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THERMAL EFFECTIVENESS OF THE THREE-FLUID CONVECTIVE SPIRAL U-TYPE RECUPERATOR

Summary. The paper contains the thermal analysis of the convective spiral U-type three-fluid parallel-flow heat exchanger. The numerical mathematical model is shown in short. This model contains the finite difference energy balance equations which have been solved using the iterative methods. The numerical results obtained are shown in a form of thermal effectiveness.

NOMENCLATURE

- a, c - auxiliary constants,
- b - mean total width of a single channel, m,
- b_1, b_2 - total width of channels, the width b_1 is referred to the fluids 1 and 3, the width b_2 is referred to the stream 2 and 4, $2b = b_1 + b_2$, m,
- f - element number,
- h - height of channels, m,
- i - coil number, $i = 0, 1, \ldots, n, n+1$,
- j - stream number, $j = 1,2,3,4$,
- $k(k_{j-k})$ - overall heat transfer coefficient (between the streams j and k, $k_{j-k} = k_{k-j}$), $W/(m^2.K)$,
- K_{j-k} - dimensionless criterion parameter, $K_{j-k} = \frac{k_{j-k}bh}{W_j}$,
- m - number of concentric elements of heat exchanger, divisible by 4,
- n - number of coils,
- NTU - Number of Transfer Units,
- r - radius, m,
- r_p - initial mean radius of the first coil, m,
- T - temperature, K,

t - dimensionless temperature,

$$t = \frac{T - T_{1d}}{T_{2d} - T_{1d}},$$

if $|T_{2d}-T_{1d}| \geqslant |T_{2d}-T_{3d}|$ then l=1, else l=3,

W_j - heat capacity of the stream j, positive when the stream j flows in the direction of increasing argument φ, W/K,

W_{max} - greater absolute value of the heat capacity $|W_2|$ and the sum $|W_1| + |W_3|$, W/K,

W_{min} - smaller absolute value of the heat capacity $|W_2|$ and the sum $|W_1| + |W_3|$, W/K,

z - dimensionless coordinate, $z = \frac{r}{4b}$.

Greek symbols

ε - thermal effectiveness,

φ - angle coordinate,

φ_o - the angle coordinate determining the position of the inlet and outlet points at the outer circumference of the heat exchanger.

The indexes

$)_{1,\dots,4}$ - stream 1,...,4, the streams and simultaneously fluids 1 and 3 flow outside the spiral U-type channel, the streams 2 and 4 flow inside the U-type passage,

$)_d$ - at the inlet,

$)_o$ - surroundings,

$)_s$ - corrected value,

$)_w$ - at the outlet,

$)_x$ - at the reverse.

INTRODUCTION

The two-fluid spiral Rosenblad heat exchanger is known in industrial practice. A lot of papers, e.g. ,[1 ÷ 8],contain thermal analyses and design descriptions for recuperators of this type. The other versions of parallel-flow spiral heat exchangers are also possible, among them parallel-flow four-stream three-fluid spiral U-type recuperator [9]. Such heat exchanger is considered in the paper.

1. DESCRIPTION OF THE PARALLEL-FLOW SPIRAL U-TYPE (PSU) HEAT EXCHANGER

The schematic view of the heat exchanger considered is shown in Fig. 1. One fluid flows inside the U-type passage of rectangular cross-section.

