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DELIVERY DAY OPTIMISATION AS A PART OF DISTRIBUTION SYSTEM DESIGN

This article deals with optimisation techniques for delivery day combination assignment to form the customer clusters, on which a vehicle routing algorithm provides good solutions. The usage of original "one cluster - one tree" approach to this problem is reported and compared to newly designed "one cluster - many trees" approach.

OPTYMALIZACJA DNIA DOSTAWY JAKO CZĘŚĆ PROJEKTU SYSTEMU DYSTRYBUCJI

Udział ten obejmuje techniki optymalizacji przypisania dnia dostawy tak, aby tworzyły one skupiska klientów pozwalające na dobre rozwiązania algorytmu wytyczania tras pojazdów. Przedstawiono porównanie oryginalnego podejścia „jedno skupisko – jedno drzewo" z nowoprojektowanym podejściem „jedno skupisko – wiele drzew".

1. INTRODUCTION

The delivery day optimisation problem emerges whenever every customer of a distribution system is not supplied daily, but for example, ones or twice a week. In this case a decision on assignment of delivery day combination to a customer must precede decisions on vehicle route forming. Let us explain consequences of admissible and inadmissible assignments of delivery day combinations to customers using the following simple example. Let us consider a part of transportation network situated in a long blind valley with customers 1 - 4 in figure 1.

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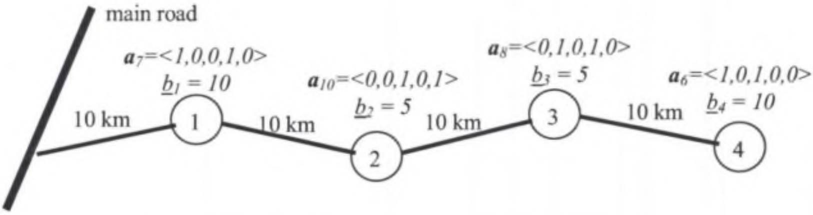

Fig.1. The original delivery day combination assignment

Each one of the customers demands a delivery of size b_j. At present, each customer is supplied in accordance to the combination, which is recorded in figure 1. One delivery day combination is described by vector $\boldsymbol{a} = <a_1, a_2, ..., a_{|T|}>$, where each component corresponds to one day of period T. If component a_t takes value 1, then it means that day t is a delivery day of the combination. Otherwise, the component takes value 0.

It is easy to order the combination in a matrix $\boldsymbol{A}$, where each row $\boldsymbol{a}_k$ corresponds to exactly one delivery day combination. A considered combination may be then referred using the number of the raw. Then, it is possible to describe each set of feasible delivery day combinations by a set of row numbers. Comparing the current combinations in Fig.1, we find that a vehicle with considered capacity *30* units has to visit the valley five times in a week. On Monday, it comes all the way up to customer 4 and it satisfies demands of customers 1 and 4 and it traverses total length of *80* km. On Tuesday, the vehicle satisfies the demand of customer 3 only and it traverses *60* km along the valley. On Wednesday, it visits customer 4 again and satisfies the demand of customer 2 also. It traverses length of *80* km again. On Thursday, it visits customer 3 and satisfies demand of customers 1 by the way. It traverses length of *60* km this day. On Friday, it visits customer 2 only and traverses only *40* km along the valley. In the week the vehicle would drive the total distance of $80+60+80+60+40 = 320$ km.

If we let unchanged the original combinations of customers 1 and 4, and if we assign to customers 2 and 3 new combinations depicted in figure 2, then it will do to visit the valley three times in a week. The delivery days would be Monday, Wednesday and Thursday. In this case, the vehicle would serve all customers on Monday and traverses distance of *80* km. On Wednesday it would serve customers 2 and 4 driving *80* km again and on Thursday it visits customers 1 and 3 traversing distance of *60* km. The new distribution plan needs traversing of $80+80+60 = 220$ km. Now, it is obvious that using admissible combination assignment, we can considerable influence the necessary length of vehicle routes in the considered period.

