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NOTCH SHAPE SENSITIVITY ANALYSIS USING FICTITIOUS STRESS METHOD

Summary. Methods for shape sensitivity analysis of the notch with indirect boundary element method for stress analysis are described briefly in this paper. Three approaches: analytical, semi-analytical and finite differences are considered.

ANALIZA WRAŻLIWOŚCI KSZTAŁTU KARBU Z WYKORZYSTANIEM METODY NAPRĘŻEŃ FIKCYJNYCH

Streszczenie. W artykule przedstawia się w skrócie metody analizy wrażliwości kształtu karbu, z wykorzystaniem do analizy naprężeń pośredniego wariantu metody elementów brzegowych . Rozpatruje się trzy metody: analityczną, półanalityczną oraz różnic skończonych.

АНАЛИЗ ЧУВСТВИТЕЛЬНОСТИ ФОРМЫ КОНЦЕНТРАТОРА НАПРЯЖЕНИЙ С ПРИМЕНЕНИЕМ МЕТОДА ФИКТИВБЫХ НАГРУЗОК

Резюме. В статье рассматриваются методы анализа чуствительности формы концентратора напряжений. К анализу напряжений используется непрямий вариант метода граничных элементов. Рассмотрены три метода: аналитический, полуаналитический и конечных разностей.

1. INTRODUCTION

An extensive literature has developed on optimization of structures and structural elements which are defined by cross-section and thickness variables (size optimization). A more important problem, from the point of view of mechanical design, is determination of the shape of 2D, or 3D structural elements (shape optimization). For such problems, the shape of the structural element must be treated as design variables.

As it is already known, the presence of holes, fillets, undercuts, cut-outs etc. (known colectively as "notches") in a structural element results in increasing the local tensions which are called stress concentrations. One of the principal features of stress concentration is so called Stress Concentration Factor (SCF). To decrease the SCF, a possibility exists to minimize stress by changing the shape of a notch. Such a class of optimization problems is reffered to as the notch shape optimization.

The notch shape optimization algorithm is based on an iterative process of optimization that includes: geometric modeling (shape definition), structural analysis by the Finite Element Method (FEM) or the Boundary Element Method (BEM) called for short analyzer, the sensitivity analysis and the optimization procedure (optimizer). Especially in structural shape optimization, the sensitivity analysis is related to the analysis method. This work mainly deals with the notch shape sensitivity analysis using the Fictitious Stress Method (FSM), the indirect variant of the BEM [6].

2. FICTITIOUS STRESS METHOD AS ANALYZER

The Fictitious Stress Method (FSM) presented in [6] is used for an analysis of the stress distribution in 2-D structural elements. This is an indirect boundary element method. Fig. 1 shows a cavity (very long in z direction) in an infinite elastic body. The boundary of the cavity is labeled C in Fig. 1a. The dashed curve C' shown in Fig. 1b has the same shape as the curve C. Both curves are approximated by straight line segments (elements), joint end to end. The difference between curves is, that curve C' represents the location of line segments in an inifinite body (without cavity), which are coincident with the real boundary C. The shear P^j_s and normal P^j_n stresses applied to the segment j induce the actual stresses σ^i_s and σ^i_n at the midpoints of each element of curve C', i=1 to n. The stresses P^j_s and P^j_n are fictitious quantities and should be determined. The relation between the actual stresses σ^i_s and σ^i_n and the fictitious stresses P^j_s and P^j_n is based on the analytical singular solution to the problem of constant normal and shear stresses applied to an arbitrarily oriented, finite line segment in an infinite body (Kelvin solution), what leads directly to the system 2n*2n equations

$$[C]\{P\} = \{b\}, \tag{1}$$

where: [C] - the influence coefficient matrix, {P} - unknown fictitious stress components, {b} - given tractions.

If the fictitious stresses are known, we can find the tangential stresses

$$\{\sigma_t\}=[A_{ts}]\{P_s\} + [A_{tn}]\{P_n\}, \tag{2}$$

where $[A_{ts}]$ and $[A_{tn}]$ - are n*n matrices of the influence coefficients for tangential stresses, $\{P_s\}$ and $\{P_n\}$ are n*1 vectors of the shear and normal fictitious stress components, respectively.

