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Daniela MARASOVÁ, Eduard ŠTROFFEK
Uniwersytet Techniczny Koszyce, Słowacja

APPLYING THE FEM IN MODELLING OF HYPERELASTIC MATERIAL

Summary. From the definition for the hyperelastic material is obvious that the function of density of deformation energy W directly corresponds with materials' characteristics. Many scholars dealing with these problems suggested for concrete types of materials various shapes of functions of density of deformation energy or their materials' models. The problem with determining the hyperelastic materials' characteristics is one of the most important factors effecting the accuracy of the calculation result.

ZASTOSOWANIE FEM (METODY ELEMENTÓW SKOŃCZONYCH) W MODELOWANIU HIPERELASTYCZNEGO MATERIAŁU

Streszczenie. Z definicji materiału hiperelastycznego wynika, że funkcja gęstości od deformacji energii W bezpośrednio odpowiada charakterystyce materiału. Wielu uczonych zajmuje się tym problemem, sugerując dla konkretnych typów materiałów różne kształty w funkcji gęstości od deformacji energii lub ich modeli materiałowych. Problem oznaczenia charakterystyk materiałów hiperelastycznych jest jednym z najważniejszych czynników wpływających na dokładność wyników obliczeń.

1. Solving non-linear problems of mechanics by the boundary-element method

In the practice the boundary-element method for many non-linear problems seems to be less suitable than other numerical methods because it requires many limitations. It gives, however, possibilities, which, on the contrary, other numerical methods do not content. That is why it is advantageous to combine the boundary-element method with other methods from the aspect, that in certain subdomains of problem's definition domain the boundary-element method and elsewhere the finite-element method, difference method or other suitable numerical method are applied.

Solving the non-linear problems can be relatively well applied by the finite-element method under the assumption that the definition domain of the problem is limited and the non-linear behaviour includes changes connected with the definition domain. There are then first of all problems concerning non-linear behaviour of model inside the definition domain.

2. Mooney-Rivlin's theory

At present one of the most spread methods being able to solve non-linear problems of the mechanics is the finite-element method. When solving problems of hyperelasticity by the finite-element method (FEM) for introducing the input materials' constants of the body investigated at present the most well-thought-out Mooney-Rivlin's theory is used. It is the method describing the behaviour of hyperelastic materials with large elastic (reversible) deformations.

The Mooney-Rivlin's theory defines the density of deformation energy for materials affected by large elastic deformations at uniaxial loading as:

$$W = A(I_1 - 3) + B(I_2 - 3), \tag{1}$$

where I_1 and I_2 are invariants of stress tensors (the first and the second) and A and B are searched materials' constants called as Mooney's or Moony-Rivlin's constants. After the adjustment of the equation the following dependence is obtained:

$$\frac{\sigma_1}{2(1-\lambda_1^{-3})} = A\lambda_1 + B, \tag{2}$$

where $\lambda_1 = 1 + \varepsilon_1$. The right side of the formula is the equation of the strait line with the gradient A and the section on the vertical axis B. The test should be carried out only in the extent of deformations being assumed at calculations. In case that the calculation will be carried out only for the compressive stressing, it is suitable to determine characteristics also from the test in compression.

3. Concrete applications of the Mooney-Rivlin's material model

The concrete applications of the Mooney-Rivlin's material model (Fig. 1) were carried out for the P2000/4 conveyor belt. The task was to determine the materials' characteristics of the mentioned conveyor belt and to verify these characteristics by modelling by the finite-element method in the ANSYS 5.5 code.

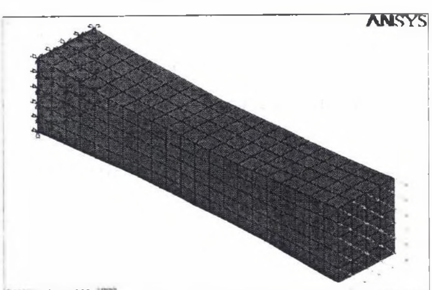

Fig. 1. Model of the testing body
Rys.1. Model testowania ciała

On the testing body in a thin part in the middle are depicted two marks perpendicularly to its longitudinal axis *(Fig. 2)* that define the measured length (l_0) of the working part. After gripping the testing body to the tensile testing g machine the test begins. The testing sample is loaded by uniaxial tension in the range from 0 to 23.5 kN with the step of 20 N or 25 N, 50N with recording the relative elongation at each increasing of the load. The range of loading depends on the ability of the tensile testing machine to hold the testing body. From these measured values there is plotted the graphical dependence (force F - elongation Δl_1) and dependence (stress σ - deformation ε).