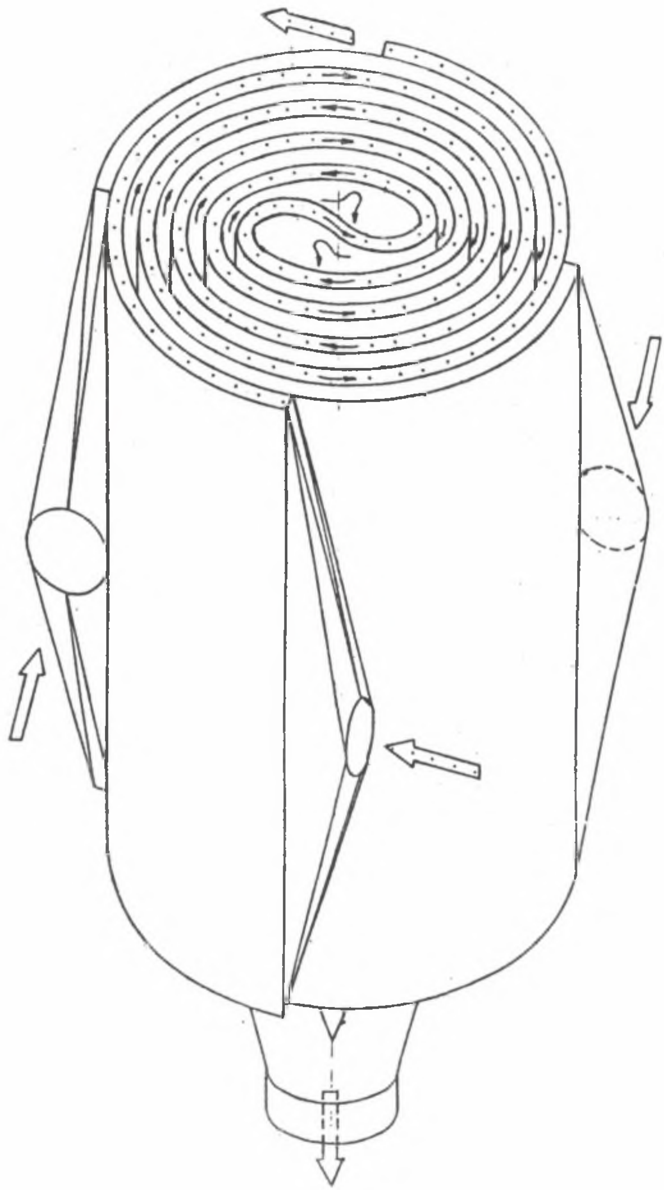

Fig. 1. The scheme of the parallel-flow spiral U-type recuperator

Rys. 1. Schemat równoległoprądowego spiralnego rekuperatora pętlicowego (PSU)

Two other fluids flow through the rectangular gaps between the wound metal strips forming the spiral U-type channel. The top and bottom flat covers enclose the fluids passages. The fluids going outside the U-type channel can flow in the direction of the inner gathering chambers or in opposite direction. These fluids are in practice the same medium but the parallel-flow spiral U-type recuperator belongs to the three-fluid heat exchangers because the three boundary conditions have a form of the given inlet temperatures.

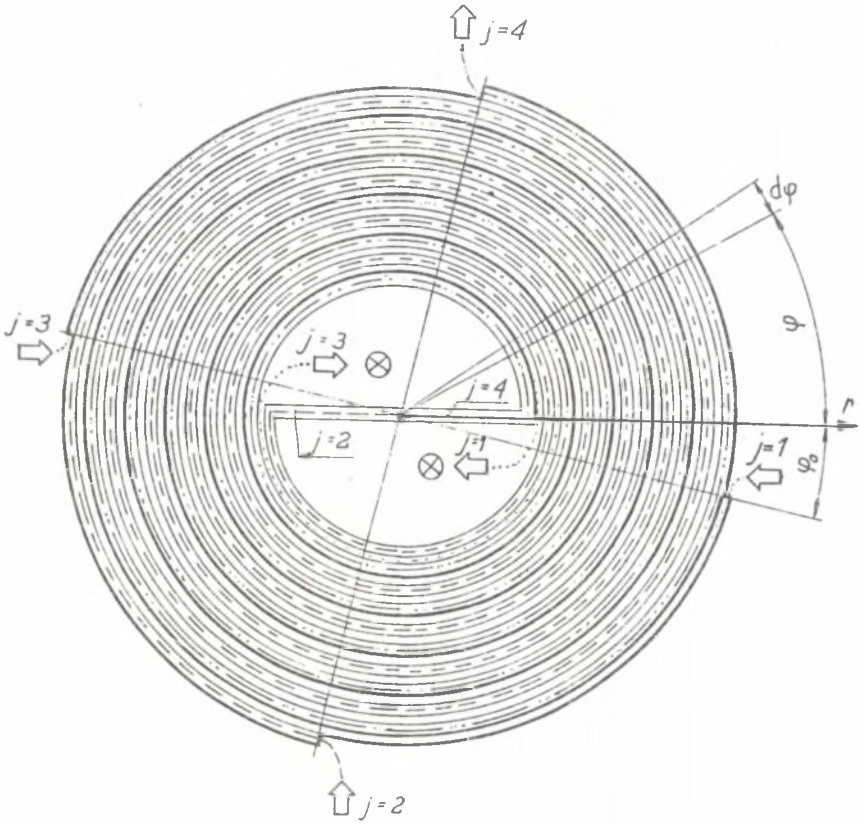

Fig. 2. Cross-section of the model PSU recuperator
Rys. 2. Przekrój modelowego rekuperatora PSU

A cross-section of the model PSU recuperator is shown in Fig. 2. The real PSU recuperator can be replaced by the model one after the correction of the overall heat transfer coefficients in the middle part of the recuperator. For the integer value of the number of coils $\varphi_o = 0$.

2. ENERGY BALANCE EQUATIONS

The thermal analysis of the model PSU exchanger is made for the presumptions generally used in the theory of the convective heat exchangers, i. e., steady state operation, negligible changes of media potential and kinetic energy, one-dimensional heat flow in the streams and in the walls, no heat sources, no radiative heat transfer, one-dimensional and entirely mixed flow of the fluids along the cross-sections of the flow.

