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THE METHOD OF MATHEMATICAL-PHYSICAL MODELLING OF CONVECTIVE FLOWS IN ROOMS

Summary. The suggested method of mathematical-physical modelling of convective flows in rooms makes possible to identify convective flows sources and to predict further evolution of these flows. It can be applied to any quasi-axisymmetrical heat source of any power generating a free plume in still environemnt and it includes the possibility of air thermal stratification occurring. The method may be used to the complex introduction of convective streams to numerical calculation of air circulation in rooms while the streams are introduced in the boundary conditions form.

1. Introduction

Theoretical prediction of thermal comfort in rooms is often difficult since there is no possibility to determine air temperature and velocity distributions. Thus, in order to compare thermal conditions and energy consumption in rooms that are heated or ventilated with the use of various equipment, it is still necessary to carry out laborious and sometimes complicated experiments, predominantly in natural scale objects [1, 2].

The air flow study aimed at determination of a ventilation problem solution, particulary in complicated cases, is often carried out by means of physical modelling in smaller scale [3, 4]. In the course of such tests the condition of flows self-similarity must be preserved. When considering rooms ventilation this condition is usually fulfilled in the case of jets or of some convective flows but it cannot always be fulfilled in secondary flows regions.

Theoretical calculation of air flows in rooms have been carried out hitherto only in the case of two-dimensional flows [5, 6, 7]. However, the employed calculation methods have not taken into account some substantial factors affecting the air circulation in rooms such as convective streams or infiltration. The basic problems in such calculation are: turbulence model and flow equations, numerical grid and boundary conditions. Strict mathematical modelling of some of the factors causing the air circulation in a room is at present practically impossible since there are not appropriate flow models and the divergence of the phenomena scales is so great that a fine numerical grid would be required.

It seems that these difficulties could be avoided when introducing some factors to calculation as a composite unity in the form of appropriately stated boundary conditions. In the case of mechanical ventilation the aerodynamic characteristic of the inlet, obtained from experiments, was introduced to calculation by Nielsen [5] in the form of a boundary condition. To this purpose he separated a region from the room, close to the inlet, assuming that the effect of the air circulation in the room on the flow in this region is negligible. The boundary conditions at the border of this region were determined with the use of experiments results (fig. 1). Similar methods may be also employed in many other cases e.g.

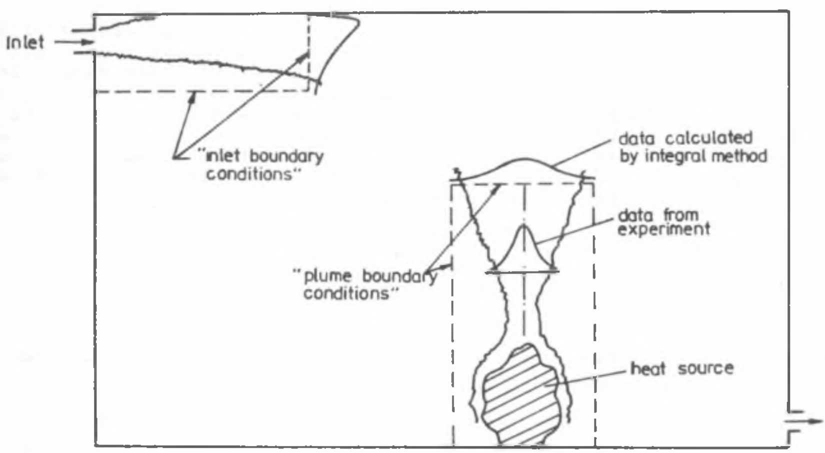


Fig. 1. Factors causing the air circulation in room in the complex form of boundary conditions

Rys. 1. Czynniki powodujące cyrkulację powietrza w pomieszczeniu w złożonej postaci warunków brzegowych

for plumes above heat sources or for infiltration. When developing the above reasonings perhaps it will be possible to calculate three-deminsional flows in rooms, the essential factors causing the air circulation included. At present, the main reason for difficulties in the calculation of three-dimensional ventilation flows is the unprofitableness of such the numerical calculation that is related to the required capacity of computer memory.

The introduction of the mentioned above local boundary conditions obtained experimentally makes a significant simplification possible thanks to the possibility of using a less dense numerical grid. To this purpose the appropriate experiments are necessary and in many cases they call for adequate new measurement methods.