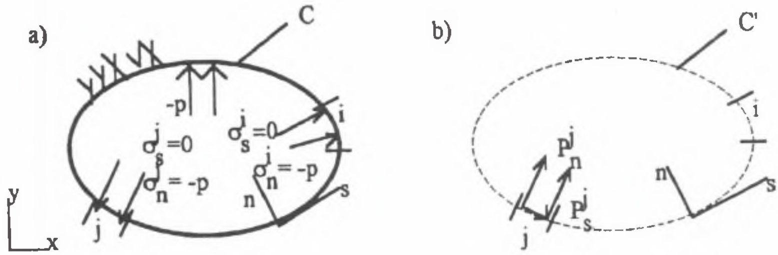

Fig. 1. Example of 2-D body : a) Real contour, b) Numerical model
Rys. 1. Przykład ciała 2-wymiarowego: a) Kontur rzeczywisty, b) Model numeryczny

Because constant stress elements are assumed to be on the boundary there is no need for numeri-cal integration. The accuracy of results can be increased by increasing the number of elements. The accuracy of the FSM for solution of stress concentration problem in 2-D machine components has been examined in [15]. It has been found that there is no remarkable difference between the results of the FSM notch stress analysis with the analytical, and numerical results widely presented in literature.

3. SENSITIVITY ANALYSIS

Although many approaches to sensitivity analysis exist, there are two fundamentally different ones, namely, the discretized approach, called the Direct Sensitivity Analysis (DSA), and the continuous approach, called the Variational Design Sensitivity (VDSA) [2-4,8,9,11,12]. The well known, the Finite Difference Method (FDM) can be ranked among the discretized approach. The VDSA method is beyond the scope of this paper.

In the case of DSA approach the sensitivities are obtained by a direct implicit differentiation of the discrete analytic equations with respect to the each design variable. There are several formulations of this method: 1) analytical techniques like: a) the Direct Differentiation Method (DDM), the Adjoint System Method (ASM), and 2) semi-analytical (quasi-analytical) technique [1,4,5,7,9,10,11]. The last one is used, when analytical derivatives are complicated, and in which the required derivatives are obtained using finite differences. The DDM method is more efficient than ASM method if the number of design variables is less than the number of constraints in the case of one load condition. This is the case in the considered notch shape optimization problems.

In a number of papers it has been reported that the semi-analytical approach may exhibit serious inaccuracies [8,13,14], and even the change of sign in derivatives, see [8] for instance,although some suggestions for improvment of accuracy of semi-analytical design sensitivities are known [13].

This work is devoted to different formulations of the implicit differentiation of the discretized boundary equations to obtain design sensitivities.

The DDM approach to design sensitivity analysis of a bondary element discretized structural element with the notch is based on the implicit differentiation of the tangential stress equation (1) and of the global equilibrum equation (2). Differentiating Eq. 2 with respect to design variable D_j, we obtain

$$\frac{\partial\{\sigma_t\}}{\partial D_i} = \frac{\partial[A_{ts}]}{\partial D_j}\{P_s\} + [A_{ts}]\frac{\partial\{P_s\}}{\partial D_j} + \frac{\partial[A_{tn}]}{\partial D_j}\{P_n\} + [A_{tn}]\frac{\partial\{P_n\}}{\partial D_j}. \tag{3}$$

The fictitious stress derivatives can be obtained by differentiation Eq. 1

$$\frac{\partial[C]}{\partial D_j}\{P\} + C\frac{\partial[P]}{\partial D_j} = \frac{\partial\{b\}}{\partial D_j}. \tag{4}$$

Rearranging yields

$$\frac{\partial\{P\}}{\partial D_j} = -[C]^{-1}\frac{\partial[C]}{\partial D_j}\{P\} + \frac{\partial\{b\}}{\partial D_j}. \tag{5}$$

If the loaded boundary is not affected by the design changes, {b} is not a function of D_j

$$\frac{\partial\{b\}}{\partial D_j} = 0, \tag{6}$$

When the above sensitivities are derived analitically by differentiation with respect to Dj then this method is called the analytical method. If the sensitivities of $[A_{ts}]$, $[A_{tn}]$and [C] are approximated by finite differences

$$\frac{\partial[A_{ts}]}{\partial D_j} \approx \frac{[A_{ts}(D_j+\Delta D_j)]-[A_{ts}(D_j)]}{\Delta D_j}, \qquad \frac{\partial[A_{tn}]}{\partial D_j} \approx \frac{[A_{tn}(D_j+\Delta D_j)]-[A_{tn}(D_j)]}{\Delta D_j}$$

$$\frac{\partial[C]}{\partial D_j} \approx \frac{[C(D_j+\Delta D_j)]-[C(D_j)]}{\Delta D_j}, \tag{7}$$

the term semi-analytical method is used.

The most straightforward and conceptually simplest way of obtaining the gradients of stress constraints is the finite difference method

$$\frac{\partial\{\sigma_t\}}{\partial D_j} \approx \frac{\{\sigma_t(D_j+\Delta D_j)\}-\{\sigma_t(D_j)\}}{\Delta D_j}, \tag{8}$$

Since stress analysis is very costly, the finite difference scheme is inefficient, because of the need to gener and to solve the system of equation n times, where n is the number of design variables. However, this approach has been found to yield good accuracy with the proper selection of the finite difference interval [1,8,11].

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