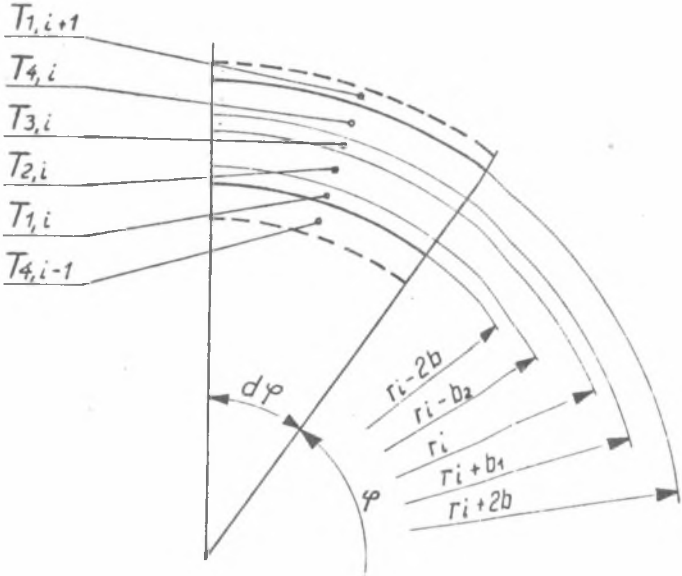

Fig. 3. The elementary portion of the coil i
Rys. 3. Elementarny wycinek zwoju i

The method using finite difference equations is adopted. Such method was used also for a thermal analysis of the typical spiral heat exchangers. The analytical methods are useful only for the simplest cases and they give an approximate form of the solutions or the solution is too complicated and unconvenient for practical applications.

The elementary portion of the coil i is shown in Fig. 3. This portion has an average radius r_i and it is closed within the elementary angle $d\varphi$.

The energy balances for each stream j have here a form as follows ($j = 1 \div 4$, $i = 1 \div n$):

$$W_j \frac{dT_{j,i}}{d\varphi} = \sum_{\substack{k=j-1,\\ j+1}} k_{j-k}\, h\, r_{j-k,i} \left(T_{k,i} - T_{j,i}\right) - 2\, k_{j-o}\, b_j\, r_{j-i} \left(T_{j,i} - T_o\right), \tag{1}$$

where

$$T_{0,i} = T_{4,i-1},\ T_{5,i} = T_{1,i+1},\ k_{1-0} = k_{1-4},\ k_{4-5} = k_{4-1}, \tag{2}$$

$$b_3 = b_1,\ b_4 = b_2$$

and $r_{j-k,i}$ is the radius of metal strip between stream j and k shown in Fig. 3 and ranging within $(r_i-2b) \div (r_i+2b)$. The average radius of the stream j channel $r_{j,i}$ ranges within $(r_i - b_2 - b_1/2) \div (r_i + b_1 + b_2/2)$. The radius r_i is given by the equation

$$r_i = r_p + 4b\,(i - 1) + \frac{2b}{\pi}\,\varphi, \tag{3}$$

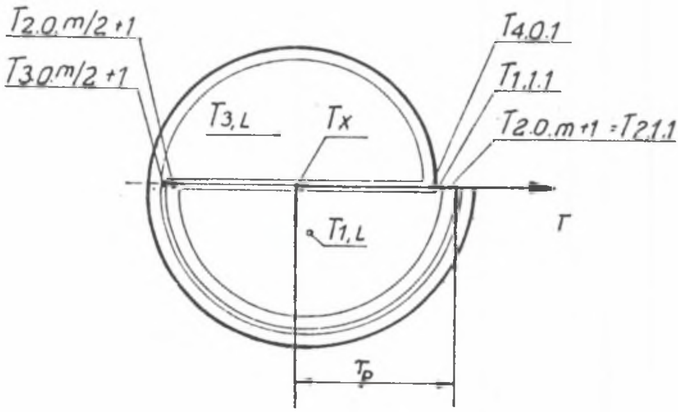

Fig. 4. The inner zone of the model PSU recuperator
Rys. 4. Wewnętrzna strefa modelowego rekuperatora PSU

where r_p is shown in Fig. 4, φ ranges within $0 \div 2\pi$ and i=0 for the coil forming the inner zone of the PSU recuperator. This zone contains two gathering chambers and the central segment of the spiral U-type channel.

The last expression in equation (1) describes the heat losses through the top and bottom covers.

