

*urban street network,
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INVESTIGATION OF STREET NETWORKS WITH VIEW POINT OF RELIABILITY THEORY

Estimation of reliability in the situation of possible random event e.g. disaster, accident, destruction of bridges is important for efficient management of transportation system. Street network is modelled by a weighted graph, where each edge is associated with a suitable probability of availability. The algorithm for calculation of connection reliability between given points in the street network has been presented. Analysis of network reliability took into consideration the following cases: location of origin and destination point, level of ring-radial network development and traffic calming in the city centre.

ANALIZA SIECI ULIC Z PUNKTU WIDZENIA TEORII NIEZAWODNOŚCI

Określenie niezawodności w sytuacji możliwych zdarzeń losowych, jak np. wypadek drogowy, klęski żywiołowe, awaria mostu jest ważne dla sprawnego zarządzania systemem transportu. Analizy zostały przeprowadzone w oparciu o model matematyczny, w którym sieć ulic jest modelowana jako graf ważony, w którym każda krawędź jest związana z określonym prawdopodobieństwem dostępności. Przedstawiono algorytm do wyznaczania niezawodności połączenia między zadanymi punktami w sieci ulic. Analizy niezawodności zostały przeprowadzone dla różnych przypadków: lokalizacji źródła i celu podróży, poziomu rozwoju sieci o charakterze promienisto-obwodnicowym i uspokojenia ruchu w centrum.

1. INTRODUCTION

In system engineering, reliability may be defines as [1] the degree of stability of the quality of service which system normally offers. In the face of increasing user demands for high levels of service, system reliability is becoming increasingly important in the planning, construction and operation of transportation networks. In situation of rapidly growing

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motorization level in Polish cities and towns, this problem is especially essential in street networks. In evaluating of network reliability, the flow of vehicles may be divided into normal and abnormal states. Abnormal events are for instance: disasters, road accident and destruction of bridge due to flood or terrorist attack, section of road under construction or repairing. Moreover, transportation network reliability has two aspects: connectivity and travel time reliability. Connectivity is [1, 2] the probability that traffic can reach a given destination at all, while travel reliability is the probability that traffic can reach a given destination within a given time. In this paper the aspect of connectivity has been taken into consideration. But increasing traffic, causing oversaturated network would claim treatment of street network in the term of travel time reliability.

2. RELIABILITY OF TRANSPORTATION NETWORKS

For the consideration of reliability aspects, the road network is modelled by a weighted graph $G=G(V,A)$, where V and A denote the set of the nodes and edges, respectively. Each edge $k \in A$ is associated with its probability of availability p_k . By m the number of edges is denoted, n denotes the number of nodes.

First of all, the definition of suitable characteristics for the network reliability is of great importance. In this paper, the pairwise passage probability between two fixed nodes u and v is considered. This probability is denoted by $P(u,v)$ and stands for the probability to reach the destination v from starting point u by some way in the graph (this means, that there is at least one path from u to v , which is not blocked).

For the sake of simplicity the assumptions are made, that each edge of the graph can either be blocked (with probability $1 - p_k$) or not (with probability p_k) and that the states of the edges are stochastically independent. For a fixed network there are several opportunities for the choice of the nodes u and v (e. g. these points could lay on the same diameter or the destination point could be in the centre) as well as for the choice of suitable probabilities p_k . Of course, it is reasonable to choose the same values of p_k for comparable edges. A suitable criterion for the choice of p_k could be for instance the number of available lanes of the associated road.

From the mathematical point of view there are several opportunities for the calculation of $P(u,v)$. If one is interested in a exact calculation (and not only an estimate) of $P(u,v)$, the corresponding algorithms are in general connected with large numerical efforts. It should be mentioned, that the ring-radius-structures, which are of interest in this paper are not decomposable in an easy way into serial or parallel connections of elementary elements. Therefore, an algorithm which bases on a complete enumeration is used.

