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MODAL ANALYSIS OF ROTATING MACHINES

Summary. Modal Analysis of rotating machines has specific aspects and requires special approach. Classical Modal Analysis elaborated for passive (nonrotating) structures does not yield satisfactory results. These aspects, as well as specific application of circular perturbation testing of rotating machines is discussed in this paper. An algorithm for identification of modal parameters of rotating machines, based on Dynamic Stiffness approach is outlined.

1. Introduction

Experimental Modal Analysis has become a popular method for studying practical vibration problems of mechanical structures. Application of Modal Testing for parameter identification and diagnostics of rotating machines, which represent an important class of mechanical structures, has several specific aspects and requires a special approach. The results and predictions obtained by applying the classical "passive structure" Modal Testing to a rotating machine are usually incomplete and not sufficiently accurate for the most important lateral modes, while providing information which is insignigicant for the rotating machine operating performance.

2. Specific aspects of Modal Analysis of rotating machines

2.1. Most of the modal identification methods and conventional procedures of modal analysis deal with structures with assumed linear behavior. The structures are modeled by self-adjoint differential operators and discretized by symmetric matrices. Rotating machines have an inherent nonsymmetric nature, due to rotation-related factors, such as gyroscopic effects and fluid dynamic forces in bearings and seals, which provide feedback-like effects. The dynamic behavior of rotating machines can adequately be represented only by the non self-adjoint differential operators. The discretization yields nonsymmetric matrices. The modal analysis has to not only determine all classical modal parameters (i.e., eigenvalues, eigenfunctions constituting the right eigenvectors and form of eigenfunctions yielding generalized/modal masses associated with each eigenmode), but also the parameters provided by the left eigenfunctions. Decoupling of precessional mode components requires the utilization of additional relations (such as biorthogonality) between left and right eigenvectors 1-5 .

2.2. Rotating machines can be modeled by linear equations in very limited ranges of deflections and velocities. The classical Modal Analysis, based on the assumption of linearity, has to be completed by taking nonlinearities into consideration. Significant nonlinear effects of geometric and physical origin in rotating machines can introduce large errors to the classical Modal Tests 6-8 .

2.3. All dynamic phenomena occuring during the performance of a rotating machine are closely related to the rotative motion of the rotor. The continuous supply of rotative energy makes the system "active", containing a feedback loop. Numerous vibrational phenomena in rotating machines occur due to the transfer of energy from rotation (main performance) to vibration (undesirable side effects). Rotation of the shaft with all mechanical parts attached to it, as well as involvement in rotation of the working fluid (in fluid-flow machines, in seals and bearings), causes important modifications in modes and natural frequencies. In large turbomachines, additional changes can be generated by thermal effects, foundation deformations, and misalignment. All these factors cause the results of Modal Testing of rotating machines "at rest" ("passive structure" approach) to differ significantly from the results of testing during machine operational conditions ("active structure" approach).

2.4. Rotors, which represent the main parts of rotating machines, are similarly constrained in two lateral directions; therefore, they exhibit vibrational motion which always has two inseparable coupled lateral components (conventionally called "vertical" and "horizontal"). The result is two-dimensional precessional motion of the rotor.

Unidirectional impulse testing, widely used in Modal Analysis, when applied to a rotating shaft, results in a response containing both vertical and horizontal components.

2.5. In practical performances of rotors, the precessional motion can contain multi-frequency components, each of which has a definite relation to the direction of rotation. In the most general case, each individual component can be either forward (direction of precession the same as direction of rotation) or backward (direction of precession opposite to rotation). Direction of precessional motion is vital to the rotor integrity. The net deformation frequency of the shaft is equal to the difference between rotative and precessional frequencies, taking into account their signs. During backward precession the shaft is, therefore, subject to high frequency deformation (sum of both frequencies). When measuring rotating machine vibrations, it is important to identify each vibrational frequency component, whether it is forward or backward. Narrow band filtering and time base/orbit analysis are extremely helpful for this purpose. In classical Modal Testing, "negative" frequencies have no meaning. Applied to rotating machines, the "negative" frequency has a direct and very significant physical interpretation related to backward precession.