Equations (1) have a dimensionless form:

$$\frac{dt_{j,i}}{dz_i} = 8\pi \sum_{\substack{k=j-1,\\ j+1}} K_{j-k}\, z_{j-k,i}\, (t_{k,i}-t_{j,i}) - $$
$$- 16\pi K_{j-o} \frac{b_j}{h}\, z_{j,i}\, (t_{j,i}-t_o). \tag{4}$$

In equations (4) the temperature derivatives may be approximated by the finite difference formula with central difference estimate. Assuming that $t_{i,j,f} = t_{i,j}(\varphi)$ and $t_{i,j,f+1} = t_{i,j}(\varphi + \Delta\varphi)$, where $\Delta\varphi = 2\pi/m$ and $f = 1,\ldots,m$, the energy balance equations for the main zone of the PSU recuperator are

$$t_{j,i,f+1} \Big(\sum_{\substack{k=j-1,\\ j+1}} a_{j-k,i,f} + c_{j,i,f} + 1\Big) +$$
$$+ t_{j,i,f} \Big(\sum_{\substack{k=j-1,\\ j+1}} a_{j-k,i,f} + c_{j,i,f} - 1\Big) - \tag{5}$$
$$- \sum_{\substack{k=j-1,\\ j+1}} a_{j-k,i,f}\, (t_{k,i,f+1} + t_{k,i,f}) = 2\, c_{j,i,f}\, t_o\,,$$

where

$$a_{j-k,i,f} = \frac{4\pi}{m} K_{j-k}\, z_{j-k,i,f}\,,$$

$$c_{j,i,f} = \frac{8\pi}{m} K_{j-o} \frac{b_j}{h}\, z_{j,i,f}\,.$$

For $i = n$ and for the stream $j = 4$ if $\varphi_o > 0$ then some of the temperatures $t_{j+1,n}$ are replaced by the temperature t_o and cryterion number K_{j-k} is replaced by the number K_{j-o} in which the overall heat transfer coefficient k_{j-o} through the side-shell appears. For the outer zone $i = n + 1$ some of the elements also have a thermal contact through the shell with the surroundings having the ambient dimensionless temperature t_o. The general form of the finite difference equations for $i = n + 1$ is as before. The equations (5) for $i = 1 \div (n + 1)$ may also be obtained directly from the energy balances for the elements inside the angle $\Delta\varphi$.

In the inner zone of the PSU recuperator there are elements having irregular form, e.g., the gathering chambers and a straight segment of the U-type passage. Then the finite difference equations for these elements can be directly obtained only from the energy balances for these elements. It is assumed that the average temperatures of the fluids inside the inner gathering chambers are equal to $\frac{1}{2}(T_{1,1,1} + T_{1,l})$ or $\frac{1}{2}(T_{3,0,\frac{m}{2}+1} + T_{3,l})$, where $l = d$ or $l = w$. For example the energy balance for the fluid 1 gathering chamber under condition of no heat loss is as follows

$$W_1(T_{1,1,1}-T_{1,l}) = \sum_{F=\frac{m}{2}+1}^{m} k_{1-2}\, h\left(r_p - 4b\,\frac{m+0{,}5-f}{m} - b_2\right)\frac{1}{2}\,\Delta\varphi \,\cdot$$

$$\cdot\,(T_{2,0,f}+T_{2,0,f+1} - T_{1,l} - T_{1,1,1}) +$$

$$+\,k_{1-4,s}\,h(r_p - b_2 - b_1)\,\frac{1}{2}\cdot(T_x + T_{4,0,1} - T_{1,l} - T_{1,1,1}) + \tag{6}$$

$$+\,k_{1-2,s}\,h\,(r_p - 2\,b_2 - b_1)\,\frac{1}{2}\cdot(T_x + T_{2,0,\frac{m}{2}+1} - T_{1,l} - T_{1,1,1}).$$

The three indexes have the meaning as before, i.e., the stream number, the coil number and the element number. For the heat exchanger shown in Fig. 2 $l = w$. The corrected values take into account the real form of the inner zone.

The energy balance equations set for the whole PSU recuperator consists of $4\,mn + 4{,}5\,m + 4 - 4\,\frac{\varphi_o m}{2\pi}$ equations which contain $4mn + 4{,}5m + 7 - 4\,\frac{\varphi_o m}{2\pi}$ values of the fluid temperatures. Three of them result from the boundary conditions which have a form for the recuperator shown in Fig. 2 if $\varphi_o = 0$:

$$T_{1,n+1,m+1} = T_{1d};\; T_{2,n+1,\frac{3}{4}m+1} = T_{2d};\; T_{3,n+1,\frac{m}{2}+1} = T_{3d}\,.$$

3. THE NUMERICAL CALCULATIONS

The numerical calculations have been carried out assuming $\varphi_o = 0$ and $b_1 = b_2 = b$ as well as constant values of the overall heat transfer coefficients and heat capacities of the streams. At first, the Gauss-Seidel iteration with successive displacements and overrelaxation was used for solring the energy balance equation set. Yet this method did not give satisfactory results and at last the classic Gauss-Seidel iteration technique with successive displacements but without over-relaxation have been applied. The energy balance equation for the total recuperator was verified and the outlet temperatures were accordingly corrected. The use of the Gauss-Seidel method enables eventual corrections for the overall heat transfer coefficients and heat capacities of the streams when they are temperature functions.