Let w_1 and w_2 be two paths in G from u to v . For the probability $P(w_j)$, that the path w_j ($j = 1,2$) is not blocked it holds

$$P(w_j) = \prod_{k \in w_j} p_k . \quad (1)$$

For the probability, that w_1 or w_2 are not blocked it follows

$$P(w_1 \cup w_2) = P(w_1) + P(w_2) - P(w_1 \cap w_2) . \quad (2)$$

The main problem is, that w_1 and w_2 have in general common edges. This has the consequence, that the corresponding events are not stochastically independent. Therefore the calculation of $P(w_1 \cap w_2)$ becomes difficult and it is convenient to consider such combinations of „selected“ and „not selected“ edges, which are mutually disjoint. Each of these combinations shall include a path from u to v and all paths from u to v shall be covered.

In principle of course it is possible to consider *all* combinations of “selected” and “not selected” edges, so the conceptually simplest exact algorithm is the complete state enumeration. Instead of this here a Branch-and-Bound-algorithm is used, which reduces the computational effort by collecting as much as possible elementary events.

To describe the used algorithm the following symbols and terms are introduced.

- $KW(i)$ is the length (i. e. the number of edges) in the shortest path in G from node i to the destination node v .
- $M(i)$ is the *current marking* of i , where it is set $M(i)=j$, if i is marked outgoing from j via edge (j,i) .

- Further on it is set $S(k) = \begin{cases} 1, & \text{if edge } k \in A \text{ is currently locked by the algorithm,} \\ 0, & \text{otherwise.} \end{cases}$

- Let EB be the considered decision tree with

$$EB(i) = \begin{cases} (k, j), & \text{if the following condition (+) is fulfilled,} \\ (-k, 0), & \text{otherwise.} \end{cases}$$

condition (+):

Node i of the decision tree was generated outgoing from node $i-1$ by including edge $k \in A$ and marking node $j \in V$.

- Let $P(a)$ be the probability belonging to branch a of the decision tree and let t be the current depth in this decision tree.

The following algorithm containing three steps is used.

Step 1:

Calculate $KW(i)$ for all $i \in V$ by a suitable algorithm. $M(u):=n+1$. $M(i):=0$ for $i \neq u$. $S(k):=0$ for all $k \in A$. $t:=1$. $EB(1):=(m+1, n+1)$. $P:=0$.

Step 2:

$t:=t+1$.

If there is no unlocked edge connecting a marked node in G with an unmarked one, go to Step 3.

Else: Choose from all unlocked edges connecting a marked node with an unmarked one an edge $k=(i,j)$ with the following property:

i is marked, j is unmarked, $KW(j)$ is minimal.

Mark $M(j):=i$. Set $EB(t):=(k,j)$.

If $j \neq v$: Repeat Step 2.

Else: Calculate $P(a)$ and $P:=P+P(a)$. Replace $EB(t):=(-EB(t)[1], 0)$. Set $M(v):=0$.

Set $S(|EB(t)[1]|):=1$. Repeat Step 2.

Step 3:

$t := t - 1$.

If $EB(t)[1] < 0$ set $S(|EB(t)[1]|) := 0$. Repeat Step 3.

Else: If $EB(t)[1] = m + 1$: Stop the algorithm.

If $EB(t)[1] \neq m + 1$: Replace $M(EB(t)[2]) := 0$. Replace $EB(t) := (-EB(t)[1], 0)$. Set

$S(-EB(t)[1]) := 1$. Go to Step 2.

When the algorithm is stopped, the value of P is the probability $P(u, v)$ which was to calculate. It is possible to show, that the summands involved in the calculation of P belong to mutually disjoint events. Each of these events is connected with a path from u to v , which is not blocked. Furthermore, the algorithm acquires all events which allow a passage from u to v and therefore the pairwise passage probability is calculated correctly.

3. INVESTIGATED STREET NETWORKS

Calculations of reliability indicator have been made for various combinations of general radial ring pattern. Values of geometrical parameters for this pattern resulted investigation [3, 6] of real street network in Polish towns. The basic pattern consist few typical elements:

- three ring roads with radius; 0.5; 2.0 and 3.5 km respectively;
- four radial streets (each 4.5 km of length), connecting city centre with outer road gates;
- four complementary radial streets (each 3.0 km of length), connecting inner ring with suburban areas.

