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# STEADY-STATE ANALYTICAL ANALYSIS OF A FOUR-SWITCH SPACE VECTOR PWM INVERTER FED INDUCTION MOTOR DRIVE

**Summary**. In many applications further cost reduction for the drive is one of the important aspect, and thus, a reduction of the number of power switches in the inverter should be a main consideration.These topologies (B4) are employing four switches and four diodes as a practical alternative to the conventional (B6) inverter with six switches and six diodes. This paper presents an analytical model and analysis of the time response of the motor currents for a four-switch inverter driving a three-phase induction machine. As the phase voltages are in principle asymmetrical ,first the asymmetrical space vectors are expressed in the symmetrical forms. Next, the mathematical model uses mixed description in the p and z domains.(The Laplace and modified z-transform) Finally, analytical results are verified experimentally on a 0.75 kW B4 inverter.

**Key words:** mathematical model, inverter, induction motor

## 1. INTRODUCTION

Variable speed AC drives employing an induction motor and pulsewidth modulation (PWM) voltage fed three-phase inverters have found wide spread applications in various forms.

In many applications a further cost reduction for the drive is one of the important aspect, and thus, a reduction of the number of power switches in the inverter should be a main consideration.

Reduction in power components is possible by considering alternatives to the conventional bridge configuration for a voltage source inverter (VSI). Regarding VSI, some component-minimised schemes have been proposed in the literature and in a practical use [1] - [3].These topologies (B4) are employing four switches and four diodes shown in Fig.1 as a practical alternative to the conventional (B6) inverter with six switches and six diodes. Also modulation of a B4 inverter has recently received more attention. The modulation strategies suggested can produce three phase balanced waveforms but at a reduced output voltage compared with the conventional B6 inverter. But some strategies which are suggested may minimise high harmonic content at the expense of increased complexity and higher switching rates. This paper deals with the mathematical model of the system in exact analytical form. Modulation strategy which is used has the same symmetry as for the B6 inverter and is simple without significant increase in switching frequency. As the phase voltages are in principle asymmetrical, first the discrete Fourier transform is used to express the voltage space vectors in symmetrical form. Then, for the analysis of linear networks with periodically operated switches - mixed description in the p and z domains is used.The solution has generalized closed-form, that makes it applicable irrespective of the number of pulses per output PWM pattern.The change of the switching instants is reflected in the solution by a change in the values $m_k$ and $a_k$.

Experimental results prove the feasibilty of the proposed mathematical model.

## 2. VOLTAGE VECTORS FOR B6 AND B4 CONFIGURATIONS

A circuit diagram of the four-switch inverter is shown in Fig.1.

The scheme employs four switches ($q_1$-$q_4$) and four diodes to generate output voltages. One phase (number 3) of the motor is connected with the centre tap of the DC voltage source.The split source may be effectively obtained by splitting a buffer capacitor into two capacitors.

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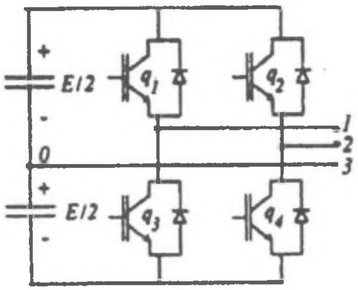



Fig. 1. Three-phase drive using B4 inverter

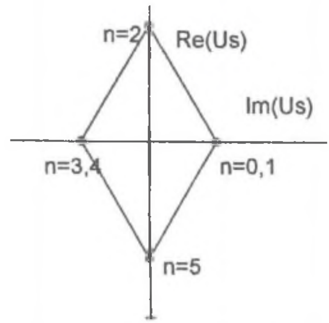



Fig. 2. Voltage vectors in B4 configuration

From the motor voltages we can calculate the voltage space vector

$$U_S(t)=2/3[u_{An}+a\ u_{Bn}+a^2\ u_{Cn}] \quad , a=e^{j2\pi/3}. \tag{2}$$

As there are four switching states we can find four possible voltage vectors V1,V2,V3 and V4 in complex $\alpha\beta$ plane as shown in Fig.2

As it is well known, for B6 inverter there are eight voltage vectors corresponding to the eight available logic states of the B6 inverter.(Six non zero vectors V1-V6,and two zero vectors V0 and V7).These vectors form hexagon on the complex plane.If we express time in a discrete form, with help of the sector number n and per unit time variable $\varepsilon$ as

$$t=(n+\varepsilon)T, \quad n=0,1,2,.....0\leq\varepsilon\leq 1 \tag{3a}$$

where T is a period of the modulation (sector), the voltage vectors in B6 configurations can be expressed as

$$U_s(n)=(2/3)E\ e^{jn\pi/3}. \tag{3b}$$

In Fig.2,for B4 configuration, it can be seen that when forming symmetrical three-phases we must use six vectors f(0)-f(5) defined as follows:

n=0, f(0)=V1, n=1, f(1)=V1, n=2,f(2)=V2, n=3,f(3)=V3, n=4, f(4)=V3, n=5,f(5)=V4. (4)

This periodic unsymmetrical sequence can be expressed by means of the discrete Fourier transform in the symmetrical form as in [5]:

$$f(n)=\frac{1}{N}\times\sum_{m=0}^{N-1} f(m)\times\sum_{i=0}^{N-1} e^{ji(n-m)2\frac{\pi}{N}} \tag{5}$$

where N is number of the vectors.

