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DECISION IN BEGINNING OF PRODUCT DESIGN

Summary. The contribution deals with the modeling of two situations in ensuring the reliability of systems. In the first case the achievement of desired reliability of a system concerns, with the aim to minimize the costs for its assurance and in the second case the system reliability is maximized at the limited costs for its assurance.

1. Introduction

The one of basic problems in beginning of product design is a problem of optimization of the relationship between the reliability and costs for its assurance. Improving of product reliability, mainly of the complex systems, is usually accompanied with an increase in the production costs, what unfavourably affects mainly the competitive abilities. Ve will present two possible approaches to solution of this problem.

2. Assurance of the desired reliability of a system with minimum costs

Let's suppose that a system consists of N subsystems, connected reliably in a series. There is necessary to determine the value of reliability index of each subsystem so that the desired level of the system reliability index would be attained at minimum costs.

If the system and subsystems reliability is expressed by the probability of survival, then the solution of this task depends on determination of such a combination of probability of survival of the subsystems which would on one hand meet the requirement laid on the value of reliability of survival of the system, and, on the other hand the cost connected with its realization would be minimum.

In order we would be able to formulate this task mathematically the following symbols will be introduced:

R_s - probability of survival of the system,

R_{sd} - desired value of probability of survival of the system,

R_{bi} - basic value of probability of survival of the i-th subsystem that will be attained without spending any additional costs,

R_i - value of probability of survival of the i-th subsystem which varies in dependence on the additionally spent costs aimed at the improvement of reliability of the i-th subsystem, whereas it is valid that $0 < R_{bi} \leq R_i \leq 1$, for $i = 1,2,\ldots,N$,

$R_i^{(o)}$ - optimum value of probability of survival of the i-th subsystem,

$F_i(R_{bi}, R_i)$ - cost function which value represents the additional cost that must be expended if we want to increase the probability of survival of the i-th subsystem from the value of R_{bi} to R_i.

The probability of survival of the system can be expressed by the formula

$$R_s = \prod_{i=1}^{N} R_i \qquad (1)$$

On the basis of the above-mentioned the supposed task can be formulated as follows:

Minimum of the following function

$$\sum_{i=1}^{N} F_i(R_{bi}, R_i) \qquad (2)$$

must be found, under the condition that

$$\prod_{i=1}^{N} R_i \geq R_{sd} \qquad (3)$$

$$0 < R_{bi} \le R_i \le 1, \; i = 1,2,\dots,N. \qquad (4)$$

So formulated broblem is the task of mathematical programming. The $R_1, R_2, \dots, R_N$ values which meet the conditions (3) and (4) are acceptable problem solutions. Those R_i $(i=1,2,\dots,N)$ simultaneously meet the conditions (3) and (4) and also minimalize the function (2) are the optimum problem solution.

If the given task is understood as an multi-stage decision -making process it can be solved as a task of dynamic programming by the method of functional equations. Principle of this method consists in the fact that the extreme of function (2) is searched after stages, whereas the number of stages equals the number of subsystems in a system, thus a N-stage decision making process is concerned. At each stage a decision is made whereas the sequence of performed decisions in the individual stages represents the decision-making strategy. The strategy that-with respect to a given optimality criterion-leads to the achievement of an optimum results is the optimum strategy (optimum solution).

If the probability of survival in the k-th stage is designated by the r_k symbol then it is valid that:

$$1 = r_N \ge r_{N-1} \ge \dots \ge r_1 \ge r_0 \ge R_{sd},$$

the recurrent relationships, by which we express mathematically the fact that the given task is understood as a N-stage decision -making process, can be written as follows:

$$f_1(r_1) = \min_{\{R_1\}} [F_1(R_{b1}, R_1) + f_0(r_0)], \; R_1 \ge R_{sd}/r_1$$

$$f_2(r_2) = \min_{\{R_2\}} [F_2(R_{b2}, R_2) + f_1(r_1)], \; R_2 \ge R_{sd}/r_2 R_1 \qquad (5)$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$f_N(r_N) = \min_{\{R_N\}} [F_N(R_{bN}, R_N) + f_{N-1}(r_{N-1})], \; R_N \ge R_{sd}/r_N R_{N-1}$$

where $f_0(r_0) = 0$, $r_N = 1$ and $r_{k-1} = r_k R_k$, for $k=1,2,\dots,N$.