2.6. Most important vibrational phenomena of rotating machines are associated with the rotor lateral vibrations (sometimes coupled lateral/torsional/longitudinal vibrations). Each mode of rotor lateral vibration contains two components (vertical and horizontal), the characteristics of which are usually slightly different as a result of elastic/mass nonsymmetry of the rotor and supporting structure in two lateral directions. Modal Testing of structures with closely-spaced modes presents numerous difficulties. Rotating machines belong to this category. It is, therefore, reasonable to define "pair modes" in rotating machines (e.g., "first mode vertical" and "first mode horizontal").

2.7. Classical Modal Testing usually, though not always, deals with a large number of modes of a structure over a wide frequency range. In the performance of rotating machines, the most important are the lowest modes and low-frequency precessional phenomena. This fact is related to the rigidity of the rotor system and to the relationship between the actual rotative speed and rotor precessional dynamic phenomena. Firstly, rigidity/mass characteristics of a rotor are always located in a lower range of frequencies than those of the supporting structure. The lowest modes of the rotating machine correspond, therefore, to the modes of the rotor itself. Secondly, the rotating machine has its own continuously active forcing function -- the unbalance, which is a inseparable feature of the rotating system. The frequency of this force is equal to the rotor s actual rotative speed. The resulting motion is referred to as synchronous precession. The operating speed of a single span machine train, even if it represents dozens or thousands of rpm, seldom exceeds third balance resonance frequency (the third lateral natural frequency); therefore, main interest is concentrated on investigating the rotor's first two or three lateral/bending modes, as the rotating machine has to survive resonances of the lowest modes during each start-up and shutdown. The amplitudes of rotor deformation at low modes are the highest, and the low modes are usually poorly damped. Therefore, they are of the greatest concern.

2.8. Another aspect of importance, focused on rotor lowest modes, is the fact that nearly all self-excited vibrational/precessional phenomena occurring during performance of a rotating machine are characterized by low frequencies, always located in the subsynchronous region (frequencies lower than synchronous frequency). The self-excited vibrations occur when rotative speed is sufficiently high, and they are often referred to as rotor instabilities, which significantly affect the machine operation. The frequency of self-excited vibrations is either equal to a fraction of the actual rotative speed -- and the same ratio to rotative speed is maintained if the rotative speed varies (oil whirl, partial rub) -- or it is equal to the rotor bending mode natural frequency (oil whip, full annular rub). Due to the specific role of internal friction, the subsynchronous vibrations of rotating machines are always characterized by much higher amplitudes than supersynchronous vibrations.

2.9. When dealing with high number of modes during classical Modal Testing, the accuracy of the phase angle readings is usually low. In rotating machines the phase angle represents an extremely important parameter. In not only gives information on the force/response relationship, but also relates the shaft lateral vibration to its rotative motion. It also yields significant information for identifying modal parameters. Limiting Modal Analysis to the lowest modes permits one to increase the accuracy of phase angle readings.

2.10. Finally, the most important aspect: The results of Modal Testing of rotors during operational conditions in low-frequency regions reveal the existence of specific modes, unknown in "passive" structures. These modes are generated by solid/fluid interaction activated by the shaft rotation, e.g., in fluid-lubricated bearings ans seals. During rotating machine performance, these modes exhibit their activity through rotor self-excited vibrations (e.g., "oil whirl" is the rotor/bearing system self-excited vibration; "oil whirl resonance" and "oil whirl mode" are modal parameters revealed by perturbation testing [9-11].

In summary, Modal Analysis of rotating machines provides a significant computational complexity due to the nonsymmetric nature of rotating structure dynamic behavior. Modal Testing of rotating machines should be focused on the lowest bending modes and applied to the rotor during normal operational conditions. The classical Modal Testing, as used in case of "passive" structures, is not the most efficient for this purpose. Better results can be obtained by applying limited frequency sweep, circular-force, perturbation testing (Fig. 1).