The introductory calculations proved that usually in practice the effect of the heat losses is negligible and that the heat transfer through the shell and covers may be neglected even for the PSU recuperators without the thermal insulation.

4. THE THERMAL EFFECTIVENESS OF THE CONVECTIVE PSU RECUPERATOR, CONCLUDING REMARKS

The thermal effectiveness of the convective PSU recuperator if the heat losses are negligible and for the constant values of heat capacities of the streams is defined here for $T_{1d} = T_{3d}$ and $|W_2| = |W_4|$, as

$$\varepsilon = \frac{|W_2|(T_{2d}-T_{4w})}{W_{min}(T_{2d}-T_{1d})} = \frac{|W_1|(T_{1w}-T_{1d}) + |W_3|(T_{3w}-T_{3d})}{W_{min}(T_{2d}-T_{1d})}. \tag{7a}$$

Usually in practice $|W_1| = |W_3|$ and then

$$\varepsilon = \frac{|W_2|(T_{2d}-T_{4w})}{W_{min}(T_{2d}-T_{1d})} = \frac{|W_1|(T_{1w}+T_{3w}-2T_{1d})}{W_{min}(T_{2d}-T_{1d})}. \tag{7b}$$

For $|W_2| \leq |W_1| + |W_3| = 2|W_1|$

$$\varepsilon = 1 - t_{4w}, \tag{7c}$$

alse

$$\varepsilon = \frac{t_{1w} + t_{3w}}{2}. \tag{7d}$$

The thermal effectiveness of the PSU heat exchanger without heat losses and with the same constant value of all of the overall heat transfer coefficients k as a function of dimensionless heat transfer surface area NTU and the ratio W_{min}/W_{max} is shown in Fig. 5. The parameter NTU may be approximately written here for the model recuperator in a form

$$NTU = \frac{k\,b\,h}{|W_2|}\left[(8n\pi + 9\pi + 4)\frac{r_p}{b} + 16n^2\pi + 16n\pi - 2{,}5\pi - 10\right]. \tag{8}$$

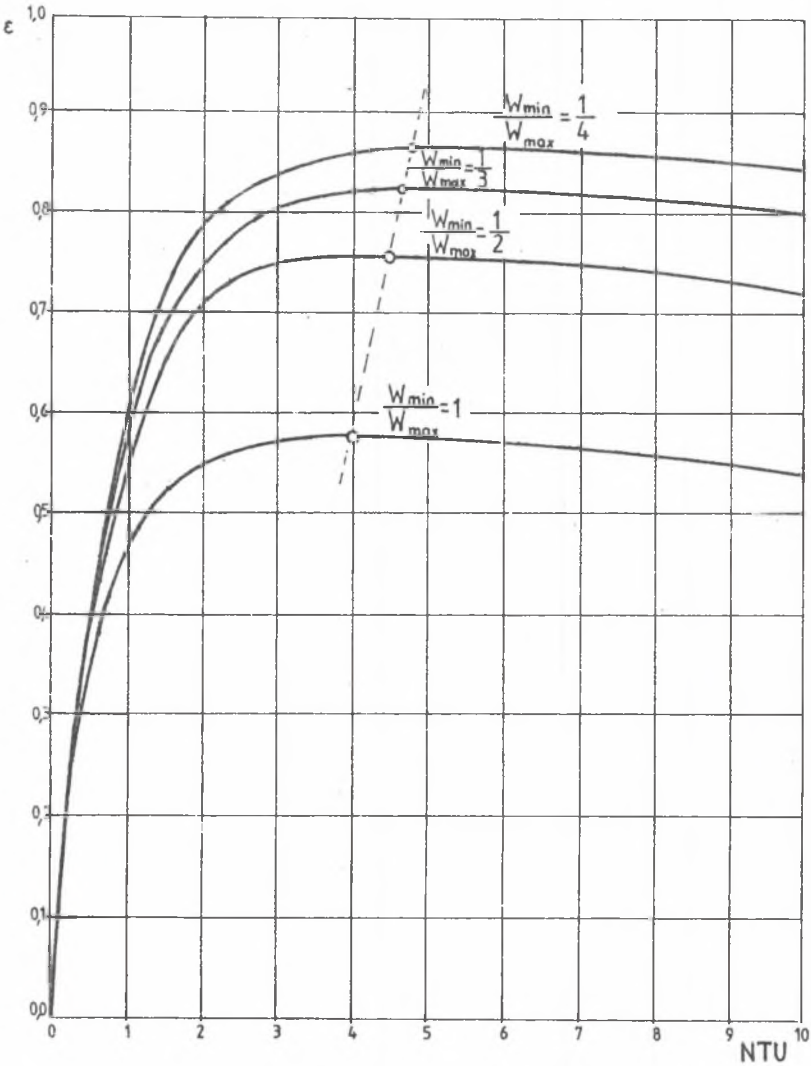

Fig. 5. The thermal effectiveness of the model PSU recuperator, $W_{min}=|W_2|$

Rys. 5. Efektywność cieplna modelowego rekuperatora PSU, $W_{min} = |W_2|$

Such an approximation introduces an error which is not practically greater than 1% (usually several times smaller). The results shown in Fig. 5 are obtained for $\frac{r_p}{b} = 5 \div 15$ and $n = 2 \div 8$. It can be seen that these parameters have no influence on the PSU recuperator thermal effectiveness. It depends only on the parameter NTU and the ratio W_{min}/W_{max}.