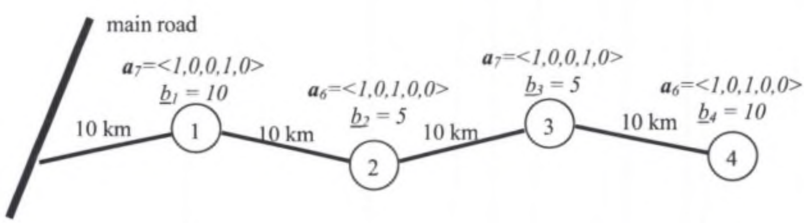

Fig.2. The optimized delivery day combination assignment

Summarizing the mentioned consequences of delivery day combination assignment, we obtain the following **period vehicle problem** [1], [2].

There is given set J of customers, which are served from depot s by a fleet of vehicles with the total capacity K_t *given by number of units of goods, which is able to be delivered by the fleet in day* $t \in T$. *T denotes short time planning period. A demand of customer* $j \in J$ *is given by size* $\underline{b}_j$ *of one delivery and by set* S_j *of feasible delivery day combinations. Each feasible combination is described by a vector of bivalent values 0 – 1 i.e.* $\mathbf{a}_k = \langle a_1, a_2, ..., a_{|T|} \rangle$ *and the distance between pair* $\langle i, j \rangle$ *of elements from* $J' = J \cup \{s\}$ *is denoted as* d_{ij}. *The objective is to assign exactly one combination from* S_j *to customer j so that daily amount of goods do not exceed day limit* K_t *and the total length of all vehicle routes be minimal.*

2. THE CLASSICAL APPROACH

The problem formulation above includes both tactical and operational decisions [3]. The tactical ones concern the delivery day assignments and the operational ones determine the daily vehicle routes. It is known that a general mathematical programming model, which comprehends both sorts of decisions, is as large that it is impossible to solve it in a sensible time by exact tools of mathematical programming. That is why we must avoid introducing the operational decision variables into model. The associated operational decision variables are used only to express the total length of vehicle routes in an objective function of a model. This can be done without these variables by using a convenient **estimation function**, which depends only on the tactical decisions.

If we model a decision on assignment combination $k \in P$ to customer $j \in J$ by variable $v_{kj} \in \{0,1\}$ and if we denote an estimation of vehicle route length on day t by term $F_t(\mathbf{v})$, then a model can be established like this:

Minimize $$\sum_{t \in T} F_t(\mathbf{v}) \tag{1}$$

Subject to $$\sum_{k \in S_j} v_{kj} = 1 \qquad \text{for } j \in J \tag{2}$$

$$\sum_{j \in J} \sum_{k \in S_j} \underline{b}_j a_{kt} v_{kj} \le K_t \qquad \text{for } t \in T \tag{3}$$

$$v_{kj} \in \{0, 1\} \qquad \text{for } j \in J, \text{ for } k \in S_j. \tag{4}$$

Let us deal with the estimation function $F_t(\mathbf{v})$. The values of variables $\mathbf{v}$ determine the current delivery day combination k for customer j. It follows that it is given if customer j should or should not be supplied in given day t.

It is awaited that function $F_t(\mathbf{v})$ should provide the vehicle route length estimation based on mutual active customer positions only. It is expected also that if the active customers are spread over smaller area, then the value of $F_t(\mathbf{v})$ is smaller than in the opposite case.

That is why the daily vehicle route length necessary for all customer service is estimated by sum of distances between active customers and key node $p \in J' = J \cup \{s\}$ and a distance between centre s and this key node. The key node is chosen so that the sum is minimum and this node is referred as **reduced median**. The estimation function can be written as

$$F_t(v) = g\,min\{d_{sr} + \sum_{j\in J}\sum_{k\in S_j} d_{rj}a_{kt}v_{kj} : r \in J'\}. \quad (5)$$

In the formulae, g is a proportional coefficient. This way we obtain non-linear discrete problem of mathematical programming, for which no exact method exists till now. Due this fact, the problem will be solved by a heuristic [1], [2], which consists of two phases. In the first one, a good initial feasible solution or a good solution with small size of infeasibility is sought for. In the second phase the heuristic makes use of exchange approach to improve the objective function value of the initial solution obtained in the first phase. During the second phase the possible infeasibility is eliminated.

