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MATHEMATICAL MODELLING OF COMPOSITE MATERIALS: REFINED MODELS FOR DYNAMIC PROBLEMS

Summary. The general modelling procedures for the investigation of non-stationary processes in periodic composites are discussed. The attention is given to new macro-models, developed recently in Poland, which take into account the effect of the microstructure length dimension on the dynamic behaviour of the body and are simple enough to be applied to the analysis of engineering problems.

MODELOWANIE MATEMATYCZNE MATERIAŁÓW KOMPOZYTOWYCH: ROZSZERZONE MODELE DLA PROBLEMÓW DYNAMICZNYCH

Streszczenie. Przedyskutowano ogólne metody formułowania modeli matematycznych, opisujących niestacjonarne procesy w kompozytach o strukturze periodycznej. Główną uwagę poświęcono nowym makromodelom, rozwijanym ostatnio w Polsce, które uwzględniają wpływ wielkości mikrostruktury na dynamikę ośrodka i są na tyle proste, że mogą być stosowane w analizie problemów inżynierskich.

MATHEMATISCHE MODELLE DER VERBUNDSTOFFE: VERBESSERTE MODELLE FÜR DYNAMISCHE PROBLEME

Zusamenfassung. Es wurden die gemeinsamen Formulierungsmethoden für Mathematische Modelle der nichtstationaren Prozessen in periodischen Verbundstoffen beschrieben. Besonders wurden die neuen Macro-Modelle, letztens in Polen entwickellt, vorgestellt. Diese Modelle können eine praktische Anwendung finden.

The general modelling procedures for the investigation of non-stationary processes in periodic composites are discussed. The attention is given to new macro-models, developed recently in Poland, which take into account the effect of the microstructure length dimension on the dynamic behaviour of the body and are simple enough to be applied to the analysis of engineering problems. The formulation of different modelling approaches in composite mechanics is motivated by the well known fact that the exact analysis of periodic heterogeneous materials in the framework of solid mechanics (viz. the micromechanics of periodic composites) can be carried out only for a few special problems. In general, the equations of micromechanics, due to the non-continuous and highly oscillating form of functions describing material properties of a composite body, cannot be taken as the basis for obtaining useful information on most of problems met in the engineering practice. That is why in composite mechanics we deal with a variety of macro-modelling methods leading to different approximate mathematical models of periodic heterogeneous materials and structures. Mathematical models of this kind are often called the macro-models and describe the effects of constituents only as averaged apparent properties of a body in the framework of the macromechanics of composite materials. Nevertheless, many macro-modelling procedures make it possible to detect also the micro-mechanical behaviour of a composite on the basis of solutions to problems of macromechanics. The list of contemporary and updated macro-modelling approaches is very extensive; in order to interrelate the new refined macro-modelling procedure, recently developed in Poland with the existing methods we outline in this summary some trends in the formulation of approximate theories for periodic composite materials.

Generally speaking, the known methods of macro-modelling can be separated into two main groups. To the first belong the general procedures, in which there are no a priori restrictions imposed on distribution of constituents within the periodicity cell. The second group consists of methods developed independently for special types of composite materials, namely for laminated composites, fibrous composites and for solids with inclusions and cavities of various shapes (i.e., for particulate composites). Thus, we can deal with general and special macro-modelling procedures. Obviously, the general macro-models have the practical meaning if they can be applied to the analysis of special types of composites. In this contribution the main attention is concentrated on the macro-modelling of nonstationary processes in periodic composites, where the effect of the length dimensions of the periodicity cell on the dynamic behaviour of a body plays an important role. That is why we discern below the length-scale macro-models and the local macro-models, in which this effect is neglected.

The main efforts in constructing the macro-models in dynamics of composites were posed on the special macro-modelling procedures. The list of references on this subject is very extensive. We can mention here the effective stiffness theories for periodically laminated elastic composites, cf. Sun, Achenbach and Hermann (1968), Achenbach and Hermann (1968), Grot and Achenbach (1970), the investigations in dynamics of fibre-reinforced composites, cf. Aboudi (1981), Toff (1983), and those related to media with voids, cf. Nunziato and Covin (1979).