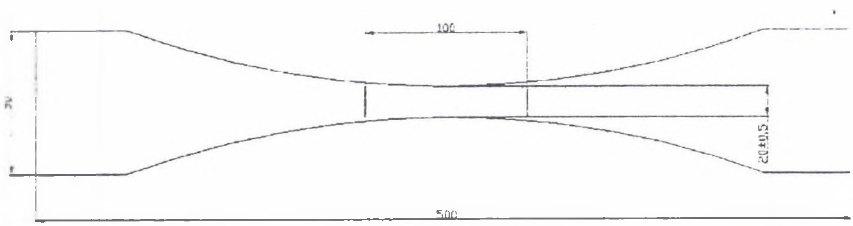

Fig.2. Testing body
Rys.2. Testowanie ciała

After determining the searched constants A and B the values $\lambda_1 = 1 + \varepsilon_1$ are determined. Afterwards the left side of the equation for each point is calculated. Along the corresponding points can be led the strait line, the gradient of which is searched constant A and the section on the vertical axis is the constant B for the Mooney-Rivlin's theory.

The measurements were carried out on six different testing samples of the ***P2000/4*** conveyor belt. The testing samples were cut in the longitudinal direction of the CB.

List of samples:

- sample No 1 CB with the protection layer
- sample No 2 CB with the protection layer
- sample No 3 CB without the protection layer
- sample No 4 CB without the protection layer, sample *t = 10 mm* with unchanged cross-section 10x50 mm
- sample No 5 CB with the protection layer, sample *t = 19 mm* with unchanged cross-section 19x50 mm
- sample No 6 **protection layer**, sample *t = 6 mm* with unchanged cross-section 6x50 mm

It is obvious that the determination of constants A and B depends on the way of plotting the strait line and on accuracy of reading the measured values at the measurement. According to the above mentioned procedure the following materials' constants were determined [1, 2]:

- sample No 1, ***A = 15 MPa, B = 31 MPa***, *Table 4 , graphs 1, 2*
- sample No 2, ***A = 5 MPa, B = 40 MPa***, *Table 4*
- sample No 3, ***A = 13 MPa, B = 57 MPa***

Table 1

Yield values of the sample No 1, A = 15 MPa, B = 31 MPa

F_1 [kN]	Δl_1 [mm] measured value	Δl_1 [mm] calculated value	Deviation [%]
3.00	2.75	3.24	*+18*
5.00	4.75	5.81	*+22*
8.00	7.75	9.37	*+21*

Table 2

Yield values of the sample No 2, A = 5 MPa, B = 40 MPa

F_2 [kN]	Δl_2 [mm] measured value	Δl_2 [mm] calculated value	Deviation [%]
3.00	2.75	3.18	*+15*
5.00	4.75	5.88	*+23*
8.00	7.75	8.91	*+15*

Table 3

Yield values of the sample for correction of material characteristics.
A = 5 MPa, B = 50 MPa

F [kN]	Δl_n [mm] measured value	Δl_v [mm] calculated value	Deviation [%]
3.00	2.75	2.73	***-0.73***
5.00	4.75	4.73	***-0.42***
8.00	7.75	7.86	***1.42***