Convective streams are a phenomenon much more complicated than jets. Their mathematical description is also adequately more difficult. However, the description of parameters in these streams is essential in order to determine the boundary conditions which in numerical calculation of air circulation in a ventilated room would represent the effect of the convective streams.

The paper presents the mathematical-physical modelling method that might be applied to the identification of plumes which are usually generated in rooms by sources of complex shapes such as human bodies, heaters, various industrial equipment etc. (see fig. 1).

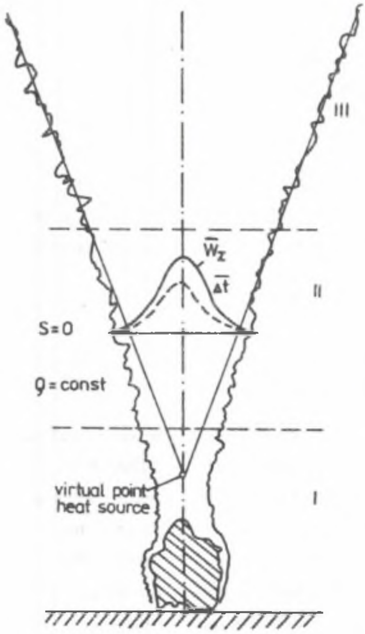
2. Air flow in convective streams

Figure 2 shows schematically the flow regions occurring in a convective stream evolving in still environment in two cases: with and without stratification ($S = 0$ and $S > 0$ respectively). Owing to the temperature difference near the heat source surface convective flow of air in the boundary layer develops. Only the flow in the boundary layer near the vertical heated plate can be considered as sufficiently well known. Basic studies in this field were carried out by Cheesewright [8].

If the source shape is of more complex type or if its temperature is differentiated the process of the convective stream generating is very complicated and practically it is impossible to predict it theoretically. All streams genereted close to the body surface mix together above the heat source and form the plume. In the instance of extensive heat sources the transition from laminar to turbulent flow may occur already in the boundary layer, in other cases only above the heat source. However, as the observation of Rouse et al. [9] has proved, even when the thermal power and the size of the source are small the convective stream, laminar in its initial stage destabilizes as well and only the length of the laminar flow section can be different.

When the elevation above the heat source increases, temperature and velocity profiles in the plume change too. The ratio of temperature and velocity profiles widths also changes. It is characteristic of flows in convective boundary layers that the velocity profile is wider than the temperature profile. In plume above a heat source the profiles widths begin to equalize.

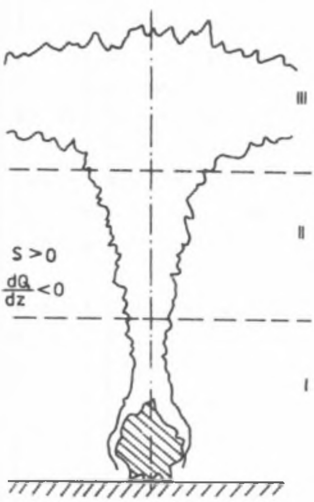
Thus, in most cases, complex aerodynamical and thermal phenomena occur near the heat source and just above it. Flow calculation in the region I (see fig. 2) of a convective stream is practically impossible. However, at certain elevation, dependent on the shape, size and thermal power of the source, the flow becomes turbulent and axisymmetrical and the shapes of velocity and temperature excess profiles become similar and they



III The region of a complete flow similarity, the flow is fully developed; the plume spreads linearly; local Archimedes number Ar and the ratio of temperature and velocity profiles widths, λ, are constant

II Self-similarity of a mean motion region, the plume is turbulent and axi-symmetrical; velocity and temperature distribution curves are of Gaussian type; the plume spreads non-linearly; the ratio of temperature and velocity profiles widths, λ, changes

I A flow in a boundary layer region; separate flows mix and form the plume; laminar flows become turbulent



III Maximum height of a plume rise region; maximum elevation is reached; air flows out horizontally

II Self similarity of a mean motion region; similar to the region II in the case when S = 0; the effect of stratification is significant

I A flow in the boundary layer region similar to the region I when S = 0; stratification affects the flow forming process in this region

Fig. 2. Schematic representation of the flow regions in the plume above heated body in two cases: with and without stratification (S = 0 and S > 0, respectively)

Rys. 2. Schematyczne przedstawienie obszarów przepływu w strudze ponad źródłem ciepła dla przypadku bez stratyfikacji i ze stratyfikacją (odpowiednio S = 0 i S>0 resemble Gaussian error curve. This elevation is the beginning of the mean motion self-similarity turbulent zone (i.e. the region II - see fig. 2). However the self-preserving flow cannot yet be observed in this zone. The profiles of flow turbulent parameters are not alike. Naturally, togehter with the elevation increase the plume width and axial values of mean temperature and velocity change too. The ratio of the temperature excess and velocity profiles widths also changes with the elevation.