The first phase is based on the fact that if there are estimated locations of reduced medians p_t for individual days $t \in T$ in advance, then it is easy to determine decrease or increase of the objective function, which follows from assigning combination k to customer j. Contribution of the assignment to the objective function value is given by following expression

$$\sum_{t\in T} d_{p_t j}a_{kt} \quad (6)$$

The second phase resides in an improvement exchange heuristic, which processes pair of customers i and j in one step and investigates consequences of simultaneous replacements of combination $k(i)$ by other combination from S_i and combination $k(j)$ by other combination from S_j for temporarily fixed locations of reduced medians p_t.

The simultaneous substitution of $u \in S_i$ for $k(i)$ and $v \in S_j$ for $k(j)$ brings saving

$$\sum_{t\in T} d_{p_t i}a_{k(i)t} + \sum_{t\in T} d_{p_t j}a_{k(j)t} - \sum_{t\in T} d_{p_t i}a_{ut} - \sum_{t\in T} d_{p_t j}a_{vt} \quad (7)$$

Besides, a value of infeasibility may be changed by the considered exchange. The value of infeasibility is defined as a sum of demands, which exceed the daily capacity K_t of the vehicle fleet. The overhangs are given by negative values of variables $\underline{K}_t$, which were introduced and initialised in the first phase. The value of infeasibility for a current solution may be expressed by $\sum_{t\in T} |min\{0, \underline{K}_t\}|$.

The change of $\underline{K}_t$, when $k(i)$ is replaced by v and $k(j)$ by u, is given by:

$$a_{ut}\underline{b}_i + a_{vt}\underline{b}_j - a_{k(i)t}\underline{b}_i - a_{k(j)t}\underline{b}_j \qquad for\ t \in T. \quad (8)$$

The value of infeasibility decreases if value of $M(i,j,u,v)$ is positive. $M(i,j,u,v)$ is given by expression

$$\sum_{t\in T}\left(min\{0, \underline{K}_t + a_{ut}\underline{b}_i + a_{vt}\underline{b}_j - a_{k(i)t}\underline{b}_i - a_{k(j)t}\underline{b}_j\} - min\{0, \underline{K}_t\}\right). \quad (9)$$

3. DRAWBACK OF ONE DAY-ONE TREE APPROACH

Analysing the estimation function $F_t(\boldsymbol{v})$ for day t and for active customers given by assignment $\boldsymbol{v}$, we can find easily that length of a set of vehicle routes satisfying all active customers is estimated by length of a simple tree in a graph of road network. This tree connects all active customers and depot to a node, which is called reduced median p_t (see Fig.3).

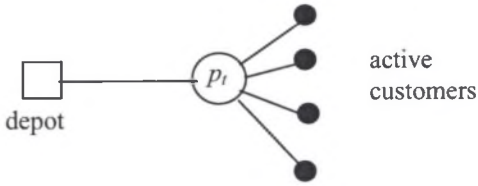

Fig.3. Reduced median

This estimation was chosen due its ability to distinguish the cases a) and b) from Fig.4 from the point of the length of the associated optimal servicing route.

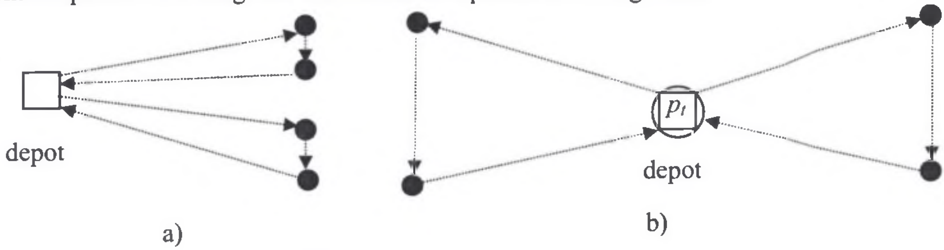

Fig.4. Routes servicing customer demands in cases a) and b)

There is considered in cases a) and b) in Fig.4 that the distances between each customer and the depot are the same and so the cases cannot be distinguished by estimation function, which takes into consideration only sum of these distances.