The equations (5) represent a system of functional equations for N-stage decision-making process. The result of solution of this system is the optimum strategy, consisting of the values $R_1^{(o)}, R_2^{(o)}, \dots, R_N^{(o)}$. The procedure for calculation of the optimum strategy can be briefly summarized as follows:

a) On the basis of relationship $r_{k-1} = r_k R_k$ the values of r_{N-1}, r_{N-2}, ... , r_0 are calculated, while the condition that r_N is followed.

b) Based on the equations (5) the values of $f_k(r_k)$, for k = 1,2, ...,N are calculated. The calculation will start from the value $f_1(r_1)$ which will be used for calculation of $f_2(r_2)$ values, etc. until the value of $f_N(r_N)$ is calculated on the basis of $f_{N-1}(r_{N-1})$ values.

c) The optimum strategy is determined. At first the $R_N^{(o)}$ value is determined. This is that value from the R_N values which corresponds to the value $f_N(r_N)$, i.e. $R_N^{(o)} = R_N(r_N)$.
Other components of the optimum strategy are determined on the basis of relationship:

$$
\begin{aligned}
R_{N-1}^{(o)} &= R_{N-1}(r_{N-1}) = R_{N-1}(R_N^{(o)}) \\
R_{N-2}^{(o)} &= R_{N-2}(r_{N-2}) = R_{N-2}(R_N^{(o)} R_{N-1}^{(o)}) \\
&\dots\dots\dots\dots\dots\dots\dots \\
R_1^{(o)} &= R_1(r_1) = R_1(R_N^{(o)} R_{N-1}^{(o)} \dots R_2^{(o)})
\end{aligned}
\qquad (6)
$$

The given relationship express the $R_i^{(o)}$ as the function r_i, for i = 1,2,...,N-1.

2. Maximization of system reliability at limited costs for its assurance

Let' s suppose that a system cosists of N subsystems, connected in series. In order to improve the system raliability there is available H volume of the financial means, which are supposed for the management of the spare subsystems. The price of a spare subsystem is designates c_i (i = 1,2,...,N). It is necessary to determine the numbers of the spare subsystems m_i (i = 1,2,...,N) for redundancy of the individual subsystems, so that at meeting the inequality

$$\sum_{i=1}^{N} c_i m_i \le H \qquad (7)$$

maximum possible system safety would be attained.

If the reliability of system and subsystems is expressed by the index of probability of survival, then the probability of the survival of system can be expressed by the formula

$$R_s = \prod_{i=1}^{N} R_i(m_i) \tag{8}$$

where $R_i(m_i)$ is the probability of survival of i-th subsystem, if is it reserved by the m_i of spare subsystems.

Based on the above-mentioned, the set task can be solved as follows: Such values of m_i $(i = 1,2,...,N)$ from the $\{0,1,2,...\}$ set must be found for which the relationship (8) will attain the maximum value and which will also comply with the inequality (7).

Its again the task of mathematical progamming. The task can be considered also as a multi-stage decision-making process and solved by the method of functional equations. Determination of the number of the spare subsystems for a N-th subsystem can be considered for the first stage, determination of the number of the spare subsystems for the (N-1)-th subsystem etc. and the determination of the numbers of the spare subsystems for the first subsystem can be considered for the N-th stage. The recurrent relationship, by which the fact that the given task is an N-stage decision-making process is expressed mathematically, can be written as follows:

$$\begin{aligned}
f_1(H) &= R_1(m_1), & 0 \le m_1 \le [H/c_1] \\
f_2(H) &= \max_m \, [R_2(m_2) f_1(H - c_2 m_2)], & 0 \le m_2 \le [H/c_2] \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\
f_N(H) &= \max_m \, [R_N(m_N) f_{N-1}(H - c_N m_N)], & 0 \le m_N \le [H/c_N]
\end{aligned} \tag{9}$$

where $[H/c_i]$ is the integer share of fraction H/c_i.

The equations (9) are a system of functional equations for a N-stage decision-making process. The system is solved in such a way that we will start with calculation of $f_1(H)$ value. On the basis of $f_1(H)$ the $f_2(H)$ is calculated etc. until the value $f_N(H)$ is calculated on the basis of $f_{N-1}(N)$. On the basis of $f_N(H)$ is calculated the $m_N^{(o)}$, i.e. the optimum decision for the first stage is determined, whereas the $m_N^{(o)}$ is the last member of the optimum strategy $(m_1^{(o)}, m_2^{(o)}, \dots, m_N^{(o)})$. The other components of optimum strategy, i.e.the optimum numbers of the spare subsystems for the individual subsystems are determined on the basis of relationships:

$$m_{N-1}^{(o)} = m_{N-1}(H - c_N m_N^{(o)})$$
$$m_{N-2}^{(o)} = m_{N-2}(H - c_N m_N^{(o)} - c_{N-1} m_{N-1}^{(o)})$$
$$\dots\dots\dots\dots\dots\dots \qquad (10)$$

$$m_1^{(o)} = m_1(H - \sum_{i=2}^{N} c_i m_i^{(o)})$$

The mentioned relationships express $m_i^{(o)}$ as the function of difference

$$H - \sum_{i=i+1}^{N} c_i m_i^{(o)}, \text{ for } i = 1,2,\dots,N-1.$$

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