Graphical dependence force – elongation Δl_1
Sample No. 1

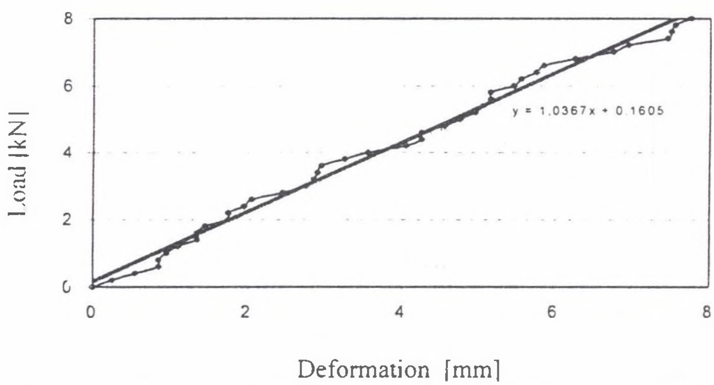

Fig.3a Grafl – Grafical dependence load x deformation, sample No
Rys.3a Zależność odkształcenia od obciążenia, przykład nr 1

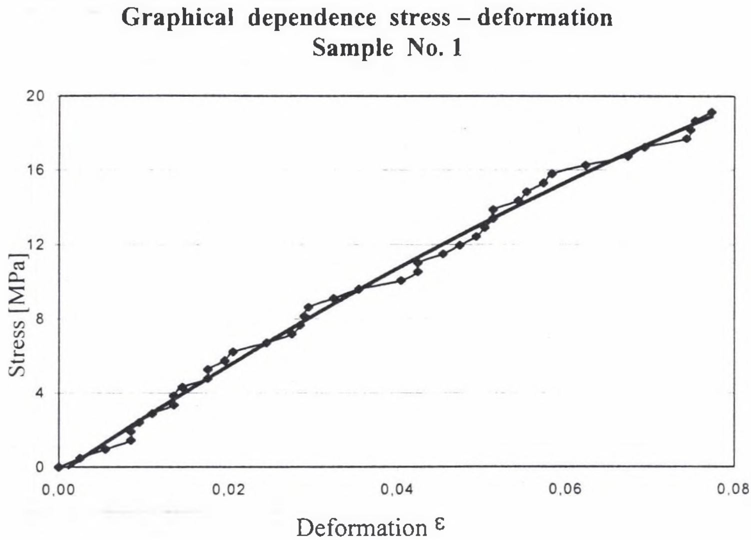

Fig.3b. Graf 2 – Graphical dependance stress x deformation, sample No.1
Rys.3b. Zależność napężenia od odkształcenia, przykład nr 1

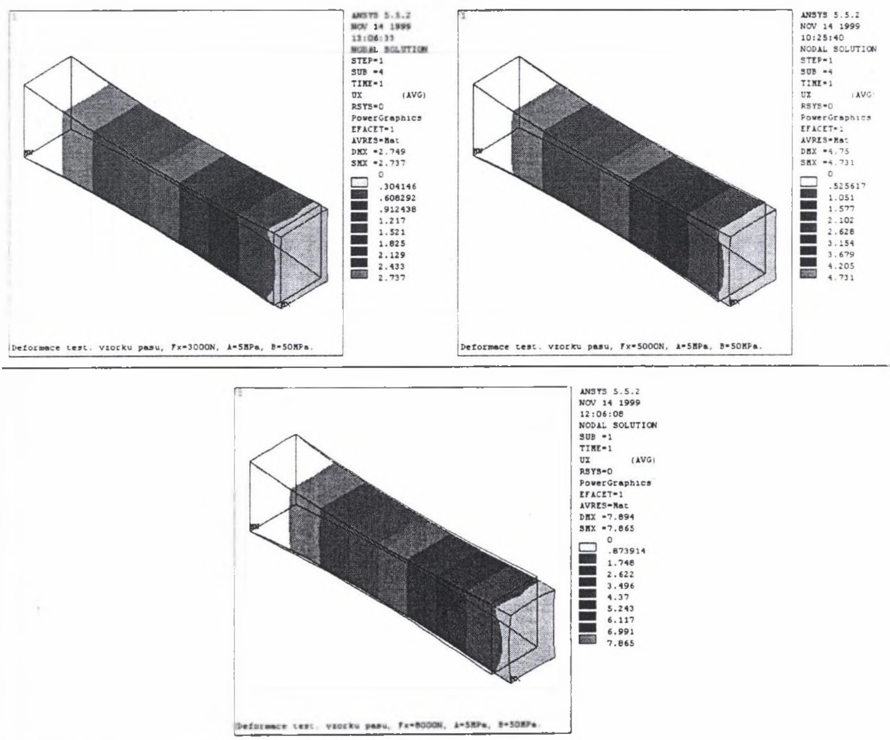

Fig.4. Distribution of deformation in the model
Rys.4. Rozchodzenie się deformacji w modelu