Among the general macro-modelling procedures we can mention those based on the asymptotic homogenization approach, cf. Bensoussan, Lions and Papanicolaou (1987), Sanchez-Palencia and Zaoui (1985), and the extensive list of references therein. The resulting macro-models are described by equations involving constant coefficients (called the effective modulae) and time dependent functions (for nonstationary processes). These mathematical objects have to be determined independently for every periodic structure by obtaining solutions to certain variational problems posed on the periodicity cell as well as certain initial value problems for materials with a memory (e.g. for visco-elastic materials). Hence, the formulation of macro-models by the asymptotic homogenization methods for any specific composite materials requires rather lengthy numerical computations. For this reason the asymptotic approach, as a rule, is restricted only to the first approximation. Within this approximation we deal with the local macro-models, in which the effect of the size of the periodicity cell on the behaviour of the body is neglected. To describe length-scale phenomena (i.e. to formulate the length-scale macro-models) by the asymptotic homogenization approach, the higher steps in the formal asymptotic procedure have to be considered. Due to considerable difficulties on the stage of a formulation of governing equations of macromechanics for a selected composite body, the above line of procedure is not accepted by most of researchers interested mainly in engineering applications of the resulting theories. Free from this drawback are general macro-modelling methods, based on theories of material continua with microstructure suggested by Mindlin (1964), Eringen and Suhubi (1964) and others. Models of this kind are called the microstructural models and belong to the length-scale macro-models. They can be formulated without any reference to the boundary-value problems on the periodicity cell. The pertinent modelling procedures are specified by certain a priori assumptions related to the expected class of micro-deformations and a certain smoothing operations. For the elastic materials they lead to systems of the second-order partial differential equations for three fields representing macro-kinematics and for extra unknown fields describing micro-kinematic behaviour of a composite. The existence of many unknown independent kinematic variables results in serious difficulties related to a complicated form of boundary-value problems. Moreover, there exist essential discrepancies between the number of boundary conditions required by the mathematical structure of the theory and the number of these conditions describing the boundary interactions for composite materials from the viewpoint of engineering applications of the theory. Hence, the microstructural models were successfully applied mainly to the investigations of the wave propagation in unbounded media. A certain alternative to microstructural models constitute macro-models based on the mixture and interacting continuum theories, developed by Green and Naghdi (1965, 1966, 1967), Green and Steel (1966), Steel (1967, 1968), Bedford and Stern (1971, 1972), Hegemier (1972), Tiersten and Jahanmir (1977) and others. They are the length-scale macro-models which are often oriented towards the investigations of selected dynamic problems.

In investigations of nonstationary processes for composites, we are often interested only in certain aspects of the micro-dynamic behaviour of periodic solids. In these cases certain special space distributions of expected micro-deformations caused by the heterogeneity of the medium are analyzed. We are also interested in the length-scale macro-models which are physically reasonable and simple enough to be applied for an analysis of engineering problems. The macro-models of this kind were recently applied to the selected dynamic problems in a series of papers by Woźniak (1993, 1994), Woźniak et al.(1993, 1994), Wierzbicki (1994), Mazur-Śniady (1993), Baron and Woźniak (1994), Michalak and Woźniak (1994), Węgrowska and Woźniak (1994), Konieczny and Woźniak (1995), Jędrysiak and Woźniak (1995), Cielecka (1995), Matysiak and Nagórko (1995). The results of the macro-modelling procedure applied in the aforementioned papers, were referred to as the refined macrodynamics of periodic materials and structures. The term refined is related to the fact that the obtained equations describe in the explicit form the effect of the microstructure length parameter (i.e. the maximum characteristic length dimension of the periodicity cell) on the dynamic behaviour of the body. In order to evaluate this effect there were introduced also local models, obtained from the refined macrodynamics by the scaling the microstructure down. These local models coincide with the macro-models analyzed in the framework of theories with microlocal parameters, developed in papers by Woźniak (1987), Matysiak and Woźniak (1987), Jakubowska and Matysiak (1987), Węgrowska (1987), Naniewicz (1987), Lewiński (1987), Kaczyński and Matysiak (1988), Matysiak and Nagórko (1989), Woźniak (1991), Matysiak (1992), Kaczyński (1993) and others.

