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INCREASE OF QUALITY OF PUBLIC TRANSPORT SERVICE PROVIDING EASY RECOGNISED TIME TABLE AND HIGH PROBABILITY OF ON TIME DEPARTURES

The main focus of the considerations of this paper is to provide constant time intervals between buses departing from bus stops. Therefore, relations between characteristics of regularly scheduled urban bus lines are studied. A buffer time between the scheduled arrival time and the departure to the next trip is considered in a deterministic as well in a stochastic framework.

WZROST JAKOŚCI OBSŁUGI KOMUNIKACJI MIEJSKIEJ POPRZEZ WPROWADZENIE CZYTELNEGO ROZKŁADU JAZDY I WYSOKIEGO PRAWDOPODOBIEŃSTWA CZASU ODJAZDU

Głównym zagadnieniem tego referatu jest wprowadzenie stałego interwału czasu między autobusami odjeżdżającymi z przystanków. Rozważane są czynniki wpływające na regularność kursowania linii autobusowych. Ważnym składnikiem obliczeń jest czas buforowy na przystankach krańcowych, który jest rozpatrywany w modelach deterministycznych i stochastycznych.

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1. INTRODUCTION

Clearly, the quality of public transport systems with respect to the comfort of the passengers has to be increased [3]. This is an evident consequence of the strong competition of public transport systems to private car traffic. There are several aspects of quality in transport systems. The main focus here is to provide constant time intervals between the buses departing from the bus stops [1, 2]. We consider a regularly scheduled urban bus line. For such a bus line we want to create a time table

- which is easy to recognise,
- which contains a reasonable compromise between economical profitability and passenger service [1] and
- which has a high reliability against tardiness.

The two first aims we will investigate by a deterministic modelling and the last aim – the reliability – we will consider within a stochastic framework.

2. DETERMINISTIC MODELLING

A time table is called *easy recognised* if the departure times at the same station have a fixed distance and recur in each hour. To describe the situation we use the following symbols and terms:

L - the length of the considered bus line given in km, i. e. the length of a tour of the bus from one terminal stop over all other stations of the line back to the same terminal stop,

v - the average velocity of a bus on this line given in km/h,

h - the tact time of buses, this means the time intervals between two one after another following buses given in minutes,

n - the number of buses which are operating on the considered line.

$2t_p$ - the buffer time per round trip given in minutes, this means that the scheduled waiting time of the bus on the both terminal stops of the line between the arrival at the stop and the start of the next trip is t_p at each terminal stop.

The scheduled time interval between two one after another starts of the same bus in the same terminal stop is then given by $\frac{60L}{v}+2t_p$ [2, 3]. This time interval is the same as $n \cdot h$, the time interval of two buses following each other multiplied with the number of operating buses on the line. Therefore we have the equation

$$n \cdot h = \frac{60L}{v} + 2t_p \,. \tag{1}$$

The number n of buses is an integer number, the time interval h (given in minutes) conveniently too. An easy recognised time table as it is defined above means that h as an integer number is a divisor of 60 (the departure times reiterate after 60 minutes). If we eliminate the divisors 1, 2, 3, 4 because they are to small then h can take one of the values

$$h \in \{5,6,10,12,15,20,30,60\} = H \tag{2}$$

Each bus has a capacity denoted by cap, i. e. the maximum number of passengers which can be carried by the bus. All the buses on the line have the same capacity because they are of the same type. Denotes p the number of passengers which enter the bus line in one hour and is h fixed then *60/h* is the number of buses per hour and *60 cap/h* is the maximum number of passengers which can be carried by the buses per hour.

Passenger service means that all passengers which want to enter the line can be carried. It means

$$60\, cap/h \geq p \qquad \text{or} \qquad h \leq 60\, cap/p = h_u, \tag{3}$$

where $h_u = 60\, cap/p$ is an upper bound for h.

Analogously we get a lower bound h_l if we consider the value *eff* as the minimal number of passengers for which a bus can operate profitable,

$$h \geq 60\, eff/p = h_l\,. \tag{4}$$

Remarks:

(a) If p is too small, for example $p < eff$ then follows from the above formula $h_l > 60$. This shows that the considered bus line can not work profitably in a one hour rhythm.