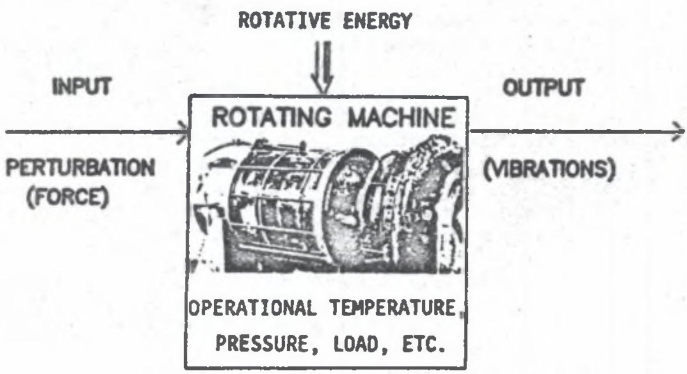

Fig. 1. Perturbation technique for Modal Testing of rotating machines at their operational conditions

3. Input forces for Modal Testing of rotating machines

Classical modal testing uses unilateral exciting forces, such as provided by impacting or sinusoidal excitation. Static structures exhibit symmetry (in terms of the mathematical model, all matrices are symmetric), which results in reciprocity of the cross-data: accelerance at point "i" when force is applied at point "k" equals the accelerance at point "k" when force is applied at point "i".

Rotating machines are not symmetric. Nonsymmetry in the system matrices results from rotation-generated tangential forces. The natural frequencies and modes differ for two different directions of rotation. They are referred to as "forward" and "backward" modes. Their corresponding natural frequencies differ in values, and these differences are functions of rotative speed (and possibly other operational factors).

When the classical unilateral excitation is applied to a rotating shaft, the forced response consists of both forward and backward modes, which are difficult to separate.

The best excitation for rotating shaft modal testing during machine operational conditions is a rotating force with the distinct direction: forward (same as rotation) or backward (opposite to rotation). This type of nonsynchronous excitation allows for easy separation of the forward and backward modes and identification of rotation-generated terms. The term "nonsynchronous" refers to the perturbation frequency which is different than the rotative speed.

The use of a circular rotating force has further advantage, namely ease of control of the force magnitude and phase by applying a controlled unbalance in the perturbation system, as well as the ease of controlling its frequency.