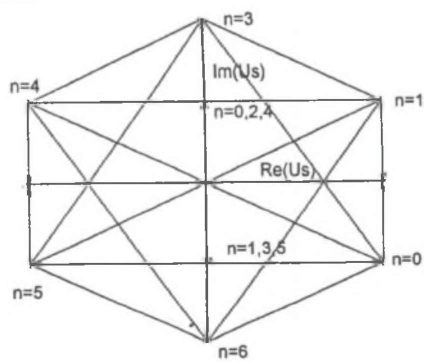



Fig.3. Symmetrical voltage vectors for B4 configuration

After substituting (4) into (5) and making some rearrangements we get for N=6

$$U_S(n)=\frac{E}{3}\left(1-\frac{j}{\sqrt{3}}\right)e^{\frac{jn\pi}{3}}+\frac{jE}{3\sqrt{3}}e^{jn\pi}=\frac{E}{3\sqrt{3}}2e^{j\left(n-\frac{1}{2}\right)\frac{\pi}{3}}+\frac{E}{3\sqrt{3}}e^{j\left(n+\frac{1}{2}\right)\frac{\pi}{3}}. \tag{6}$$

As can be seen from (6) asymmetrical sequence (4) can be decomposed into two symmetrical ones. The first sequence has vectors which are mutually shifted by $60^0$ as in the conventional B6 inverter, and the second one has vectors shifted by $180^0$ and its amplitudes are one half of those of the first sequence These two symmetrical sequences are shown in Fig.3. As can be seen from Fig.3, the second sequence lies on the imaginary axis so it cannot influence the phase A (which is connected with centre tap of DC). Voltage and current triple harmonics can arise in phases B and C. These harmonics in the both phases must have an opposite direction as still is valid

$$i_A+i_B+i_C=0. \tag{7}$$

## 2.1. Space vectors with Space Vector PWM

The equation (6) is valid for space vectors without PWM (square-wave operation). In case of modulation (6) will be dependent also on per unit time ε.

Modulation of a B4 inverter has recently received more attention. The modulation strategies suggested can produce three phase balanced waveforms but at a reduced output voltage compared with B6 inverter.But some strategies may minimise high harmonic content at the expense of increased complexity and higher switching rates. Space vector modulation (SVM) [6], has recently grown as a very popular pulsewidth modulation(PWM) method for voltage fed inverters because of its superior harmonic quality and extended linear range of operation. This type of modulation is periodical with T(or π/3 radian). Switching times are given by the well known formula [6].

In B6 mode when using SV PWM the sequence of the vectors in the first sector is as follow: $V_0,V_1,V_2,V_7$ (000,100,110,111).$V_0$ and $V_7$ are the zero vectors. After switching over to the B4 mode with the same algorithm of modulation we must use the vectors $V_0,V_0,V_1,V_2$ (00,00,10,11).

With respect to the switching times $\Delta\varepsilon_k$ and angle vectors which are used for modulation $\alpha(k)$ ,we can define the switching function $f(\varepsilon,k)$,which determines the beginning and end of the application of the k-th vector. With regard to time dependency of the switching function we can write Eq.(6) as

$$U_S(n,\varepsilon)=\sum_{k=1}^{M}\frac{E}{3}\left(\frac{2}{\sqrt{3}}.e^{j(n-1/2)\pi/3}.f(\varepsilon,k).e^{j\pi\alpha\alpha(k)/3}\right)+\sum_{k=1}^{M}\frac{E}{3}\left(\frac{1}{\sqrt{3}}.e^{j(n+1/2)\pi}.f(\varepsilon,k).e^{j\pi\alpha(k)}\right). \tag{8}$$

M is the number of pulses within the sector period T.

