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INTRINSIC TIMES OF RHEOLOGICAL BODIES AND MECHANICAL PROCESSES

Streszczenie. W pracy przedstawiono koncepcję zastąpienia zwykłego czasu chronologicznego przez parametry rozwoju, grające rolę własnych czasów ciał reologicznych i procesów mechanicznych. Czasy własne są mierzone przez kolejność procesów mechanicznych i deformacyjnych. Koncepcja miar czasów własnych wynika z porównań zmian i ruchów ciał i materiałów.

Резюме. В статье представлено методы замены обыкновенного хронологического времени параметрами розвития представляющих собствнные времена реологических тел и механических процессов. Собственные времена измеряются последовательхостями механических и деформационных процессов. Концепцию мер собственных времён следует из сравнения изменений и движений тел и материалов.

Summary. The paper deals with the approaches replacing the usually chronological time by the parameters of evolution playing the role of intrinsic times of rheological bodies and mechanical processes. The intrinsic times are measured by the sequences of mechanical and deformational processes. The concept of the intrinsic time measures follows from comparisons of changes and motions of bodies and materials.

1. INTRODUCTION

The paper is concerned with a new concept of the role which the time plays in movements and deformations of bodies and materials. The author replaces the usual chronological time by the parameters the intrinsic time measures of mechanical and rheological processes. He introduces the concept of the mechanical and deformational relativity which follows from the relationships between the mutual motions of bodies and between the changes of strains and various scalar, vector-valued and tensor valued rheological variables.

The intrinsic times proceed when the body deforms or changes its position and stand when the body or its position remain unchanged, they represent a more realistic basis for the description of the mechanical and rheological processes than the usual chronological time which is defined by the movement of the earth or by the duration of periods of radiation of the atoms of cesium and measured by a clock.

Since the mechanical and rheological processes are not usually connected with the movement of the earth, the intrinsic time measures relating the component variables with various fundamental mechanical and rheological variables can sometimes be of practical interest.

2. CHRONOLOGICAL AND INTRINSIC TIME DERIVATIVES

Comparing the velocities $\dot{u}$ and $\dot{v}$ of two bodies, we can see that the chronological time t is reduced and we have the relation:

$$\frac{\dot{u}}{\dot{v}} = \frac{\frac{du}{dt}}{\frac{dv}{dt}} = \frac{du}{dv}\,. \tag{1}$$

In the foregoing relation, the variable v plays the role of the intrinsic time of the body. The author distinguishes the chronoligical time derivative expressed by

$$\dot{u} \equiv \frac{du}{dt} = \frac{du}{dv}\frac{dv}{dt} \tag{2}$$

and the intrinsic time derivative defined by

$$\frac{du}{dv} = \frac{du}{dt}\frac{dt}{dv} \tag{3}$$

Differentiating this expression by v yields

$$\frac{d^2u}{dv^2} = \frac{1}{\left(\frac{dv}{dt}\right)^2}\left(\frac{d^2u}{dt^2} - \frac{du}{dv}\frac{d^2v}{dt^2}\right). \tag{4}$$

From this expression, we obtain the chronological time derivative:

$$\frac{d^2u}{dt^2} = \frac{d^2u}{dv^2}\left(\frac{dv}{dt}\right)^2 + \frac{du}{dv}\frac{d^2v}{dt^2}. \tag{5}$$

3. RELATIVITY OF TIME AND MOTION

The time, displacement, velocity, acceleration and higher time derivatives of the displacement are represented by relative variables and can be measured by comparison of the motions of various bodies. The general motion of two bodies A_1 and A_2 is determined by twice linear components of velocities $\dot{x}_1$, $\dot{y}_1$ and $\dot{z}_1$ as well as $\dot{x}_2$, $\dot{y}_2$ and $\dot{z}_2$ in the directions of the coordinate axes x, y, z and by the angular velocities $\dot{\phi}_{1x}, \dot{\phi}_{1y}, \dot{\phi}_{1z}, \dot{\phi}_{2x}, \dot{\phi}_{2y}$, and $\dot{\phi}_{2z}$ about these axes. Comparing the corresponding components of velocities yields