A drawback of the analysed estimation function emerges, when we realize that such a practical instance may arise, in which the active customers are located at the opposite sides of the depot (see Fig.4 b and Fig.5).

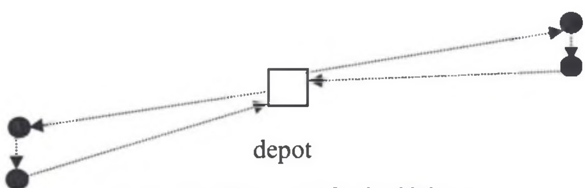

Fig.5. Servicing routes for the third case

In this cases the reduced medians lie at a position of the depot and the estimated function $F_t(v)$ cannot distinguish the case depicted in Fig 4 b) from the case in Fig.5. It means that the classical algorithm can produce sometimes very ineffective day clusters.

This drawback could be avoided using a generalization of the originally used estimation function, where the expected length of future vehicle routes is estimated by total length of a given number of trees. The number of trees may correspond to number of used vehicles or at least to the number of main roads, which coincide with the depot in the road network.

4. CONCEPT OF ONE DAY-MANY TREES APPROACH

While $F_t(v)$ evaluation asks only for simple going through set J', the exact evaluation of the P-tree estimation with capacity limit on the number of customers in a tree leads to necessity to solve an integer programming problem. Let Q be the capacity limit and J_t be set of customers, which are active on day t. Having introduced variables $y_r \in \{0, 1\}$ to model a decision on reduced median location for some tree at node $r \in J'$ and variables $x_{jr} \in \{0, 1\}$ deciding on assignment of customer $j \in J_t$ to a tree, reduced median of which is at $r \in J'$, we can formulate the associated optimisation problem as:

$$\text{Minimize} \quad \sum_{r \in J'} d_{sr} y_r + \sum_{j \in J_t} \sum_{r \in J'} d_{rj} x_{jr} \tag{10}$$

$$\text{Subject to} \quad \sum_{r \in J'} x_{jr} = 1 \qquad \text{for } j \in J_t \tag{11}$$

$$\sum_{j \in J_t} x_{jr} \le Q y_r \qquad \text{for } r \in J' \tag{12}$$

$$\sum_{r \in J'} y_r \le |P| \tag{13}$$

$$y_r, x_{jr} \in \{0, 1\} \qquad \text{for } r \in J' \text{ and } j \in J_t, \tag{14}$$

where $|P|$ denotes maximal number of trees. We note that zero-one obligatory constraints on x_{jr} may be relaxed without loss of generality.

When we decide to connect each tree p from set P with particular capacity K_p, which restricts amount of goods delivered to the customers of the tree, then the original one day-one tree problem (1) – (40) can be reformulated to the one day-$|P|$ trees via following way.

We introduce linear ordered set of pairs $[t,p]$ from $T \times P$ (day-tree pairs) and describe one delivery day-tree combination by vector $\boldsymbol{e} = \langle e_{11}, e_{12}, ..., e_{1|P|}, e_{21}, e_{22}, ..., e_{|T||P|} \rangle$, where each component corresponds to one day-tree pair of $T \times P$. If a component takes value 1, it means that in accordance to the combination a customer should be supplied at the day t and should be included into tree p. Former delivery day combination $\boldsymbol{a} = \langle a_1, a_2, ..., a_{|T|} \rangle$ from set S_j of feasible day combinations is mapped to set of vectors $\{e \in \{0,1\}^{|T|.|P|} : \Sigma_{p \in P} e_{tp} = a_t \text{ for } t \in T\}$. This way, every set S_j for $j \in J$ can be replaced by set, which consists of day –tree combinations. The cardinality of $\underline{S}_j$ can be considerable large, because every day combination, which has f active days, results in $|P|^f$ day-tree combinations. This number can be cut down considerably for practical reason, if each day-tree combination, capacity of which is less than customer's daily demand, will be excluded. Another possibility of cutting down the number of new day-tree combinations originates in rule introducing, under which if a customer is once assigned to p-th tree for a day of a combination, then it must be assigned to the p-th tree in other active days of the combination. This rule may reduce cardinality of the set of resulting day tree combinations to number $|P|.f$. Another weaker reduction can be obtained by application the above-mentioned rule on the trees only, which have the same capacity limits.