Now, we can derive Laplace transform of (6).As this equation is a function of n and ε we must use relation between the Laplace and modified Z-transform [5]

$$U_S(p)=T\int_0^1 U_S(e^{pT}).e^{-pT\varepsilon}.d\varepsilon., \quad epT=z\ . \tag{9}$$

By substituting (8) into (9) ,with regard to SVPWM strategy mentioned,we get

$$U_S(p)=\frac{1}{p}\frac{2.E}{3\sqrt{3}}\left[\frac{e^{pT}.e^{-j\pi/6}}{e^{pT}-e^{j\pi/3}}\sum_{k=1}^{M}e^{j\pi\alpha(k)/3}(e^{-pT\varepsilon_{kA}}-e^{-pT\varepsilon_{kB}})\right]+\frac{1}{p}\frac{E}{3\sqrt{3}}\left[\frac{e^{pT}.e^{j\pi/2}}{e^{pT}-e^{j\pi}}\sum_{k=1}^{M}e^{j\pi\alpha(k)}(e^{-pT\varepsilon_{kA}}-e^{-pT\varepsilon_{kB}})\right]. \tag{10}$$

The equation (10) is the Laplace transform of the stator space voltage vectors.$\varepsilon_{kA}$ and $\varepsilon_{kB}$ are the beginning and the end of the application of the k-th vector in sector n. This equation can be used to express Laplace transform of the currents.

## 3. STEADY-STATE MOTOR CURRENT

The mathematical model is described in [7] and uses mixed p-z description for the networks with periodically operated switches.Using the system of induction motor equations in the stator α,β frame,the Laplace transform of the stator and the rotor current can be expressed as:

$$I_S(p) = U_S(p)\frac{A_S(p)}{B_S(p)} = U_S(p)\frac{(k_R + p - j\omega)}{\sigma L_S(p-p_1)(p-p_2)},$$

$$I_R(p) = U_S(p)\frac{A_R(p)}{B_R(p)}\left[= -U_S(p)L_m\frac{(p - j\omega)}{(\sigma \cdot L_S L_R)(p-p_1)(p-p_2)}\right], \tag{11}$$

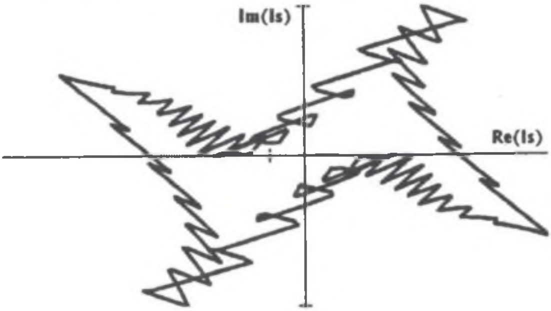
$p_{1,2}$ are the roots of the characteristic equation B(p)=0

$$p_{1,2} = \frac{-(k_S + k_R - j\sigma\omega)}{2\sigma} \pm \sqrt{\frac{(k_S + k_R - j\sigma\omega)}{2\sigma} + \frac{j\omega k_S - k_S k_R}{\sigma}}. \tag{12}$$

In the foregoing equations $R_S$ is the stator resistance, $R_R$ the rotor resistance, $L_S$ the stator self-inductance, $L_R$ the rotor self-inductance, $L_m$ the mutual inductance, $\omega$ the electrical rotor angular velocity and

$$\sigma = 1 - L_m^2/L_S L_R, \qquad k_S = R_S/L_S, \; k_R = R_R/L_R.$$

By substituting (10) into (11) we arrive at the Laplace transform of the space vectors of the stator and rotor currents, respectively.



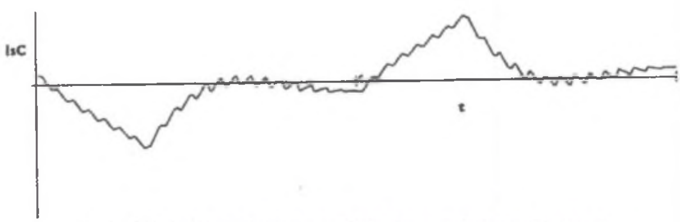




Fig.4. Stator steady-state current trajectory and phase C current

But inverse Laplace transform of (11) cannot be done in a direct way owing to the infinite number of poles in (11).
If we use in Eq.(11) the Heaviside theorem

$$L_1\{A(p)/pB(p)\} = A(0)/B(0) + \sum A(p_k)\, e^{p_k t} / (p_k B'(p_k))$$

where $B'(p_k)=(dB/dp)p=p_k$, $L_1$ means the inverse Laplace transform, then (11) can be transformed into modified z-transform. Having done so, and using the translation theorem we can find inverse z-transform /original function/ in term of variable n,ε.The solution contains both steady-state and transient components. As we consider steady-state we get for $n \to \infty$