(b) The value p as number of passengers which enter the line per hour is introduced intuitively. It can be stated more precisely in various manner. The stops of the line may be numbered by 0, 1, 2, ... , k, where 0 and k are the terminal stops. Denotes *p(i,j)* the number of passengers per hour which enter the bus in stop i and leave the bus in stop j and if we assume that on an terminal stop all passengers leave the bus then the number of passengers which are inside the bus when the bus departs from the station q is given by

$$\sum_{i=0}^{q} \sum_{j=q+1}^{k} p(i,j) \quad \text{and} \quad \sum_{i=q}^{k} \sum_{j=0}^{q-1} p(i,j), \quad \text{respectively.} \tag{5}$$

The first term stands for the journey from 0 to k, the other term for the return journey from k to 0. The value

$$p = \max\left(\max_{q=0,\dots,k-1} \sum_{i=0}^{q} \sum_{j=q+1}^{k} p(i,j),\ \max_{q=1,\dots,k} \sum_{i=q}^{k} \sum_{j=0}^{q-1} p(i,j) \right) \tag{6}$$

is then a more precise definition of the passenger number. It guarantees that the bus is not overloaded on some parts of the journey and can be used in the formula for the upper bound h_u given above. In the formula for the lower bound h_l this definition is disadvantageously. Here it is better to calculate with

$$p = \frac{1}{2k}\left(\sum_{q=0}^{k-1} \sum_{i=0}^{q} \sum_{j=q+1}^{k} p(i,j) + \sum_{q=1}^{k} \sum_{i=q}^{k} \sum_{j=0}^{q-1} p(i,j) \right) \tag{7}$$

as an averaged passenger number on all parts of the journey.

The question is now, how to determine the values of h, n and $2t_p$ for given L and v. If we require in addition that the buffer time in one terminal stop t_p does not exceed the tact time h then we have the following system of requirements – equations (1), (2) and:

$$h_l \leq h \leq h_u \tag{8}$$

$$0 \leq 2t_p \leq 2h \tag{9}$$

If we choose a value h from H we can always determine an integer number m and a real number r with

$$\frac{60L}{v} = m \cdot h + r, \quad \text{where} \quad 0 \leq r < h. \tag{10}$$

From the equations (1) and (10) we get

$$2t_p = (n-m) \cdot h - r, \tag{11}$$

and from (9) and (11):

$$0 \leq (n-m) \cdot h - r < 2h, \quad \text{i. e.} \quad m + \frac{r}{h} \leq n < m + \frac{r}{h} + 2. \tag{12}$$

These formulas allow to determine $2t_p$ and n in an easy way if h is fixed at one value of H.

Example: Assume $L = 15$ [km], $v = 20$ [km/h], $h_l = 5$, $h_u = 20$ then it follows $60L/v = 45$ and we get the solutions shown in Table 1.

Table 1

Values for h, m, r, n and $2t_p$ in the parameter setting of the example

h [min]	m	r [min]	n	$2t_p$ [min]
5	9	0	9	0
			10	5
6	7	3	8	3
			9	9
10	4	5	5	5
			6	15
12	3	9	4	3
			5	15
15	3	0	3	0
			4	15
20	2	5	3	15
			4	35

3. EFFICIENCY OF DELAY COMPENSATION EFFECTS

In this chapter, the efficiency of delay compensation effects is studied in a stochastic framework. Therefore, one bus and a fixed scheduled departure time of it (called the initial time) at one of the terminal stops is considered.

Assume, that the bus is behind schedule and for this reason not able to start the trip on time. The initial delay is denoted by d, $d > 0$. The travel times T_i $(i=1, 2, \ldots)$ of the bus in the i-th trip (counted from the initial time) are modelled as random variables, where the expectation ET_i is chosen according to Section 2 as $m=30L/v$ (note, that Ti is the travel time from the from one terminal stop to the other one). For the sake of simplicity it is assumed, that these travel times are stochastically independent and identical distributed. Deviations from this assumption may be implemented, however the main focus of interest shall be here to study delay compensation effects in principle and to support the decision of the choice of the parameters described in the previous section.

The event, that the initial delay vanishes after i trips $(i \geq 1)$ can be described with the help of the random events A_i, which are defined as

A_i - after the i-th trip a departure at scheduled time is realised for the first time after the initial time.

Written as formulas, this means

$$A_1 = \{d + T_1 \leq m + t_p\}, \tag{13}$$

$$A_2 = \{d + T_1 > m + t_p\} \cap \{d + T_1 + T_2 \leq 2(m + t_p)\}, \tag{14}$$

...

$$A_i = \bigcap_{j=1}^{i-1} \left\{ d + \sum_{k=1}^{j} T_k > j(m + t_p) \right\} \cap \left\{ d + \sum_{k=1}^{i} T_k \leq i(m + t_p) \right\} \tag{15}$$

Obviously, it holds

$$\bigcup_{i=1}^{\infty} A_i = \left\{ \exists\, j : d + \sum_{k=1}^{j} T_k \leq j(m + t_p) \right\}, \tag{16}$$

which is the event, that the initial delay vanishes somewhere in the future, admittedly an event of more theoretical than practical interest (in practical situations especially the events A_1 and A_2 will be of interest). However, from the mathematical point of view, clearly it holds

$$\mathrm{P}\left(\overline{\bigcup_{i=1}^{\infty} A_i} \right) = \mathrm{P}\left(d + \sum_{k=1}^{j} T_k > j(m + t_p)\ \forall j \right) = \mathrm{P}\left(\frac{1}{j} \sum_{k=1}^{j} T_k > m + t_p - \frac{d}{j}\ \forall j \right). \tag{17}$$