This network pattern composes the following reduced street network models:

- lack of outer ring;
- lack of inner ring;
- lack of inner and intermediate rings
- lack of mentioned four complementary radial streets.

Each model was investigated with possibility of passing the central point or not. The last case could be realised by replanning of network, cutting of diametric connections at the city centre and/or withdrawing of private through traffic from circulation in inner street ring. Both cause the situation with traffic calming in the city centre. All analysed cases of street network models have been shown at the figure 1

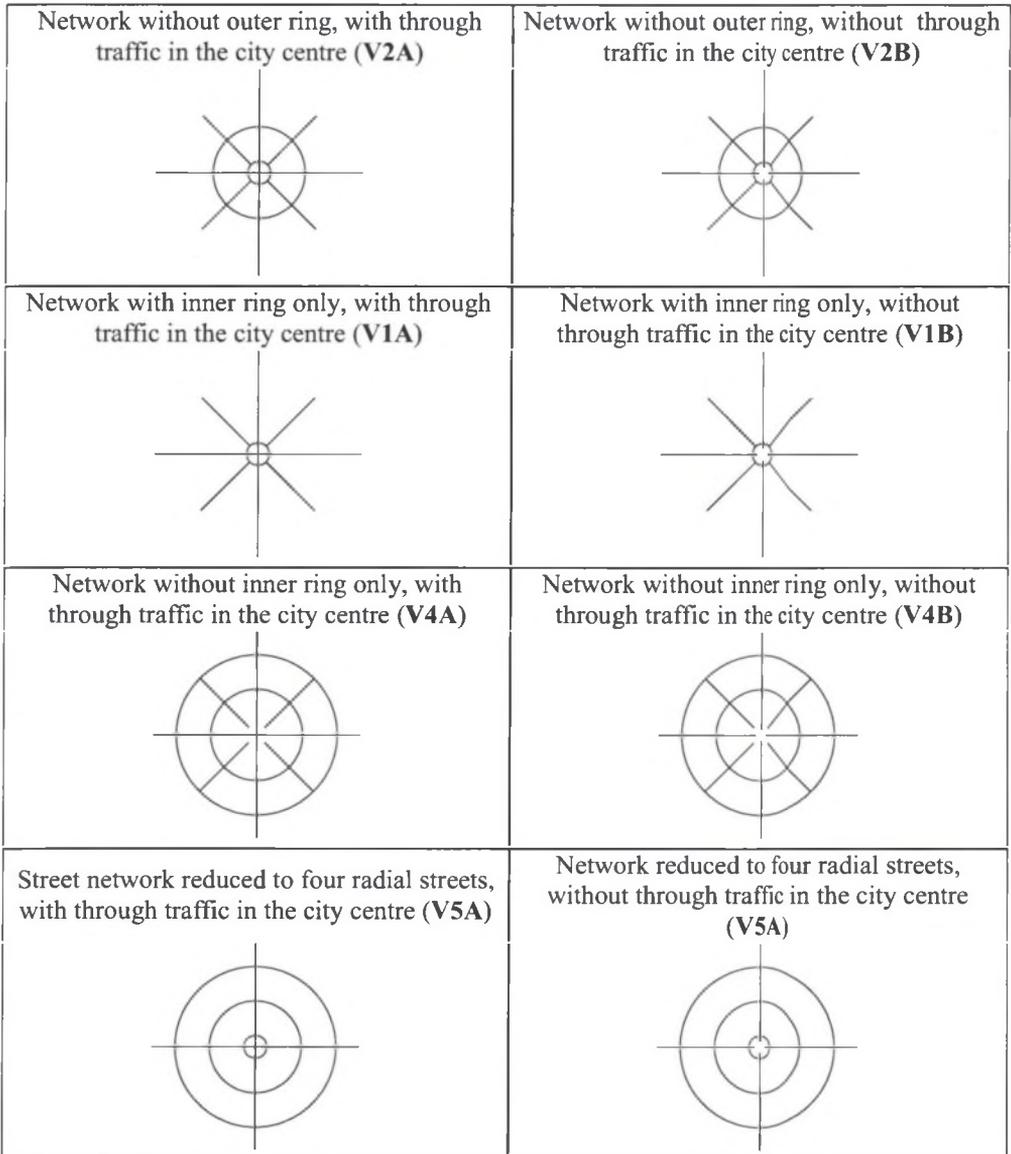


Fig. 1. Models of street networks taken into the consideration

4. RESULTS OF RELIABILITY CALCULATION

The analysis has been realized for all mentioned models, using of computer software basing on the algorithm described in the chapter 2. Because of complicated procedure of reliability calculation, the model with full three ring road has not analysed. The calculations were made for three cases of reliability values (table 1)..

Table 1

Reliability values p_k of respective section in street network for investigated cases

	Case 1	Case 2	Case 3
Inner ring	0.9	0.95	0.95
Intermediate ring	0.95	0.95	0.98
Outer ring	0.98	0.98	0.98
Radial section inside inner ring	0.9	0.95	0.95
Radial section between inner ring and intermediate ring	0.95	0.98	0.95
Radial section outside intermediate ring	0.98	0.98	0.98