The curves shown in Fig. 5 have a maximum. Hence, increasing the heat transfer surface area may sometimes result in diminishing the heat flux

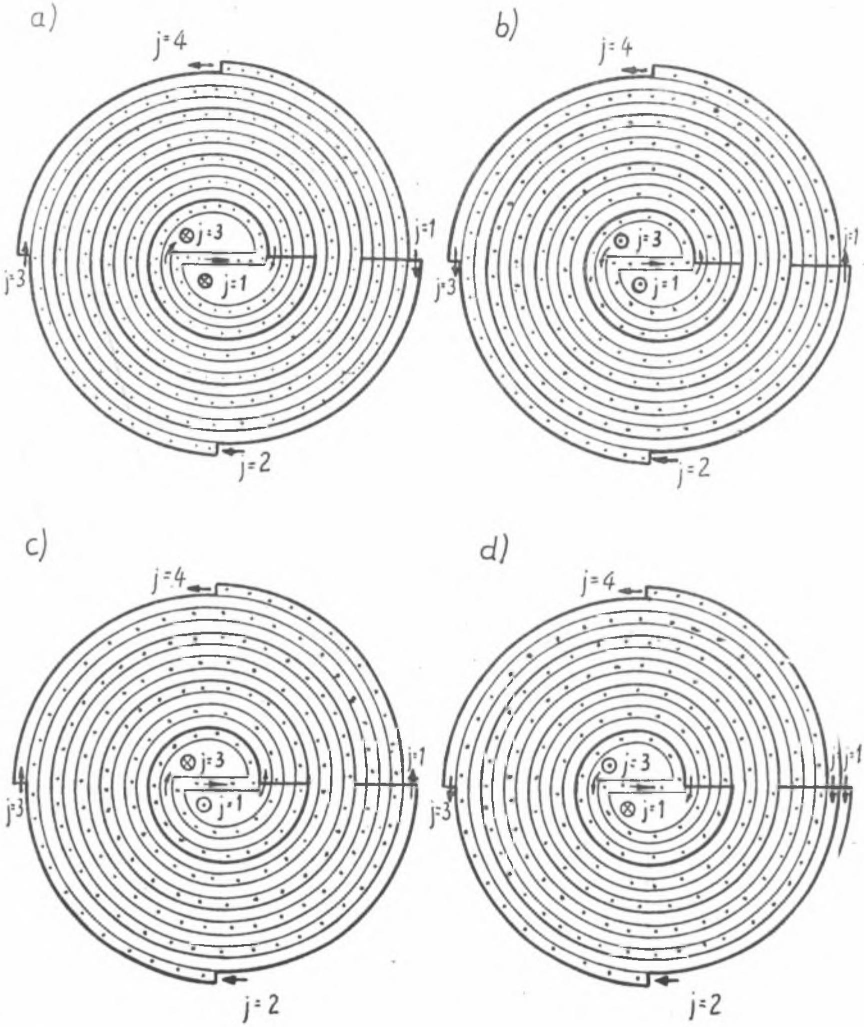

Fig. 6. The typical alterations of the PSU recuperators
Rys. 6. Typowe odmiany rekuperatora PSU