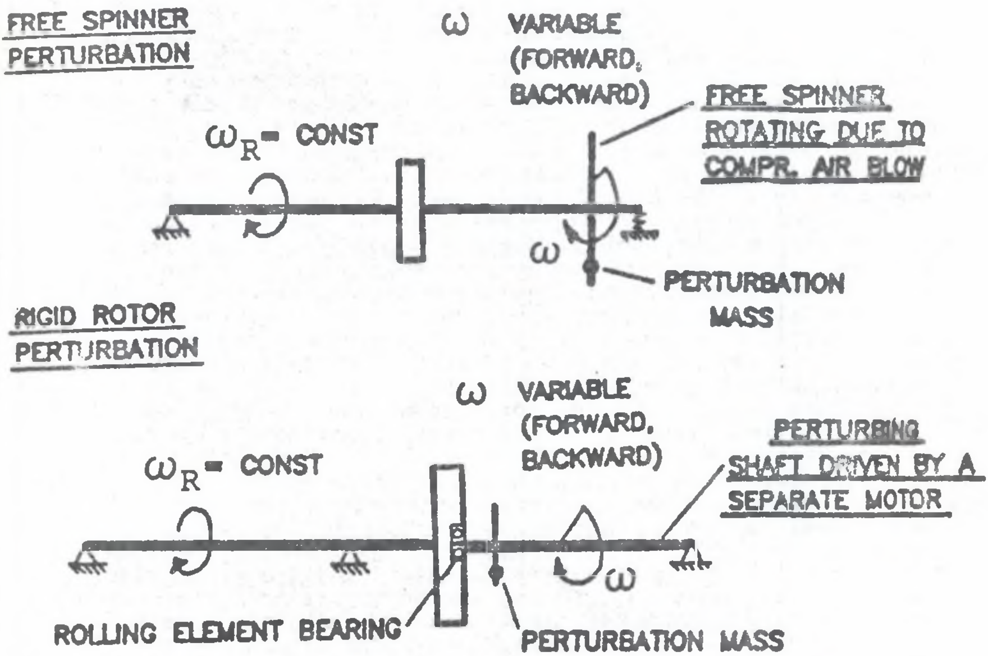

Fig. 2. Perturbation testing of rotating machines using sweep frequency circular forces generated by rotating unbalance at (A) free speener, or (B) rigid auxiliary rotor. Rotative speed of the main shaft ($_R$) maintained constant

Various types of perturbation systems generating rotating forces can be used. An unbalanced rotating free spinner mounted on the shaft (driven, for instance, by an air jet flow) or an unbalanced auxiliary shaft attached to the main rotating machine shaft through a pivoting bearing and driven by a separate motor, both allow for "nonsynchronous" shaft perturbation (Fig. 2). Two electromagnetic actuators in X-Y configuration and generating sinusoidal forces with 90 degree phase shift can also be applied as perturbation input force devices. For all these systems, the frequency (angular speed) of the perturbing force is entirely independent from the rotative speed of the main shaft: The latter rotates at a chosen constant speed, while the perturbation device provides the perturbation force with sweep frequency. The shaft can be perturbed either in a forward or a reverse direction. These perturbation systems also yield very good results in "passive" cases, i.e., when the shaft does not rotate.

4. Response measurements

Most popular transducers used in classical modal testing are accelerometers. The results are presented in terms of accelerances (or "inertances") representing response acceleration vector to input force vector ratios. Modal analysis of passive structures deals usually with high number of modes with natural frequencies located in relatively high frequency range. Accelerometers are the most appropriate instruments for this purpose.

In a rotating machine the modes of highest interest are these of the rotor itself. Most often the rotor modes correspond to the lowest modes of the entire structure. The first natural frequency may occur in the range of 5 to 15 Hz. In this range of frequencies accelerometers perform very poorly. The best transducer in the low frequency range is the displacement proximity probe. When mounted in casings or bearings, the proximity probe provides relative measurements (shaft motion relative to support motion). For machines with very soft supports the proximity probe can be complemented by a seismic probe providing casing absolute measurements (for instance in a form of a dual probe).

Results of modal testing using displacement transducers are usually presented in term of receptances (with equally used names such as "admittances", compliances", "dynamic flexibilities"), which are the ratios of the displacement response vectors to input force vectors. Receptance vectors are widely used, for instance, in rotor balancing. (They are often called "influence coefficients", but more property are "influence vectors").

Using accelerometers, velocity pickups, or proximity transducers in measurements of mechanical structure vibrations is not only a matter of rational choice, corresponding the best to the type of encountered conditions. It is also a matter of philosophy. Although popular in modal analysis applications, accelerometers are not widely used in on-line diagnostics of rotating machinery malfunctions. Rotor displacements, not accelerations, are the most meaningful signals for operating personnel. Specific changes in rotor displacements (vibration amplitudes as well as static positions) directly indicate what type of malfunctions the machine developes. Changes in the static positions indicate changes in alignment state. This data assists in prediction of oil whirl/whip self-excited vibrations, as well as shaft crack prevention. A specific content of vibration signals indicate presence of unbalance, rotor-to-stator rubs, loose parts, shaft cracking, and other malfunctions.

The most harmful vibrations for the integrity of the rotating machine are low frequency, subsynchronous vibrations, resulting usually from an instability action transferring rotational energy into vibrations. They have a "self-excited" nature. Internal/structural friction forces in rotating elements at the subsynchronous frequency range act against external damping forces. This results in lowering the level of the system "positive", stabilizing damping, and consequently, leads to high amplification of subsynchronous vibration amplitudes independently of the original source of vibration. The rotor lowest frequency vibrations are usually characterized by the highest amplitudes.