$$\frac{\dot{x}_1}{\dot{x}_2} = \frac{\partial x_1}{\partial x_2}, \qquad \frac{\dot{y}_1}{\dot{y}_2} = \frac{\partial y_1}{\partial y_2}, \qquad \frac{\dot{z}_1}{\dot{z}_2} = \frac{\partial z_1}{\partial z_2}. \tag{6}$$

$$\frac{\dot{\phi}_{1x}}{\dot{\phi}_{2x}} = \frac{\partial \phi_{1x}}{\partial \phi_{2x}}, \qquad \frac{\dot{\phi}_{1y}}{\dot{\phi}_{2y}} = \frac{\partial \phi_{1y}}{\partial \phi_{2y}}, \qquad \frac{\dot{\phi}_{1z}}{\dot{\phi}_{2z}} = \frac{\partial \phi_{1z}}{\partial \phi_{2z}}. \tag{7}$$

In this case, the relative or intrinsic time expressed by the components of motion of the body A_2 is given by the following row matrix

$$\tau = \quad x_2, \; y_2, \; z_2, \; \phi_{2x}, \; \phi_{2y}, \; \phi_{2z} \tag{8}$$

The components x_2, y_2 and z_2 have the dimension of length and other components ϕ_{2x}, ϕ_{2y} and ϕ_{2z} are expressed by the angles.

However, every velocity component of the body A_1 can be referred to all velocity components of the body A_2 so that we furthermore obtain other relations:

$$\frac{\dot{x}_1}{\dot{y}_2} = \frac{\partial x_1}{\partial y_2}, \quad \frac{\dot{x}_1}{\dot{z}_2} = \frac{\partial x_1}{\partial z_2}, \quad \frac{\dot{x}_1}{\dot{\phi}_{2x}} = \frac{\partial x_1}{\partial \phi_{2x}}, \quad \frac{\dot{x}_1}{\dot{\phi}_{2y}} = \frac{\partial x_1}{\partial \phi_{2y}}, \quad \frac{\dot{x}_1}{\dot{\phi}_{2z}} = \frac{\partial x_1}{\partial \phi_{2z}}, \tag{9}$$

$$\frac{\dot{y}_1}{\dot{x}_2} = \frac{\partial y_1}{\partial x_2}, \quad \frac{\dot{y}_1}{\dot{y}_2} = \frac{\partial y_1}{\partial y_2}, \quad \frac{\dot{y}_1}{\dot{z}_2} = \frac{\partial y_1}{\partial \phi_{2z}}, \quad \frac{\dot{y}_1}{\dot{\phi}_{2x}} = \frac{\partial y_1}{\partial \phi_{2x}}, \quad \frac{\dot{y}_1}{\dot{\phi}_{2y}} = \frac{\partial y_1}{\partial \phi_{2y}}, \,. \tag{10}$$

The classical chronological time is referred to the movement of the earth namely to its rotation on its axis or to its circulation on the axis passing through the sun. These axes have different direction. In fact, the time can be represented by the angle and the motion is then defined as follows

$$\frac{\dot{x}_1}{\dot{\phi}} = \frac{dx_1}{d\phi}, \quad \frac{\dot{y}_1}{\dot{\phi}} = \frac{dy_1}{d\phi}, \quad \frac{\dot{z}_1}{\dot{\phi}} = \frac{dz_1}{d\phi}, \quad \frac{\dot{\phi}_{x1}}{\dot{\phi}} = \frac{d\phi_{x1}}{d\phi}, \quad \frac{\dot{\phi}_{y1}}{\dot{\phi}} = \frac{d\phi_{y1}}{d\phi}, \quad \frac{\dot{\phi}_{z1}}{\dot{\phi}} = \frac{d\phi_{z1}}{d\phi} \tag{11}$$

Since the force is given by the product of mass and absolute or relative acceleration the author accordingly distinguishes the absolute m_t and relative mass m_v. Introducing into the equation for the force

$$F = m_t \frac{d^2u}{dt^2} = m_v \frac{d^2u}{dv^2} \tag{12}$$

the expression (4) for the relative acceleration d^2u/dv^2 yields the relation between both masses:

$$m_v = \frac{m_t \left(\frac{dv}{dt} \right)^2}{1 - \frac{du}{dv} \frac{\frac{d^2v}{dt^2}}{\frac{d^2u}{dt^2}}} . \tag{13}$$

As may be seen, the masses m and m have different physical dimensions.