The refined macromechanics have some advantages compared to the general macro-models of the asymptotic as well as microstructural type outlined above. First, it is able to describe nonlinear problems related to the finite deformations of an arbitrary simple material. Secondly, the formulation of governing equations of the pertinent refined macro-models do not require any solutions to the boundary-value problem on the basic cell as well as solutions to the initial-value problems (if we deal with viscoelastic materials). Thirdly, the extra unknown fields in the refined macromechanics (i.e., the fields which do not have their counterparts in solid mechanics of heterogeneous media) are governed by relations involving exclusively time derivatives and/or time dependent functionals. It follows that the extra unknowns do not enter the boundary conditions and play a role of certain dynamical internal variables. This fact is very essential in the applications of the theory, since the boundary-value problems of the refined macrodynamics are similar to those met in solid mechanics. At last, using the refined models we are able to evaluate a posteriori the relation between the size of the periodicity cell and the accuracy of the obtained solution. The refined macro-models can be formulated on the various levels of accuracy and hence are useful for the modelling procedures of the adaptive type. The main drawback of the refined macro-models is the restriction of microdynamics to the postulated a priori class of micro-motions, which is often based on the intuition of the researcher.

REFERENCES

(Refs [1-12] are related to problems of the refined macrodynamics.)

[1] KONIECZNY S., WOŹNIAK M.: On the wave propagation in microinhomogeneous media, J. Theor. Appl. Mech., 33, 1995

[2] MAZUR-ŚNIADY K.: Macro-dynamics of micro-periodic elastic beams, J. Theor. Appl. Mech., 31, 781-793, 1993

[3] WIERZBICKI E.: On the wave propagation in micro-periodic elastic media, Bull. Pol. Ac. Sci., Tech. Sci., 41, 323-327, 1993

[4] WIERZBICKI E.: Non-linear macro-micro dynamics of laminated structures, J. Theor. Appl. Mech., 33, 1995

[5] WIERZBICKI E., WOŹNIAK CZ., WOŹNIAK M.: Finite rotations in the refined macrodynamics of elastic composites, J. Theor. Appl. Mech., 33, 1-11, 1995
[6] WOŹNIAK CZ.: Refined macro-dynamics of periodic structures, Arch. Mech., 45, 295-304, 1993
[7] WOŹNIAK CZ.: Nonlinear macro-elastodynamics of micro-periodic composites, Bull. Pol. Ac. Sci., Tech. Sci., 41, 315-321, 1993
[8] WOŹNIAK CZ.: Macro-dynamics of elastic and viscoelastic micro-periodic composites, J. Theor. Appl. Mech., 31, 763-770, 1993
[9] WOŹNIAK CZ.: Micro-macro dynamics of periodic material structures, in: Structural Dynamics - EURODYN'93, Moan et al. (eds), Balkema, Rotterdam, 573-575, 1993
[10] WOŹNIAK CZ., WOŹNIAK M. and KONIECZNY S.: A note on dynamic modelling of periodic composites, Arch. Mech., 45, 779-783, 1993
[11] WOŹNIAK CZ., WOŹNIAK M.: On the micro-modelling of dynamic response for thermoelastic periodic composites, 1994, in: Proc. IUTAM Symposium on Microstructure-Property Interactions in Composite Materials, August 23-25, Aalborg, Denmark, 1994
[12] WOŹNIAK M., WOŹNIAK CZ.: Laminated media with interface defects, ZAMM 75, SI, S171-S172, 1995
[13] ABOUDI J.: Int. J. Solids Structures, 17, 1005-1018, 1981
[14] ACHENBACH J. D., HERRMANN G.: Wave motions in solids with lamellar structuring, in: Dynamics of Structured Solids (ed. G. Herrmann), Am. Soc. of Mech. Eng., New York, 1968
[15] BAKHVALOV N. S., PANASENKO G. P.: Processes averaging in periodic media (in Russian), Nauka, Moscow, 1984
[16] BEDFORD A., STERN M.: A multi-continuum theory for composite elastic materials, Acta Mech., 14, 85-102, 1972
[17] BENSOUSSAN A., LIONS J. L. and PAPANICOLAU G.: Asymptotic analysis for periodic structures, North Holland, Amsterdam, 1978
[18] ERINGEN A. C., SUHUBI E. S.: Nonlinear theory of simple elastic solids, Int. J. Engng Sci., 2, 189-203, 389-404, 1964
[19] GREEN A. E., NAGHDI P. M.: A dynamical theory of interacting continua, Int. J. Engng Sci., 3, 231, 1965
[20] GREEN A. E., STEEL T. R.: Constitutive equations for interacting continua, Int. J. Engng Sci., 4, 483, 1966
[21] GROT R. A., ACHENBACH J. D.: Large deformations of a laminated composite, Int. J. Solids Structures, 6, 641, 1970
[22] HEGEMIER G. A.: On a theory of interacting continua for wave propagation in composites, in: Dynamics of Composite Materials, (ed. E. H. Lee), Am. Soc. of Mech. Eng., New York, 1972
[23] JAKUBOWSKA M. E., MATYSIAK S. J.: Propagation of plane harmonic waves in periodic multilayered elastic composites, Studia Geotech. and Mech., 9, 17-25, 1987
[24] KACZYŃSKI A.: Three-dimensional thermoelastic problems of interface cracks in periodic two-layered composites, Eng. Frac. Mech., 48, 783-800, 1994