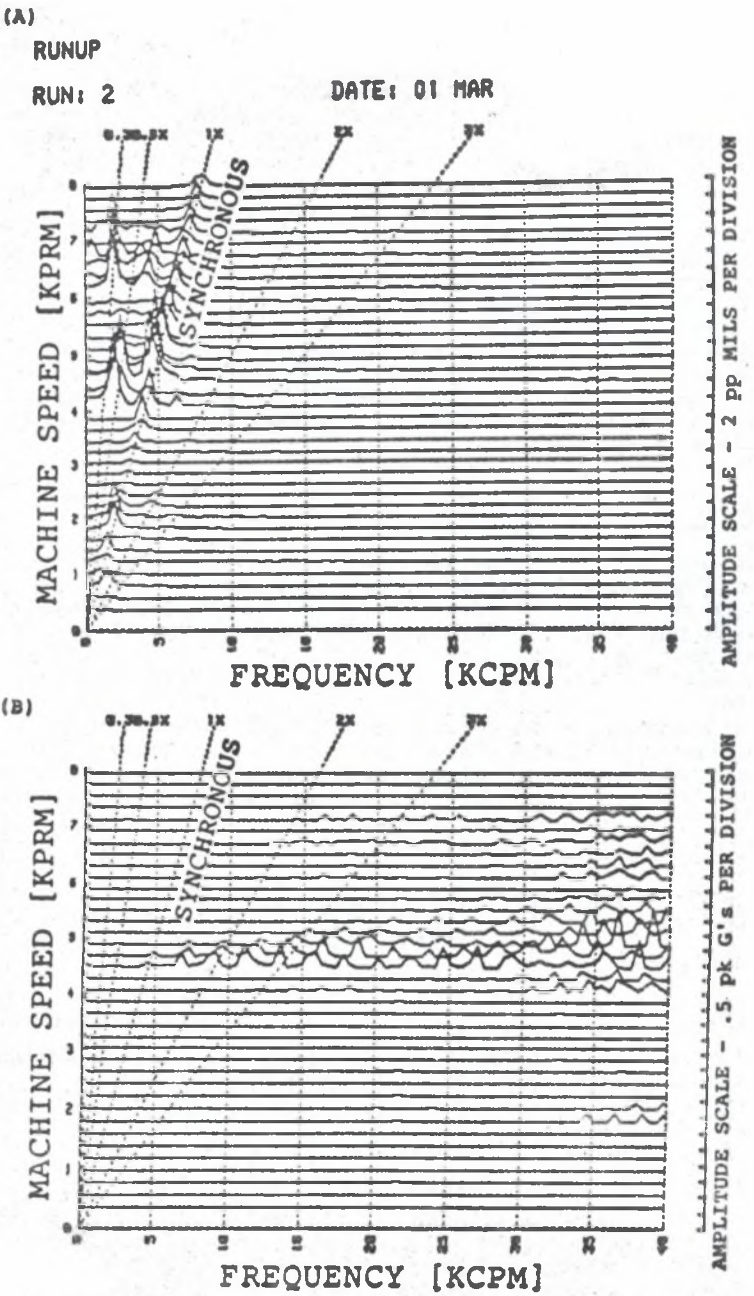

Fig. 3. Spectrum cascade plots of the rubbing rotor vibrational response during start-up measured by (A) displacement proximity transducer and (B) accelerometer

Acceleration amplitude is proportional to the square of vibration frequency. This means that when measuring rotor vibration by using accelerometers, the higher frequency components become dominant, indicating high amplitudes. The low frequency components look insignificantly small, even though they have high amplitudes in terms of rotor displacements. In addition, accelerometers are most often installed outside the rotor casing; thus they measure vibrations of the outside structure, not vibrations of the rotor, which is the main source of vibration. By transmission through the structure, vibration becomes attenuated.