$$
\begin{aligned}
Iy(n,\varepsilon) = \sum_{k=1}^{M}\Bigg[\frac{A_y(0)}{B_y(0)}\Bigg[&\frac{2U}{3\sqrt{3}}e^{j\pi/3(n+1/2+\alpha(k))}(e^{-j\pi m_k/3}-e^{-j\pi n_k/3})(\frac{1}{e^{j\pi/3}-1})++\frac{U}{3\sqrt{3}}e^{j\pi(n+3/2+\alpha(k))}(e^{-j\pi m_k}-e^{-j\pi n_k})(\frac{1}{e^{j\pi}-1})\Bigg]+\\
\sum_{s=1}^{2}\frac{A_y(p_s)}{p_s B'(p_s)}e^{p_s T\varepsilon}&\left[\begin{array}{l}\frac{2U}{3\sqrt{3}}e^{j\pi(n+1/2+\alpha(k))/3}(e^{-j\pi m_k/3+p_s T(m_k-\varepsilon_{kA})}-e^{-j\pi n_k/3+p_s T(n_k-\varepsilon_{kB})})\frac{1}{(e^{j\pi/3}-e^{p_s T})}+\\ +\frac{U}{3\sqrt{3}}e^{j\pi(n+3/2+\alpha(k))}(e^{-j\pi m_k+p_s T_s(m_k-\varepsilon_{kA})}-e^{-j\pi n_k+p_s T(n_k-\varepsilon_{kB})}\frac{1}{(e^{j\pi}-e^{p_s T})}\end{array}\right],
\end{aligned}
\tag{13}
$$

where parameters $m_k$ and $n_k$ define prepulse,inside-pulse and postpulse instants.

$$
m_k = \begin{cases} 1 & \text{for } \varepsilon<\varepsilon_{kA} \\ 0 & \text{for } \varepsilon\geq\varepsilon_{kA} \end{cases}, \quad n_k = \begin{cases} 1 & \text{for } \varepsilon<\varepsilon_{k} \\ 0 & \text{for } \varepsilon\geq\varepsilon_{kB} \end{cases}
\tag{14}
$$

$I_y(n,\varepsilon)$ are the steady-state components of the stator (y=S) or the rotor (y=R) space vector currents,respectively.

As can be seen from (13) steady-states current components can be expressed as

$$I_y(n,\varepsilon)=I_y(e^{jn\pi/3},\varepsilon)+I_y(e^{jn\pi},\varepsilon). \tag{15}$$

The steady-states currents are formed by two components. The first one has symmetry $\pi/3$ as in B6 inverter and second has symmetry $\pi$ . This second vector sequence lays in the imaginary axis and so it cannot influence the phase A.Fig.4 shows the trajectory of the steady-state stator current and the phase C current given analytically by (13) and (14).

## 4. EXPERIMENTAL RESULTS

To verify the proposed mathematical model the laboratory implementation of the B4 inverter discussed previously was carried out with the scheme sketched in Fig.1.The drive system is composed of a static B4 inverter and 0.75kW ,4-pole 50-Hz three-phase induction machine($R_s$=9,8 $\Omega$,$R_R$=11,8 $\Omega$)

Fig.5 shows the phase C voltage and stator steady-state current.

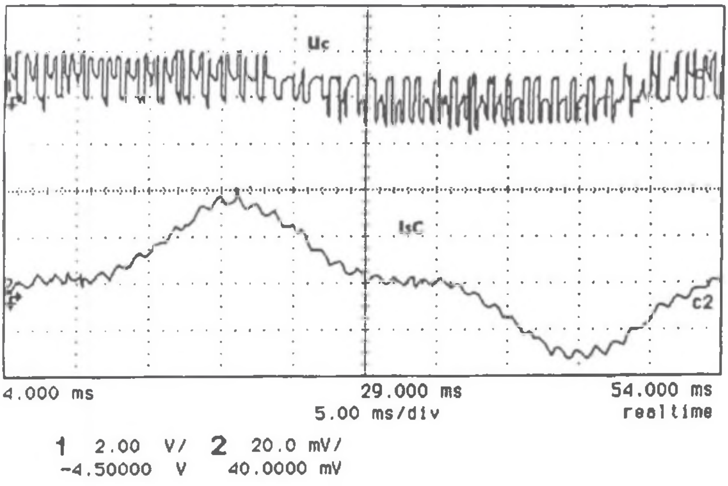



Fig.5. Experimental results: phase C voltage and steady-state stator current in B4 configuration

This figure is in good agreement with the analytically derived waveform presented in Fig.4.

## 5. CONCLUSIONS

Reducing the number of power electronic switching devices in a converter can lead to an overall cost reduction of the unit and to the reduction of switching losses. In comparison with a B6 inverter a semiconductor reduction of 1/3 can be noticed. So, under the assumption that the main losses are due to conduction, the losses in the inverter (independent of current) could be reduced by approximately 67 procent. When the machine fed from the B6 inverter is connected in star, it can be connected in delta for supply from the B4 inverter.
Since the B4 topology in not symmetrical, it is more sensitive to unequal sharing of the DC link voltage between capacitors. As this topology does not automatically eliminate the triple harmonics, the switching frequency has to be increased compared to a conventional inverter.
Analytical relations have been derived for the investigation of the B4 inverter topology .in the time domain.The validity of the mathematical model was proved also by experiments.

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