4. DEFORMATIONAL RELATIVITY

The plastic and viscous flow are represented by the general tensor relation between the stress σ_{kl} and strain-rate tensor $\dot{\varepsilon}_{ij}$:

$$\dot{\varepsilon}_{ij} = f_{ij}(\sigma_{kl}). \tag{14}$$

The corresponding volumetric deformation is given by

$$\dot{\varepsilon}_v = f_v(\sigma_{kl}) \tag{15}$$

Dividing Eq. (14) by Eq. (15), we can reduce the chronological time and get

$$\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_v} = \frac{d\varepsilon_{ij}}{d\varepsilon_v} = \frac{f_{ij}(\sigma_{kl})}{f_v(\sigma_{kl})}.$$

Multiplying Eq. (14) by itself yields the scalar relation for the second-order mean value of the strain-increase tensor:

$$\dot{\varepsilon}_{sc} = \sqrt{\frac{1}{3} \dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{\frac{1}{3} f_{ij}(\sigma_{kl}) f_{ij}(\sigma_{kl})} \tag{17}$$

Dividing Eq. (14) by Eq. (17), we have

$$\frac{\dot{\varepsilon}_{ij}}{\dot{\varepsilon}_{sc}} = \frac{d\varepsilon_{ij}}{d\varepsilon_{sc}} = \frac{f_{ij}(\sigma_{kl})}{\sqrt{\frac{1}{3} f_{ij}(\sigma_{kl}) f_{ij}(\sigma_{kl})}}. \tag{18}$$

Multiplying Eq. (14) by the stress tensor σ_{ij} yields the invariant relation for the strain power:

$$\dot{W} = \sigma_{ij}\dot{\varepsilon}_{ij} = \sigma_{ij}f_{ij}(\sigma_{kl}). \tag{19}$$

Dividing Eq. (14) by Eq. (19) yields

$$\frac{\dot{\varepsilon}_{ij}}{\dot{W}} = \frac{d\varepsilon_{ij}}{dW} = \frac{f_{ij}(\sigma_{kl})}{\sigma_{ij}f_{ij}(\sigma_{kl})}. \tag{20}$$

The rheological model of the Zener body consists of the Hookean elastic element $H(E_1)$ and the Maxwell viscoelastic group $M((E_2,\lambda)$ connected in parallel. For the three-dimensional state of this body, the author has derived the following tensor relation:

$$(E_1+E_2)\dot{\varepsilon}_{ij} + \frac{E_1E_2}{\lambda}\varepsilon_{ij} = \frac{E_2}{\lambda}[(1+\mu)\sigma_{ij} - 3\mu\sigma_M\delta_{ij}] + (1+\mu)\dot{\sigma}_{ij} - 3\mu\dot{\sigma}_M\delta_{ij}, \tag{21}$$

where E_1, E_2 - the moduli of elasticity,
λ - the coefficient of viscosity,
μ - the Poisson ratio,
σ_M - the mean stress,
δ_{ij} - the Kronecker delta.

Multiplying W_q. (21) by the stress tensor σ_{ij} yields the rate of strain energy:

$$\sigma_{ij}\dot{\varepsilon}_{ij} = \frac{dW}{dt} = \frac{3E_2}{\lambda(E_1+E_2)}[(1+\mu)\sigma_S^2 - 3\mu\sigma_M^2] -$$

$$- \frac{E_1E_2}{\lambda(E_1+E_2)}\sigma_{ij}\varepsilon_{ij} + \frac{1+\mu}{E_1+E_2}\sigma_{ij}\dot{\sigma}_{ij} - \frac{9\mu}{E_1+E_2}\sigma_M\dot{\sigma}_M. \tag{22}$$

Comparing Eqs. (21) and (22), we obtain the relativistic constitutive equation of the Zener body:

$$\frac{d[(E_1+E_2)\varepsilon_{ij} - (1+\mu)\sigma_{ij} + 3\mu\sigma_M\delta_{ij}]}{d[(E_1+E_2)W - \frac{1+\mu}{2}\sigma_{i}\,\sigma_{ij} + \frac{9}{2}\mu\sigma_M^2]} = \frac{E_1E_2\varepsilon_{ij} - E_2[(1+\mu)\sigma_{ij} - 3\mu\sigma_M\delta_{ij}]}{E_1E_2\sigma_{ij}\varepsilon_{ij} - 3E_2[(1+\mu)\sigma_S^2 - 3\mu\sigma_M^2]} \tag{23}$$

where

$$\sigma_S = \frac{1}{\sqrt{3}}\sqrt{\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2\ 2(\sigma_{12}^2\ \sigma_{23}^2\ \sigma_{31}^2)}$$

is the second-order mean stress.