The evolution of a convective stream depends on thermal stratification of the environment and practically also on velocities of external cross-flows of air and on the room height. If there were no stratification (S = 0) the plume would reach the state of flow complete similarity, the ratio of temperature and velocity profiles widths as well as Archimedes number value would become constant and the stream would spread linearly. However, this is a rather theoretical case and it is difficult even to get it in practice [10]. In most cases there is stratification in rooms and therefore the plume reaches the maximum height of its rise and the air flows out horizontally or impinges on the room ceiling or, at last, is disintegrated by outer cross-currents of the air.

3. The method of parameters distributions calculation in convective streams

In practice, temperature and velocity distributions have been described hitherto by means of the model of plume generated by a point heat source [11, 12, 13]. Such a mathematical model was presented for the first time by Schmidt [14] and then by Rouse et al. [9]. Its analysis was also presented by Popiołek in the Appendix 7.1 of his report [15]. However, this model in its present form arouses many reservations e.g. it has not been found out so far at what height above real heat sources convective flows can be approximated in this way. The ambient air stratification is neither taken into account and, furthermore, the model of the plume generated by a point heat source has not been sufficiently well verified experimentally. When considering this model we may assume that it can be applied only to the zone III of the plume which is seldom found in practice and only when there is no stratification, S = 0, (see fig. 2.).

The method enabling the flow calculation in the region II i.e. in the self-similarity of mean motion zone, would be of much greater practical significance since such flow conditions can be found in many instances of room ventilation.

In the region III, when S = 0, flow parameters may be explicitly determined on the basis of the values of the plume enthalpy and the distance from the heat source (the position of a virtual point heat source must be known as well). However, even if this values are known, they are not sufficient to determine flow parameters in the region II since in this region it is necessary to know some other parameters which must be obtained from experiments.

In order to calculate the flow in the self-similarity of mean motion zone it is necessary to know the values of four independent flow parameters in any beginning cross-section of this zone. These may be such parameters as $\overline{W}_{zmo}$, $\Delta\overline{t}_{mo}$, R_{wo} and R_{to}, obtained directly from measurements or other parameters sets e.g. Q_o, Ar_o, λ_o, R_{wo}, or Q_o, V_o, I_o, λ_o. If only the profiles of velocity and temperature excess in plume are known, the values of all parameters can be determined on the basis of four independent parameters values. These profiles can be known in the region II and III at S = 0 since, as it is found experimentally they are very similar to the following Gaussian error curves:

$$\overline{W}_z = \overline{W}_{zm}\, e^{-\left(\frac{r}{R_w}\right)^2} \; ; \quad \Delta\overline{t} = \Delta\overline{t}_m\, e^{-\left(\frac{r}{R_t}\right)^2} \tag{1}$$

In order to calculate the flow when stratification S>0 it is necessary to know this stratification value expressed in K/m or in dimensionless form given by teh relation

$$s = S\,\frac{R_{wo}}{\Delta\overline{t}_{mo}} \tag{2}$$

In order to calculate flow parameters in both regions II and III it is necessary to determine experimentally:

1) the elevation above the heat source at which the region II i.e. the self-similarity of mean mation zone begins,

2) the values of four independent flow parameters e.g. Q_o, Ar_o, λ_o, R_{wo} in any beginning cross-section of this zona,

3) vertical distribution of air temperature near the plume.

It seems that the most useful method to calculate flows in the regions II and III would be so-called integral method [15, 18]. This results from the fact that Gaussian shape of velocity and temperature profiles may be assumed for calculation. Temperature excess profile is rather significantly different from Gaussian error curve only in the zone III when S > 0. Thus, applying the method, it will not be possible to determine precisely the maximum elevation of plume if stratification exists, though it will be possible to estimate this elevation value.

The integral method is commonly applied to flows calculation in vertical buoyant jets in the environment with and without stratification

[16, 17]. The difference in the method application is that in the case of vertical buoyant jets the calculation is begun at the outlet cross-section of the nozzle in which flat hat-top profile of velocity and low turbulence are found whereas at the beginning of the zone II of plume velocity and temperature excess profiles are of Gaussian shape.