An example is shown in Fig. 3: Due to severe rotor-to-stator rub, subsynchronous vibrations at frequencies around 2 kCPM exhibiting amplitudes above 6 mils pp do not generate any meaningful signal when measured by an accelerometer. Vibrations with 6 mils pp at 2 kCPM represent only 0.34 g. With accelerometer sensitivity of 10.2 mV/g it gives a weak signal of 3.5 mV. On the other hand, the eighth harmonic (8x) at 38 kCPM shows the high acceleration amplitude of 1.5 g (15.3 mV), which represents, however, a meaningless motion of 0.073 mils pp.

5. Identification of modal parameters of a rotating machine

In the classical Modal Testing identification of the structure modal parameters are usually based on curve fitting of results presented in the form of Receptances (Fig. 4). Much better identification results are, however, obtained when Dynamic Stiffness Approach is applied. Straight lines are definitely the best to fit (Fig. 4). An algorithm for identification of modal parameters of a rotating machine is outlined below.

Rotor responses to rotating force excitation bring meaningful data for identification of the system parameters. The identification procedure involves matrice inversions, thus identification of higher number of modes requires a computer for experimental data adquisition and processing. The identification algorithm is outlined below.

Consider a nonsymmetric (laterally anisotropic) rotor with 2n degrees of freedom:

$$\begin{bmatrix} [M_1] & [M_{12}] \\ [M_{21}] & [M_2] \end{bmatrix} [\ddot{z}] + \begin{bmatrix} [D_1] & [D_{12}] \\ [D_{21}] & [D_{22}] \end{bmatrix} [\dot{z}] + \begin{bmatrix} [K_1] & [K_{12}] \\ [K_{21}] & [K_2] \end{bmatrix} [z] = \tilde{F}$$

where $z = \mathrm{col}\left[x_1,\ldots,x_p,\ldots,x_n,y_1,\ldots,y_p,\ldots,y_n\right]$ represents rotor horizontal and vertical deflections at its $p=1,\ldots,n$ axial locations. The system parameters are represented by the matrices: $[M_q]$, $[M_{q,3-q}]$, $[D_q]$, $[D_{q,3-q}]$, $[K_q]$, $[K_{q,3-q}]$, $q=1,2$ are inertia, damping, and stiffness matrices correspondingly. The matrices $[D_{q,3-q}]$ contain gyroscopic effects and "cross" damping terms; matrices $[K_{q,3-q}]$ contain elements of "cross" stiffness type. They strongly depend on the rotative speed. The function $\tilde{F}$ represents an excitation vector. For identification of the system parameters, a rotating perturbation (excitation) force is applied consecutively at "r" $(r = 1,\ldots,n)$ axial locations of the rotor in either forward $(s=1)$ or backward $(s=2)$ direction. The n pairs of displacement transducers in X-Y configuration are mounted at "p" $(p=1,\ldots,n)$ axial locations of the rotor (Fig. 5). At the experiment when the force is applied at r-th location, the excitation vector is, therefore,