Complementary radial sections	0.95	0.95	0.95

Case 1 reflects a network with good traffic conditions at outer ring, but with overcrowded city centre.

Case 2 reflects a network with good traffic conditions at outer ring and at main radial streets.

Case 3 reflects a network with good traffic conditions at outer ring, at intermediate ring and at main radial streets.

Performed calculations are continuation of former researches, which have been presented in [5]. Former calculation and analysis referred to reliability of selected connection in inner traffic. Current analysis refers connections for outer traffic, both through traffic and traffic involved with the town. Reliability of connections between pairs of an origin and a destination were calculated as at figure 2 (presented on next page).

Results of calculation have been listed in the table 2.

Table 2

Calculation results of reliability $P(u,v)$ for investigated connections

Symbol of street network	Connection of outer gate with:				
	the other outer gate	central point	inner ring	intermediate ring	outer ring
V1A	0.8257	0.9084	0.9004	0.8554	0.8212
V1B	0.7462	-	0.8357	0.7939	0.7621
V2A	0.9221	0.9602	0.9599	0.9601	0.9217
V2B	0.9217	-	0.9594	0.9598	0.9214
V4A	0.9604	0.9796	0.9309	0.9799	0.9800
V4B	0.9604	-	0.9309	0.9799	0.9800
V5A	0.9603	0.9799	0.9798	0.9800	0.9798
V5B	0.9603	-	0.9780	0.9799	0.9797

The value set of section reliability has given for the case 1 (see table1).