The basic equation set of the integral method is as follows [15]:

mass balance equation

$$\frac{d}{dz}(\bar{W}_{zm}\, R_w^2) = 2\alpha\bar{W}_{zm}R_w \tag{3}$$

conservation of vertical momentum equation

$$\frac{d}{dz}(\bar{W}_{zm}^2 R_w^2) = 2\beta g\Delta\bar{t}_m R_w^2 \lambda^2 \tag{4}$$

heat conservation equation

$$\frac{d}{dz}\left[\frac{\lambda^2}{2(1+\lambda^2)}\,\Delta\bar{t}_m\bar{W}_{zm}R_w^2\right] = -\frac{dt_\infty}{dz}\,\frac{\bar{W}_{zm}^2 R_w^2}{2} \tag{5}$$

This set of equations must be completed by two additional equations describing the process of air entraining from the plume surroundings and the variability of the ratio of temperature and velocity profiles widths. There may be several ways of this equation set closure. It was thus necessary to analyse this problem carefully [18]. As the result of the analysis the following closure equations were suggested:

Fox's formula [16]:

$$\alpha = \alpha_j + \left(2 - \frac{3}{\lambda^2 + 1}\right)\lambda^2 Ar \tag{6}$$

where

$$\alpha_j = \frac{3}{4}\, Ar_p\lambda_p^2\, \frac{3 - \lambda_p^2}{1 + \lambda_p^2} \tag{7}$$

and a new equation describing the variability of the profiles widths ratio, λ:

$$\frac{d}{dz}\left(\Delta\bar{t}_m\,\frac{\lambda^2+1}{\lambda^2+2}\right) + \left[2 - \frac{(\lambda^2+1)^2}{\lambda^2(\lambda^2+2)}\right]\frac{dt_\infty}{dz} + \frac{\Delta\bar{t}_m}{R_w}\,\frac{\lambda^2+1}{\lambda}\,\gamma = 0 \tag{8}$$

where

$$\gamma = \frac{5}{2}\, Ar_p \, \frac{\lambda_p^3}{\lambda_p^2 + 2} \tag{9}$$

In order to describe temperature changes in the plume surroundings the following equation was suggested

$$t_{\infty} = t_{\infty o} + \Delta \bar{t}_{mo}\, s\left(\frac{z}{\bar{R}_{wo}}\right)^{N} \tag{10}$$

The foregoing equations were transformed into the dimensionless ones on the basis of which the calculation algorithm was stated [18]. The algorithm was additionaly completed with a special method of results normalization for the case when S = 0. Owing to this it was possible to determine the elevation, beginning from which the flow in a plume might be approximated by the model of plume above a point heat source. It was also possible to determine the position of the virtual point heat source.

4. Experimental tests of flow in plumes

In the course of the investigation flow measurements in plumes above heat soruces were carried out. The sources dimensions varied from 3.5 to 70 cm and their power was between several and several hundred watts. Flows measurements in plumes above standing and sitting man were also carried out. The results of some measurements were already published [10, 13, 15]. The method of temperature and velocity measurements as well as the method of turbulent parameters measurements in convective flows were developed in the course of tests.

The temperature excess in plumes disappears very quickly. Owing to this, even if the heat source temperature is high and amounts at several hundred °C, the temperature excess decreases already at the relatively small distance from the heat source. Particularly in zones II and III it reaches the values of several K or may be often even only of the order of a few tenths of K. If a temperature profile is to be measured with the accuracy of the order of a few percent of the temperature excess in the plume axis the required thermometer accuracy ought to be of the order of 0.01 K. Such an accuracy of temperature measurement cannot be secured with the use of typical measurement equipment such as thermocouples or resistance thermometers.

In the case of rooms ventilation the air flow velocity in plume is low, usually much lower than 1 m/s. The flow is characterized by high intensity of turbulence and it is nonisothermal. Thus, only a laser anemometer properties are adequate to the measurments of flows of this type without any restrains. A vibration anemometer properties (such as DISA LVA) are similar but the range of measured velocities is limited to 0.3 m/s.