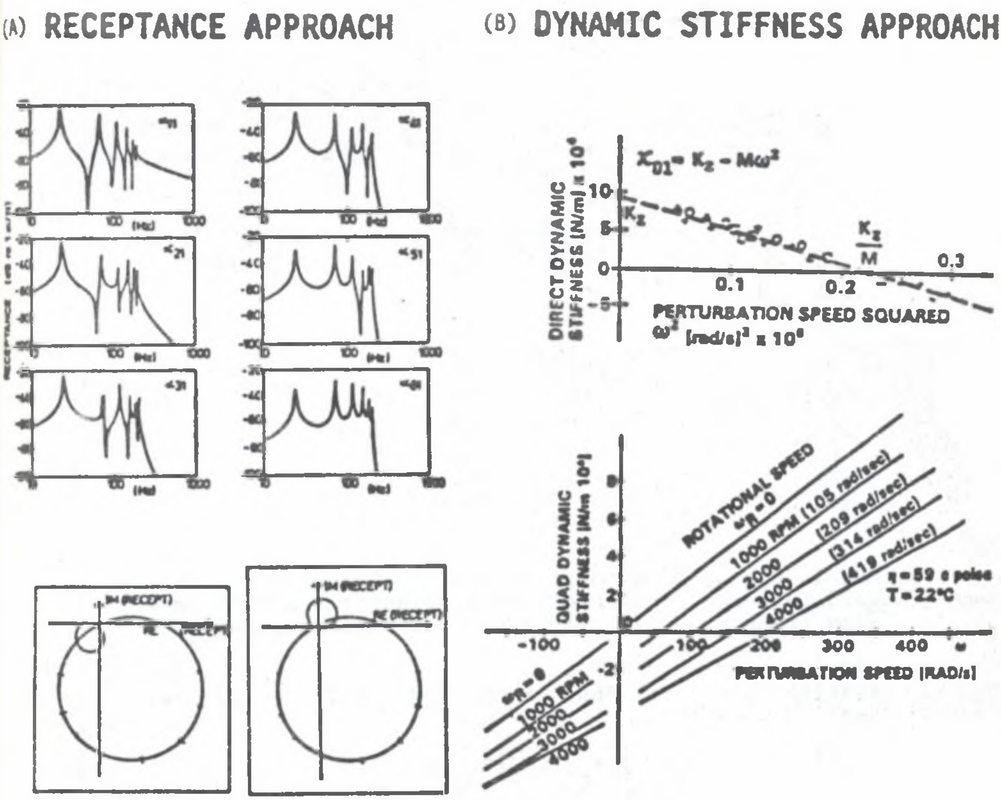

Fig. 4. Identification of modal parameters: (A) curve fitting of receptances, 12 , (B) straight line fitting of dynamic sitffness results 11

$$\tilde{F} = \mathrm{col}\left[0,\ldots,0,F_{sr}\cos(\omega t+\delta_{sr}),0,\ldots 0,(-1)^{s+1}F_{sr}\sin(\omega t+\delta_{sr}),0,\ldots,0\right],\quad s=1,2.$$

where ω is perturbation frequency, F_{sr} and δ_{sr} are the perturbation force amplitude and phase respectively. Rotor responses measured at "p" axial locations and filtered to the components with frequency ω are as follows:

$$\left[A_{sq}\right] = \begin{bmatrix} \bar{A}_{sq11} & \cdots & \bar{A}_{sq1n} \\ \vdots & & \vdots \\ \bar{A}_{sqn1} & \cdots & \bar{A}_{sqnn} \end{bmatrix} = \left[A_{sqpr}e^{j\alpha_{sqpr}}\right], \quad s,q = 1,2$$

where "r" indicates the exciting force location, A_{sqpr} is the response amplitude, α_{sqpr} is the corresponding response phase (angle between force and displacement). In this presentation the comples number formalism is used $(j=\sqrt{-1})$.

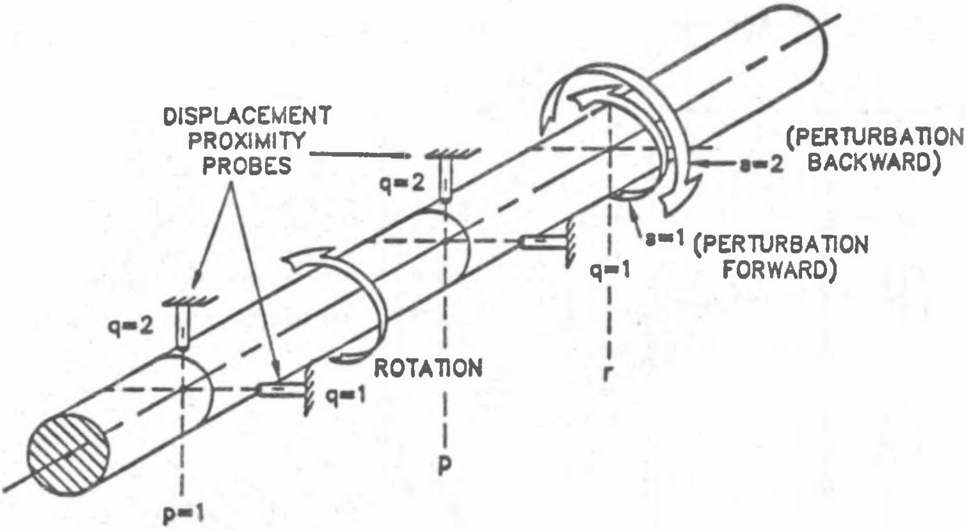

Fig. 5. Modal testing of a rotating shaft using circular forward and backward perturbation forces and displacement noncontacting proximity transducers for vibration response measurements