The varying plastification in a general plane can be expressed by the tensor relations with the plastification tensor ϑ_{pq} which can be regarded as a tensor-valued intrinsic time measure defined by the directions r_q of strains and stresses as well as by the normals n_p to the planes of individual cross-sections. This tensor-valued time measure corresponds to the general deformational relativity. The relations between the stress components in the course of plastification have the following form

$$\sigma_{ij} = \sigma_{Ei\lambda}(\sigma_{\lambda j} - \vartheta_{\lambda j}) + \sigma_{Pi\lambda}\vartheta_{\lambda j}, \tag{25}$$

where σ_{ij} - the nominal stress components referred to the total cross-sectional area,

$\sigma_{Ei\lambda}$ - the work-hardening tensor with the components on the elastic region of the cross-sectional area,

$\sigma_{Pi\lambda}$ - the tensor of yield stresses or the stress components satisfying the yield criterion.

From Eq. (25), we obtain the strain tensor:

$$\varepsilon_{kl} = a_{i\lambda kl}(\sigma_{ij} - \sigma_{Pi\mu}\vartheta_{\mu j})(\delta_{\lambda j} - \vartheta_{\lambda j})^{-1}. \tag{26}$$

Differentiating this equation with respect to the components of the plastification tensor yields the generalized flow rule without strain hardening:

$$\frac{d\varepsilon_{kl}}{d\vartheta_{\lambda j}} = a_{i\lambda kl}(\sigma_{ij} - \sigma_{Pij})(\delta_{\lambda j} - \vartheta_{\lambda j})^{-2}. \tag{27}$$

There exist differences between the usual chronological time and the intrinsic times since the earth always rotates whereas a body need not always deform. In the course of deformation, the parameters of intrinsic times can vanish or have the negative values. They cannot vanish or change the sign if all cycles of loading, unloading and reverse loading are analyzed separately and the parameters of evolution are taken by their absolute values. Therefore, the author distinguishes the orientated time which can have different signs and senses and the expanded time, which is always positive and non-vanishing when the intrinsic intervals of individual cycles are put after each other. The usual chronological time is an expanded time. When the hands of a clock come in the same position, the chronological time does not vanish but twelve hours have gone or when the earth has once rotated on its axis, a day is over and when the earth has once rotated on the sun, a year is over.

REFERENCES

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CZASY WŁASNE CIAŁ REOLOGICZNYCH I PROCESÓW MECHANICZNYCH

Autor wprowadza miary czasów własnych ciał reologicznych i względnych czasów w mechanice ruchu ciał sztywnych. Czasy te przedstawiają parametry ewolucji odkształceń albo zmian pozycji i ruchu ciał. Miary czasów własnych albo względnych wynikają z porównania postępujących zmian. Ogólna miara ruchu ciała ma sześć składowych i określa się równaniem (8). Według przyspieszenia względnego i bezwzględnego, danego równaniami (4) i (5), autor rozróżnia masę względną m_v i bezwzględną m_t. Związek między nimi przedstawia równanie (13).

Związki względności deformacyjnej płynięcia plastycznego przedstawiają równania (16), (18) i (20). Miarami czasów własnych są odkształcenia objętościowe ε_v, średnia wartość drugiego rzędu tensora przyrostu odkształceń ε_{sc} lub prędkość zmiany energii odkształcenia w.

Wprowadzenie parametrów ewolucji w reologii doprowadza do wzorów bez zwykłego czasu chronologicznego. Przykładem jest równanie (23) dla trójwymiarowego stanu lepkosprężystego ciała Zenera.

W końcu autor wprowadza w równaniu (25) tensor uplastycznienia, który jest tensorową miarą czasu wewnętrznego. Stąd wynika równanie względności (27).