It appears that it is not easy to choose suitable measurement transducers to measure convective flows. Furthermore, the large scale instabilities found in plume are another difficulty. As Popiołek`s measurements have shown [15] the instabilities result from the plume axis wandering and their effect on the measurements results may be eliminated when the actual position of the plume axis is determined from simultaneous measurements in various points.

Thus, it is necessary to use multipoint measurement systems in order to measure convective flows. The way in which such the temperature measurement was carried into effect by means of copper-constantan thermocouples was presented in [15]. This measurement ensured the temperature measurement resolution equal to 0.01 K. When applying a multipoint temperature and velocity measurement method the measurement time is much shorter. It is of great importance when tests are not carried on in a climatic test chamber but in ordinary rooms where the temperature in the plume surroundings and the value of stratification change with time.

The research on multichannel system of velocity measurement is presently carried on in the Institute of Heating, Ventilation and Air Protection in the Silesian Technical University. In its first stage, which has already been finished, an omnidirectional anemometer with automatic temperature compensation was worked out. The anemometer has been designed with the purpose of air flow velocity measurements in rooms [19]. In the course of further study an anemometer for one component of velocity measurement and a main unit for control and signals measurement are to be worked out.

5. Concluding remarks

Experimental test of plumes above heat sources yielded data necessary to verify the suggested method of mathematical-physical modelling of convective flows. Such the verification was carried out by means of the comparison of measurements results and the results obtained from calculation with the use of the integral method. In the zone II the differences between the results comprised within the range of 5-15%.

Numerical calculation of plume was performed for various values of these flows beginning parameters. The calculation has yielded:

- the information about the value of elevation, beginning from which the flow may be approximated by the model of plume above a point heat source and about the position of the virtual point heat source in the case when $S = 0$,

- the information about the way in which stratification affects plume parameters and about the value of stratification which already has the essential influence on the plume evolution,
- the information about the approximate value of the maximum height of plume rise and about the factors on which this height depends.

A detailed report on this stage of the research will be published next year by the Royal Institute of Technology in Stockholm.

6. Acknowledgments

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7. Nomenclature

$Ar = \beta g \Delta \bar{t}_m R_w \bar{W}_{zm}^{-2}$, Archimedes number
g, acceleration due to gravity
I, momentum flux
N, exponent of stratification changes
Q, enthalpy flux
s, dimensionless stratification
S, stratification
t, temperature
$\Delta \bar{t}$, mean temperature excess
V, volume flux
$\bar{W}$, mean velocity
z, elevation

Greek symbols

α, entrainment factor
β, thermal expansion coefficient
γ, factor characterizing heat turbulent diffusion
$\lambda = R_t/R_w$, ratio of temperature and velocity profiles widths

Indices

j, jet
m, maximum value (in the plume axis)
o, at the beginning crosssection
p, plume from point heat source
w, velocity
t, temperature
z, vertical component
∞, surroundings

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METODA MATEMATYCZNO-FIZYKALNEGO MODELOWANIA PRZEPŁYWÓW KONWEKCYJNYCH

S t r e s z c z e n i e

Proponowana metoda matematyczno-fizykalnego modelowania przepływów konwekcyjnych umożliwia identyfikację źródeł przepływów konwekcyjnych, jak również przewidywanie dalszego rozwoju tych przepływów. Może być stosowana dla dowolnego quasi-osiowosymetrycznego źródła ciepła o dowolnej mocy, generującego strugę konwekcją w spokojnym otoczeniu, uwzględnia również możliwość wystąpienia cieplnej stratyfikacji powietrza.

Metoda może być użyta do kompleksowego wprowadzenia strug konwekcyjnych do obliczeń numerycznyh cyrkulacji powietrza w pomieszczeniach, gdzie strugi wprowadza się w postaci warunków brzegowych.

МАТЕМАТИКО-ФИЗИЧЕСКИЙ МЕТОД МОДЕЛИРОВАНИЯ КОНВЕКЦИОННЫХ ТЕЧЕНИЙ

Р е з ю м е

Предлагаемый метод математико-физического моделирования конвекционных течений разрешает идентифицировать источники конвекционных течений, а также предусматривать дальнейшее развитие этих течений. Этот метод может применяться для любого квазиосево-симметрического источника тепла с любой мощностью, создающего конвекционную струю в спокойной среде, а также учитывает возможность возникновения тепловой стратификации воздуха.

Предлагаемый метод может применяться в комплексном введении конвекционных струй для численных расчётов циркуляции воздуха в помещениях, в которых струи вводятся в виде граничных условий.