From the Law of large numbers it follows

$$\mathrm{P}\left(\lim_{j \to \infty} \frac{1}{j} \sum_{k=1}^{j} T_k \neq m \right) = 0 \tag{18}$$

and therefore $\mathrm{P}\left(\bigcup_{i=1}^{\infty} A_i\right)=1$ for $t_p>0$, i. e. every delay can and will be compensated somewhere in the future by a buffer time $t_p>0$.

Of course this mathematical result is not very useful for practical applications, one is more interested in statements like "With probability p the initial delay d can be compensated after i trips (i = *1, 2, ...*)". If the distribution of the travel times is known, for such calculations on the one hand the way of Monte Carlo methods stands open. On the other hand analytical results are available. Under the assumption of a normal distribution, $T_i \sim N(m,\sigma^2)$, such results are presented in the next sections.

3.1 PROBABILITY $\mathrm{P}(A_1)$

Under the assumption of normal distribution it holds

$$\mathrm{P}(A_1)=\mathrm{P}(d+T_1\le m+t_p)=\mathrm{P}(T_1\le m+t_p-d)=\Phi\left(\frac{t_p-d}{\sigma}\right), \tag{19}$$

where Φ denotes the CDF (cumulative distribution function) of a standard normal distribution. Table 2 shows values of $\mathrm{P}(A_1)$ for the parameter situation of the example in Section 2.

<u>Example:</u> Given $L=15$ [km], $v=20$ [km/h], $\sigma=4$ [min], the following probabilities to compensate a delay of $d=5$ [min] can be calculated for different buffer times (see the example in Section 2). Moreover, the values calculated by the above formula are compared with the result of a Monte Carlo simulation (10.000.000 realisations). Here, for instance the value $\mathrm{P}(A_1)=0.4503$ means, that with probability 45,03 % an initial delay of 5 minutes can be compensated by the buffer time $2t_p$ of 9 minutes after one trip.

Table 2

Probability $\mathrm{P}(A_1)$ for different buffer times

buffer time $2t_p$	0	3	5	9	15	35
$\mathrm{P}(A_1)$ (exact)	0.1056	0.1908	0.2660	0.4503	0.7340	0.9991
$\mathrm{P}(A_1)$ (Monte Carlo)	0.1056	0.1908	0.2662	0.4502	0.7339	0.9991

3.2. PROBABILITY $P(A_2)$

It holds

$$P(A_2) = P(\{T_1 > m + t_p - d\} \cap \{T_1 + T_2 \le 2(m+t_p) - d\}) = \int_{-\infty}^{m+t_p} \int_{m+t_p-d}^{2(m+t_p)-d-t} f_{T_1}(s)\, f_{T_2}(t)\, ds\, dt\,, \tag{20}$$

where f_{T_1} and f_{T_2} denote the probability density functions of T_1 and T_2, respectively. Using the property of normal distribution, straightforward calculation gives

$$P(A_2) = \int_{-\infty}^{t_p/\sigma} \phi(t)\, \Phi\left(\frac{2t_p - d - \sigma t}{\sigma}\right) dt - \Phi\left(\frac{t_p - d}{\sigma}\right) \Phi\left(\frac{t_p}{\sigma}\right), \tag{21}$$

with the probability density function ϕ of the standard normal distribution.

Example: Given the same values for L, v and σ as in the example in Section 3.1, the following probabilities to compensate a delay of $d = 5$ [min] can be calculated for different buffer times. Here, for instance the value $P(A_2) = 0.3316$ means, that with probability 33,16 % the initial delay of 5 minutes can not be compensated by the buffer time $2t_p$ of 9 minutes after one trip, but after two trips (that means, after one round trip).

Table 3

Probability $P(A_2)$ for different buffer times

buffer time $2t_p$	0	3	5	9	15	35
$P(A_2)$ (exact)	0.1178	0.2093	0.2694	0.3316	0.2318	0.0009
$P(A_2)$ (Monte Carlo)	0.1178	0.2095	0.2692	0.3316	0.2319	0.0009

3.3 PROBABILITIES $P(A_i), i \ge 3$

The probabilities $P(A_i)$, $i \ge 3$, can be written as

$$P(A_i) = P\left(\bigcap_{j=1}^{i-1} \left\{d + \sum_{k=1}^{j} T_k > j(m+t_p)\right\} \cap \left\{d + \sum_{k=1}^{i} T_k \le i(m+t_p)\right\}\right). \tag{22}$$