The identification procedure is converged into the following expressions:

$$\left[K_q\right] - \omega^2\left[M_q\right] = \mathrm{Re}\left\{\left[W_{1_q}\right] - (-1)^q\left[W_{2_q}\right]\right\} \tag{1}$$

$$\omega\left[D_q\right] = \mathrm{Im}\left\{\left[W_{1_q}\right] - (-1)^q\left[W_{2_q}\right]\right\} \tag{2}$$

$$\omega\left[D_{q,3-q}\right] = -\mathrm{Re}\left\{(-1)^q\left[W_{1_q}\right] - \left[W_{2_q}\right]\right\} \tag{3}$$

$$\left[K_{q,3-q}\right] = \mathrm{Im}\left\{(-1)^q\left[W_{1_q}\right] - \left[W_{2_q}\right]\right\}, \quad q = 1,2 \tag{4}$$

where

$$\left[W_{sq}\right] = \left[F_s\right]\left\{\left[A_{sq}\right] - \left[A_{3-s,q}\right]\left[A_{3-s,3-q}\right]^{-1}\left[A_{s,3-q}\right]\right\}^{-1}, \quad s,q=1,2$$

$$F_s = \mathrm{diag}\left[F_{s1},\ldots,F_{sn}\right]$$

The symbols "Re{ }" and "Im{ }" denote the real and imaginary parts of the corresponding expressions. The power "-1" indicates matrix inversion.

With the frequency sweep excitation (ω variable from zero to ω_{max}) the results (1) to (4) are graphically presented versus ω or ω^2. The latter is used for Eq. (1): each element of the matrix $\left[K_q - \omega^2 M_q\right]$

will then be represented in the plane $(\omega^2, K_{qsr} - \omega^2 M_{qsr})$ by a straight line, thus allowing easy identification of the elements of the matrices $[K_q]$ and $[M_q]$. Each stiffness K_{qsr} can be read at intersection with the vertical axis and the mass M_{qsr} as a slope of the corresponding line (Fig. 48). The elements of the matrics $[D_q]$ and $[D_{q,3-q}]$ are calculated as slopes of the straight lines versus yielded by E_{qs}, (2) and (3).

The described methodology was successfully used for one- and two-mode identification of rotor/bearing system [9-11].

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О МОДАЛЬНОМ АНАЛИЗЕ ТУРБОМАШИН

Р е з ю м е

Модальному анализу турбомашин свойственны специфические аспекты и требует он специального подхода.

Классический модальный анализ, разработанный для пассивных невращающихся конструкций, не даёт положительных результатов.

Эти аспекты, как и применение исследования колесных нарушений в турбомашинах, представлены в настоящей работе. В анализируемом случае, базируя на элементах матрицы динамической жёсткости, сформулирован алгоритм идентификации модальных параметров турбомашин.

O ANALIZIE MODALNEJ MASZYN WIRNIKOWYCH

S t r e s z c z e n i e

Analiza modalna maszyn wirnikowych ma specyficzne aspekty i wymaga specjalnego podejścia.

Klasyczna analiza modalna, która została opracowana dla biernych niewirujących konstrukcji, nie daje zadowalających rezultatów.

Te aspekty, jak również zastosowanie badania kołowych zaburzeń w maszynach wirnikowych,przedstawiono w tej pracy. W omawianym przypadku bazując na elementach macierzy sztywności dynamicznych sformułowano algorytm identyfikacji parametrów modalnych maszyn wirnikowych.

Recenzent: prof. dr hab. inż. J. Wojnarowski

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