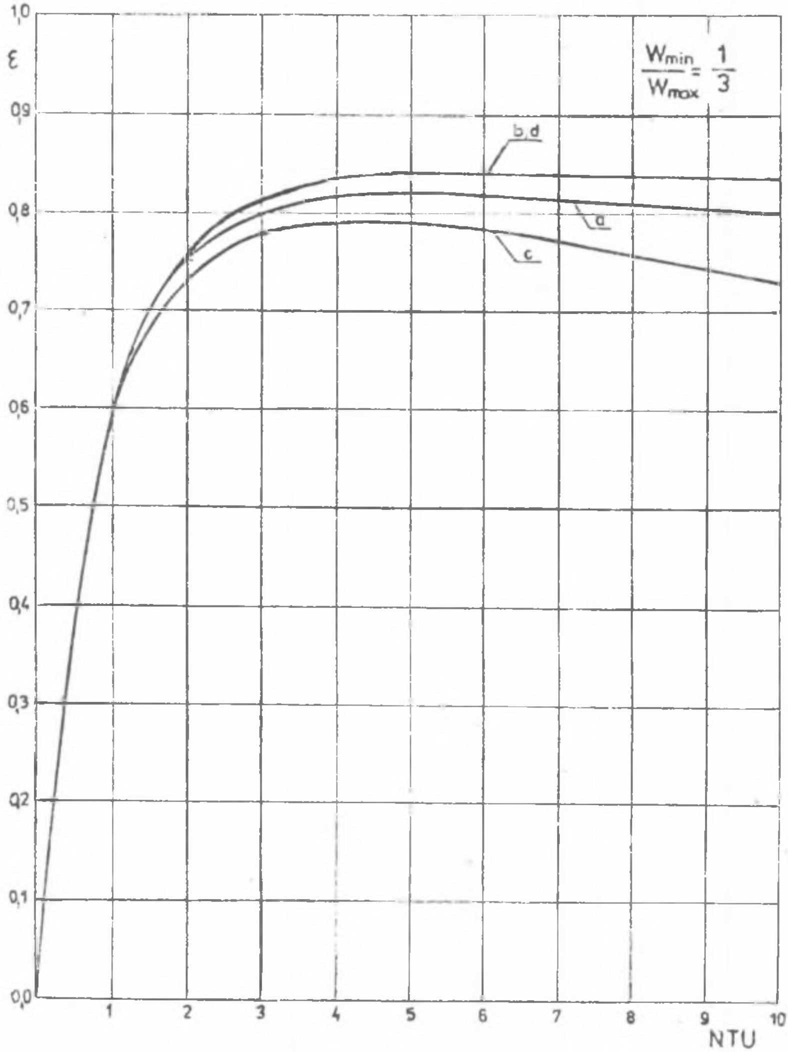

Fig. 7. The results of comparative calculations for the typical four alterations of the model PSU recuperator

Rys. 7. Wyniki obliczeń porównawczych dla typowych 4 odmian modelowego rekuperatora PSU being transferred. This is the same effect as known for some sytems of heat exchangers [10, 11].

Generally, there are four typical cases of the PSU recuperatrs, for which

a: $W_1 < 0$, $W_2 < 0$, $W_3 < 0$, $W_4 > 0$,

b: $W_1 > 0$, $W_2 < 0$, $W_3 > 0$, $W_4 > 0$,

c: $W_1 > 0$, $W_2 < 0$, $W_3 < 0$, $W_4 > 0$,

d: $W_1 < 0$, $W_2 < 0$, $W_3 > 0$, $W_4 > 0$.

These cases are shown in Fig. 6. The case considered is seen inFig. 6a. This case has the greatest practical significance. The results o the comparative calculations for the four cases mentioned above are shon in Fig. 7. It can be seen that each of the four typical cases of the PSUrecuperator gives similar thermal output for the lower values of the NTUparameter.

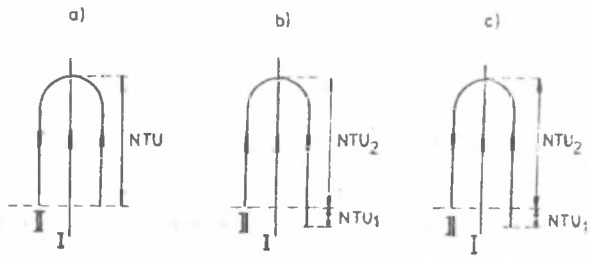

Fig. 8. The schemes of the standard parallel - flow U-type heat exchangers

Rys. 8. Schematy klasycznych równoległoprądowych pętlicowych wymienników ciepła

The results of the calculations which have been performed according to the numerical model shown in the paper briefly have been compared with results obtained for the standard parallel-flow U-type recuperators. Three cases of such recuperators have been considered. The schemes of these cases are given in Fig. 8. The case a in Fig. 8 is the typical one, the case b is equivalent to the PSU recuperator as in Fig. 6a while the stream 3 does not exist. Adequately the case c in Fig. 8 is equivalent to the PSU recuperator while the stream 1 does not exist. For the cases b and c in Fig. 8 $NTU = NTU_1 + NTU_2$. The results of the comparative calculations for the PSU recuperator type a in Fig. 6 and for the heat exchangers as in Fig. 8 are shown in Fig. 9. Similarly as before it was assumed that $|W_2| = W_4 = W_{min}$. For the recuperators b and c in Fig. 8 and equivalent PSU recuperators it was assumed $NTU_1/NTU = 0{,}0325$. The calculations for the cases shown in Fig. 8 were performed using simple analytical formulae. For the case c in Fig. 8 it is not difficult to prove analytically or by physical analysis that the thermal effectiveness as a function of NTU pa-