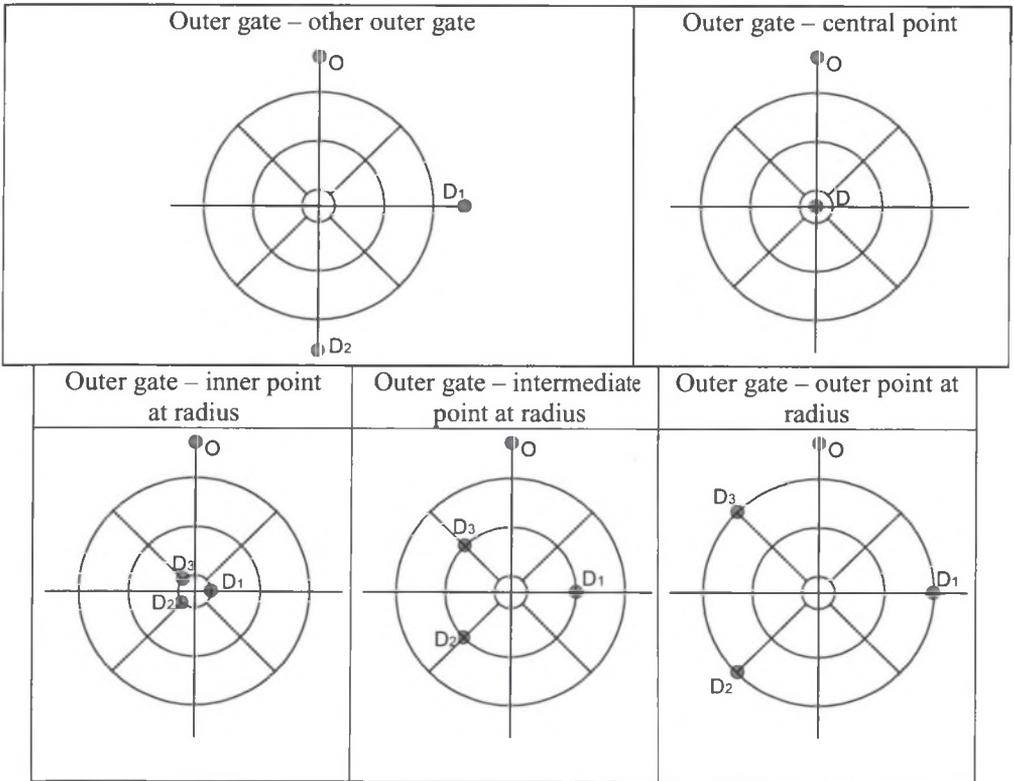


Fig. 2. Location of origin (O) and destination (D) pairs, selected for analysis of network reliability

On the base of presented and other results the following statements can be concluded:

- For connections of outer gates in more developed street networks, elimination of central point in network does not decrease of reliability. Decreasing of reliability this connections is essential for street network with inner ring only, especially for the lower values of section reliability.
- Reliability values of sections for more developed network influence slightly reliability of connections between outer gates. Essential influence has been proved for network with inner ring only, due to lack of alternative paths.
- Supplement of the intermediate ring to the network with the inner ring only causes reliability increase:
 - from 5 to 7 % for connections between outer gate and city centre;
 - about 11 % for connections between two outer gates;
 - from 11 to 13 % for connections between outer gate and the points located in grater distance from the city centre.

- The same effects have been recognised for network with traffic calming in the city centre.
- Supplement of the intermediate ring to the network with the inner ring only, but with traffic calming in city centre causes reliability increase:
 - from 7 to 18 % for connections between outer gate and city centre;
 - from 12 to 26 % for connections between two outer gates
 - from 12 to 25 % for connections between outer gate and the points located in greater distance from the city centre.

5. GENERAL CONCLUSIONS

With viewpoint of network reliability, development of street network should consist more in supplement of new sections than increasing of reliability for existing sections. Effects in reliability improvement given connection depend on location origin and destination of travel. The more outside of a city centre this points of travel are located, the effect of street development is more seen in the term of reliability.

BIBLIOGRAPHY

- [1] BELL M.G.H., IIDA Y. – Transportation Network Analysis; Ed. Chichester, Wiley, 1997
- [2] BELL M.G.H., IIDA Y. – The Network Reliability of Transport; Proceedings of the 1st International Symposium on Transportation Network Reliability (INSTRE); ed. Pergamon, Amsterdam, 2003
- [3] DUDEK M. - Współzależność struktury i funkcji sieci ulic miasta. Politechnika Krakowska (praca doktorska), 1998.
- [4] DUDEK M., RICHTER M. – Zuverlässigkeit und strukturelle Parameter von Verkehrsnetzen. Internationales Kolloquium über Anwendungen der Informatik und Mathematik in Architektur und Bauwesen IKM2003, Weimar, 2003.
- [5] DUDEK M., RICHTER M. - Niezawodność sieci ulicznej a jej parametry strukturalne. Konferencja Naukowo-Techniczna „Systemy Transportowe – teoria i praktyka”. Zeszyty Naukowe Politechniki Śląskiej, seria Transport, zeszyt 46, 2003
- [6] PIĘCINSKI W. - Metody analizy przestrzennej struktury sieci ulic na przykładzie wybranych miast. Prace Naukowe Politechniki Lubelskiej 214, Budownictwo 39, 1991.

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