The calculation of these probabilities requires the consideration of multidimensional integrals of the corresponding density functions. However, the following estimates hold, which give upper bounds for $P(A_i)$. Examples show, that the estimates have sufficient quality in practical situations. It holds

$$\mathrm{P}(A_i) \le \mathrm{P}\left(\left\{d + \sum_{k=1}^{i-1} T_k > (i-1)(m+t_p)\right\} \cap \left\{d + \sum_{k=1}^{i} T_k \le i(m+t_p)\right\}\right) \tag{23}$$

and similar calculations as in Section 0 lead to

$$\mathrm{P}(A_i) \le \int_{-\infty}^{t_p/\sigma} \phi(t)\, \Phi\left(\frac{it_p - d - \sigma t}{\sigma\sqrt{i-1}}\right) dt - \Phi\left(\frac{(i-1)t_p - d}{\sigma\sqrt{i-1}}\right) \Phi\left(\frac{t_p}{\sigma}\right). \tag{24}$$

This estimate is the basis of the example in this section. It should be mentioned, that it follows the (much more weaker) estimate

$$\mathrm{P}(A_i) \le \Phi\left(\frac{t_p}{\sigma}\right)\left(1 - \Phi\left(\frac{(i-1)t_p - d}{\sigma\sqrt{i-1}}\right)\right), \tag{25}$$

which can be obtained without large computational efforts.

Example: Given the same values for L, v and σ as in the previous examples, the following upper bounds for the probabilities to compensate a delay of $d = 5$ [min] firstly after i trips can be calculated for different buffer times. Moreover, these upper bounds are compared with the result of a Monte Carlo simulation to check the quality of the estimates. The estimates work quite sufficiently. Moreover, it can for instance be seen, that with a buffer time of 35 minutes in the given parameter constellation in almost all cases the initial delay is compensated after one round trip (compare also the values in Tables 2 and 3).

Table 4

Upper bounds for the probabilities $\mathrm{P}(A_i)$, $i = 3, 4, 5$ for different buffer times

buffer time $2t_p$	0	3	5	9	15	35
upper bound for $\mathrm{P}(A_3)$	0.0961	0.1599	0.1823	0.1462	0.0339	0.0000
$\mathrm{P}(A_3)$ (Monte Carlo)	0.0864	0.1436	0.1632	0.1302	0.0299	0.0000
upper bound for $\mathrm{P}(A_4)$	0.0826	0.1282	0.1292	0.0671	0.0051	0.0000
$\mathrm{P}(A_4)$ (Monte Carlo)	0.0642	0.0992	0.0994	0.0512	0.0037	0.0000
upper bound for $\mathrm{P}(A_5)$	0.0733	0.1063	0.0946	0.0318	0.0008	0.0000
$\mathrm{P}(A_5)$ (Monte Carlo)	0.0495	0.0714	0.0631	0.0209	0.0005	0.0000

For summarising the considerations of Section 3 and for pointing up the influence of different buffer times to delay compensation, the results of the example in Tables 2 - 4 are combined in the following Table 5. Here upper bounds for the probability, that the first compensation of the initial delay of 5 minutes takes places within the first j trips ($j = 1,2,\ldots,5$) (in other words upper bounds for the sums $\sum_{i=1}^{j} \mathrm{P}(A_i)$) are shown

Table 5

Upper bounds for the cumulative probabilities for different buffer times

buffer time $2t_p$	0	3	5	9	15	35
$P(A_1)$	0.1056	0.1908	0.2660	0.4503	0.7340	0.9991
$P(A_1)+P(A_2)$	0.2235	0.4001	0.5354	0.7818	0.9658	1.0000
upper bound for $\sum_{i=1}^{3} P(A_i)$	0.3196	0.5600	0.7177	0.9281	0.9996	1.0000
upper bound for $\sum_{i=1}^{4} P(A_i)$	0.4021	0.6883	0.8469	0.9952	1.0000	1.0000
upper bound for $\sum_{i=1}^{5} P(A_i)$	0.4755	0.7946	0.9415	1.0000	1.0000	1.0000

4. CONCLUDING REMARKS

In our investigations we have assumed that the passenger number is constant over the day. Indeed, in the rush-hour it is significantly larger. It is to compensate by additional buses which depart to the same times as the regular buses. The possible increasing of travel times we have neglected. However, rush-hour effects can be taken into account by suitable choices of the standarddeviation σ of the travel times in the considerations of Section 3.

The finalchoose of a solution will be a compromise between the various evaluations of the solutions in each case. Perhaps it is possible to choose the tact time in such a way that in the rush-hour the additional buses depart in the gaps symmetrically to the normal departures.

The method given above is extendable also in the case of two or more bus lines. Then we need additional requirements in connection with the switch stations and with the limited number of avaiable buses for all lines.

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