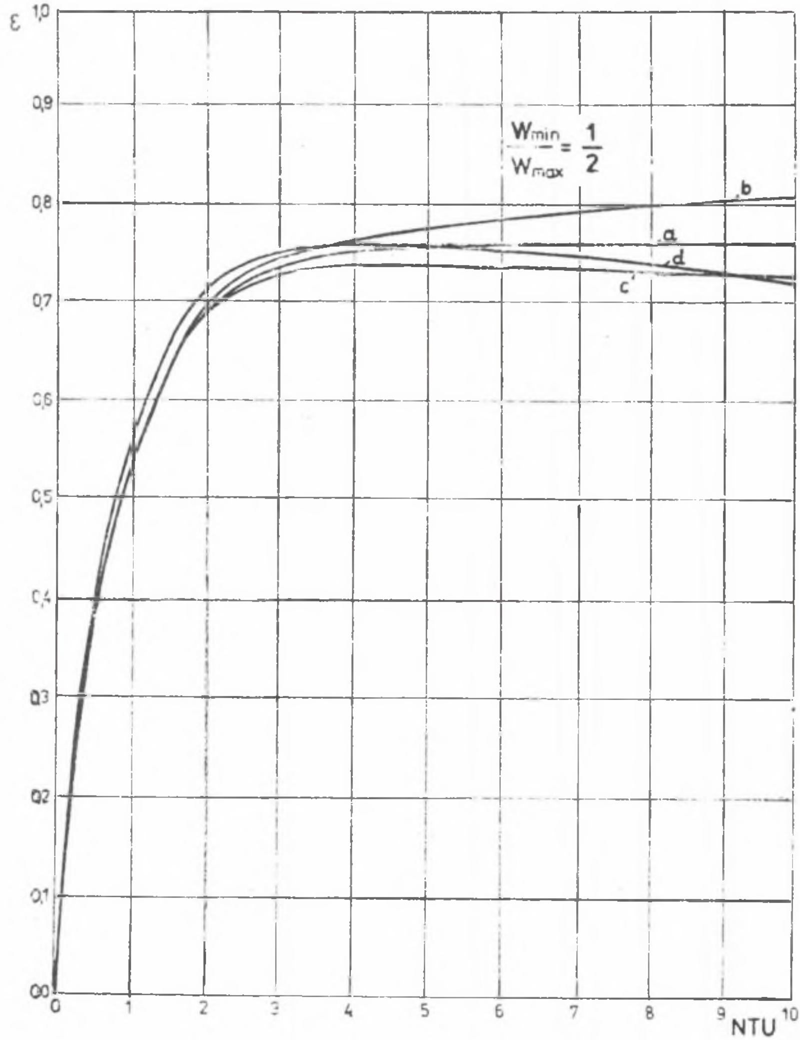

Fig. 9. The results of comparative analytical and numerical calculations a) case a in Fig. 8, $W_{II} = W_{min}$, b) case b in Fig. 8 and case a in Fig. 6 for $|W_2| = W_{II}$, $W_1 = W_I$ and $W_3 = 0$, c) case c in Fig. 8 and case a in Fig. 6 for $|W_2| = W_{II}$, $W_3 = W_I$ and $W_1 = 0$, d) case a in Fig. 6 for $W_1 = W_3$

Rys. 9. Wyniki analitycznych i numerycznych obliczeń porównawczych a) przypadek a na rys. 8, $W_{II} = W_{min}$, b) przypadek b na rys. 8 i przypadek a na rys. 6 dla $|W_2| = W_{II}$, $W_1 = W_I$ i $W_3 = 0$, c) przypadek c na rys. 8 i przypadek a na rys. 6 dla $|W_2| = W_{II}$, $W_3 = W_I$ i $W_1 = 0$, d) przypadek a na rys. 6 dla $W_1 = W_3$

rameter has a maximum. It can be seen that the analytical and numerical calculations for equivalent cases produced the same results. It also can be seen in Fig. 9 that for low values of the NTU parameter all the cases have similar thermal effectiveness.

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EFEKTYWNOŚĆ CIEPLNA TRÓJCZYNNIKOWEGO KONWEKCYJNEGO SPIRALNEGO REKUPERATORA PĘTLICOWEGO

S t r e s z c z e n i e

W pracy przedstawiono analizę cieplną konwekcyjnego spiralnego trójczynnikowego równoległoprądowego wymiennika ciepła pętlicowego. Przedstawiono w skrócie model matematyczny urządzenia. Model ten zawiera równania różnicowe bilansu energii rozwiązywane metodą iteracyjną. Wyniki obliczeń przedstawiono w formie wykresów efektywności cieplnej.

ТЕПЛОВАЯ ЭФФЕКТИВНОСТЬ ТРЁХФАКТОРНОГО КОНВЕКЦИОННОГО СПИРАЛЬНОГО ПЕТЛЕВОГО РЕКУПЕРАТОРА

Р е з ю м е

В работе приведён тепловой анализ токопараллельного трёхфакторного спирально-теплового теплообменника.

В сокращении показана математическая числовая модель. Эта модель содержит дифференциальные уравнения энергетического баланса решённые итерационным методом.