[25] KACZYŃSKI A., MATYSIAK S. J.: Plane contact problems for a periodic twolayered elastic composite, Ing. Arch., 58, 137-147, 1988
[26] LEWIŃSKI T.: On the interrelation between microlocal parameter and homogenization approaches to the periodic elastic solids, Bull. Pol. Ac. Sci., Tech. Sci., 35, 383-391, 1987
[27] MATYSIAK S. J., NAGÓRKO W.: Micro-local parameters in the modelling of microperiodic plates, Ing. Arch., 59, 434-444, 1989
[28] MATYSIAK S. J., WOŹNIAK CZ.: Micromorphic effects in a modelling of periodic multilayered elastic composites, Int. J. Engng Sci., 25, 549-559, 1987
[29] MINDLIN R. D.: Microstructure in linear elasticity, Arch. Rat. Mech. Anal., 16, 51-78, 1964
[30] NANIEWICZ Z.: On the homogenized elasticity with microlocal parameters, Bull. Pol. Ac. Sci., Tech. Sci., 35, 1987
[31] NUNZIATO J. W., COWIN S. C.: A non-linear theory of elastic materials with voids, Arch. Rat. Mech. Anal., 72, 175-201, 1979
[32] SANCHEZ-PALENCIA E.: Non-homogeneous media and vibration theory, Lecture Notes in Physics, vol. 127, Springer, Berlin-Heildelberg-New York, 1980
[33] SANCHEZ-PALENCIA E., ZAOUI A.: Homogenization technics for composite media, Lecture Notes in Physics, vol. 272, Springer, Berlin-Heildelberg-New York, 1985
[34] STEEL T. R.: Applications of theory of interacting continua, Quart. J. Mech. Appl. Math., 20, 51, 1967
[35] SUN C. T., ACHENBACH J. D., HERRMANN G.: Time-harmonic waves in a stratified medium propagating in the direction of the layering, J. Appl. Mech., 35, 408-411, 1968
[36] TIERSTEN H. F., JAHANMIR M.: The theory of composites modelled as interpenetrating solid continua, Arch. Rat. Mech. Anal., 65, 154, 1977
[37] TOLF G.: On dynamical description of fibre-reinforced composites, in: New Problems in Mechanics of Continua, Proc. Third Swedish-Polish Symp. on Mechanics, Jabłonna, Poland, May 10-14 (ed. O. Bralin, R. K. T. Hsieh), Univ. of Waterloo Press, 145-159, 1982
[38] WĘGROWSKA M.: Certain solutions of axially -symmetric problems in the linear elasticity with microlocal parameters, Bull. Pol. Ac. Sci., Tech. Sci., 35, 3-5, 1987
[39] WOŹNIAK CZ.: A nonstandard method of modelling of thermoelastic periodic composites, Int. J. Engng Sci., 25, 483-498, 1987

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