unsteady states based on the GODUNOV scheme has been described and applied to the examination of cascade flows. In the course of testing an calculation algorithm, main attention has been paid to the explanation of the influence of a shape of the trailing edge on calculation results. Some consideration was given to the auxiliary solution of the initial hyperbolic problem which was formulated for the unsteady Euler motion equations.

It has been determined that the assumption of results obtained from the water method to be initial data reduces the computational time.

Despite its deversification in particular channel regions, the conformity of numerical calculations with experimental data is not on the whole worse than the accuracies reached by means of other conventional methods of examining transonic flows in blade-to-blade channels [4, 5]. The algorithm and calculation programme worked out here can in a relatively simple way be expanded so as to cover the analysis of axis-symetric and three-dimensional flows through the blade cascade.

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Recenzent: Doc. dr inż. Jan RADWAŃSKI

Wpłynęło do Redakcji w marcu 1987 r.

NUMERYCZNE ROZWIĄZANIE ZADANIA ANALIZY TRANSONICZNEGO PRZEPŁYWU PRZEZ PALISADĘ ŁOPATKOWĄ

S t r e s z c z e n i e

Opisano zastosowanie metody kolejnych stanów ustalonych do określenia parametrów przepływu transonicznego przez palisady łopatkowe. Zostało sformułowane pomocnicze zadanie początkowo-brzegowe, które w obszarze całego kanału, bez względu na wartość prędkości jest typu hiperbolicznego. Pomocnicze zadanie początkowo-brzegowe utworzono na podstawie układu nieustalonych równań zachowania masy pędu i energii, równanie stanu, odpowiedni warunek początkowy oraz odpowiadające stanowi ustalonemu warunki brzegowe.

Zostały przedyskutowane warunki brzegowe w przekroju wlotowym i wylotowym.

Schemat różnicowy utworzono wykorzystując algorytm Godunowa i jego współpracowników. W ramach obliczeń numerycznych badano wpływ sposobu kształtowania siatki w obrębie krawędzi wylotowej na rezultaty obliczeń. Przedstawiono wyniki obliczeń otrzymanych dla profilu VKI 1 o różnym stopniu wydłużenia krawędzi opływu.

ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧИ АНАЛИЗА ТРАНСЗВУКОВОГО ТЕЧЕНИЯ В РЕШЁТКАХ

Р е з ю м е

В работе предложен численный метод расчёта плоских трансзвуковых невязких течений в решётках турбомашин. Решение задачи получено методом установления с применением явной схемы первого порядка точности на равномерных сетках (схема С.К. Годунова). В статье приведены результаты расчёта двухмерного течения при различных формах кромочной части профилей.