Having formed sets of feasible day-tree combinations $\underline{S}_j$ for each $j \in J$ and having introduced variables $z_{kj} \in \{0, 1\}$ deciding on assignment of day-tree combination $\underline{k} \in \underline{S}_j$ to customer $j \in J$, we can state the associated model as:

$$\text{Minimize} \quad \sum_{[t,p]\in T\times P} F_{tp}(z) \tag{15}$$

$$\text{Subject to} \quad \sum_{\underline{k}\in \underline{S}_j} z_{\underline{k}j} = 1 \qquad \text{for } j \in J \tag{16}$$

$$\sum_{j\in J}\sum_{\underline{k}\in \underline{S}_j} \underline{b}_j e_{\underline{k}[t,p]} z_{\underline{k}j} \le K_p \qquad \text{for } [t,p] \in T\times P \tag{17}$$

$$z_{\underline{k}j} \in \{0, 1\} \qquad \text{for } j \in J, \text{for } \underline{k} \in \underline{S}_j. \tag{18}$$

where the estimation function can be written as

$$F_{tp}(v) = g\,min\{\, d_{sr} + \sum_{j\in J}\sum_{\underline{k}\in \underline{S}_j} d_{rj} e_{\underline{k}[t,p]} z_{\underline{k}j} : r \in J' \,\}. \tag{19}$$

This model has the same structure as model (1)-(4) and so the assocated problem can be solved using the same two-phase heuristic, which was described in Section 2.

5. PRELIMINARY NUMERICAL EXPERIMENTS

To verify and compare the both approaches, the associated algorithm was implemented using Delphi 5 programming environment. The implementation was completed in two versions, separately for one day-one tree and one day-many tree strategies, even if the solving procedure was the same. This approach was used for more comfortable preparing of the input data, especially the generalized day-tree combinations.

To perform the numerical experiments, PC 256 MB, 1300 MHz was used. The computation times of the heuristics were negligible, that is why they are no reported here. To obtain exact solution of model (10) – (14) for the comparison, MP-solver XPRESS was used.

The preliminary experiments were performed with three regions, Prešov (Pre), Banská Bystrica (Ban) and Nitra (Nit), which were part of a real distribution systen working in five-day period. The numbers of customers connected with these regions were 18, 35 and 36 respectively and most of them were supplied twice in the five-day period.

Both approaches of one day- one tree (1-1) and one day-many trees (1-P) were used on each of the regions with the daily capacities C_D and tree capacities C_T reported in items in table 1. The results of the both heuristics contained assignment of feasibe day or day-tree combinations to the particular customers and this way a list of active customers with their demands is given for each region (Pre, Ban, Nit), each approach ((1-1) and (1-P)) and each working day. This data were completed by number of trees P and tree capacity Q given in number of customers according to the last two rows of table 1. These thirty mathematical programming problems described by model (10)-(14) were solved by MP-solver [4].

Table 1

Input parameters of the preliminary experiments

	Pre	Ban	Nit
C_D [items]	400	672	840
C_T [items]	200, 200	224, 224, 224	400, 220,220
Q [customers]	5	5	6
P	2	3	3

The resulting values of objective function for each region and each approach summed over whole considered period are reported in table 2, together with corresponding sums of values (10) over the period of five working days.

Table 2

Objective function (median) values

	Pre	Ban	Nit
1-1	1929	3297	3312
MP	1407	2299	2173
1-P	1120	2039	2470
MP	1385	2088	2380

6. CONCLUSIONS

We have shown a way, how to obtain better assignment of feasible delivery day combination to supplied customer of a distribution system. This generalized one day-many trees approach enables to form more efficient clusters, which can be served each by one vehicle route.

The reported preliminary experiments indicated improvement of the MP-objective function used for comparison in two of three cases. In the third case, the generalized approach didn't yield any improvement, what can be caused by non-uniformity of the tree capacities, which input into the (1-P) heuristic and which was not considered in the associated MP model, where uniform capacity Q= 6 was used. Next development of more precise way of outputting cluster comparison will be part of our future research.

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