Table 4

Measured values force x deformation of sample No 1 and 2

F_1 [kg]	F_1 [kN]	l_1 [mm]	Δl_1 [mm]	F_2 [kg]	F_2 [kN]	l_2 [mm]	Δl_2 [mm]
0	0.00	100.25	0.00	0	0.00	99.00	0.00
20	0.20	100.50	0.25	25	0.25	99.50	0.50
40	0.40	100.80	0.55	50	0.50	100.00	1.00
60	0.60	101.10	0.85	75	0.75	100.10	1.10
80	0.80	101.10	0.85	100	1.00	100.25	1.25
100	1.00	101.20	0.95	125	1.25	100.30	1.30
120	1.20	101.35	1.10	150	1.50	100.30	1.30
140	1.40	101.60	1.35	175	1.75	100.50	1.50
160	1.60	101.60	1.35	200	2.00	100.60	1.60
180	1.80	101.70	1.45	225	2.25	101.00	2.00
200	2.00	102.00	1.75	250	2.50	101.10	2.10
220	2.20	102.00	1.75	275	2.75	101.50	2.50
240	2.40	102.20	1.95	300	3.00	101.75	2.75
260	2.60	102.30	2.05	325	3.25	102.00	3.00
280	2.80	102.70	2.45	350	3.50	102.10	3.10
300	3.00	103.00	2.75	375	3.75	102.40	3.40
320	3.20	103.10	2.85	400	4.00	102.80	3.80
340	3.40	103.15	2.90	425	4.25	103.00	4.00
360	3.60	103.20	2.95	450	4.50	103.10	4.10
380	3.80	103.50	3.25	475	4.75	103.40	4.40
400	4.00	103.80	3.55	500	5.00	103.80	4.80
420	4.20	104.30	4.05	525	5.25	104.10	5.10
440	4.40	104.50	4.25	550	5.50	104.20	5.20
460	4.60	104.50	4.25	575	5.75	104.30	5.30
480	4.80	104.80	4.55	600	6.00	104.70	5.70
500	5.00	105.00	4.75	625	6.25	105.00	6.00
520	5.20	105.20	4.95	650	6.50	105.20	6.20
540	5.40	105.30	5.05	675	6.75	105.80	6.80
560	5.60	105.40	5.15	700	7.00	105.90	6.90
580	5.80	105.40	5.15	725	7.25	106.00	7.00
600	6.00	105.70	5.45	750	7.50	106.10	7.10
620	6.20	105.80	5.55	775	7.75	106.10	7.10
640	6.40	106.00	5.75	800	8.00	106.20	7.20
660	6.60	106.10	5.85	825	8.25	106.30	7.30
680	6.80	106.50	6.25	850	8.50	106.50	7.50
700	7.00	107.00	6.75	875	8.75	106.80	7.80
720	7.20	107.20	6.95	900	9.00	107.00	8.00
740	7.40	107.70	7.45	925	9.25	107.10	8.10
760	7.60	107.75	7.50	950	9.50	107.30	8.30
780	7.80	107.80	7.55	975	9.75	107.70	8.70
800	8.00	108.00	7.75	1000	10.00	107.80	8.80

4. Conclusions

By the finite-element method in the ANSYS 5.5 code were obtained these results:

- Stressing of the model of part of conveyor belt with the thickness t = 6 mm with materials' constants A = 0.1 MPa and B = 1.6 MPa (Mooney-Rivlin's model). For this 3D problem were used the HYPER86 elements. The force acting at the belt's edge is F_{y1} = 150 N and is evenly distributed to 12 nodal points. The point of application of force is close behind the support (roller). The second point of application of force F_{y2} = 30 N was chosen between supports (rollers). The distribution of principal stresses is shown in Fig. 5

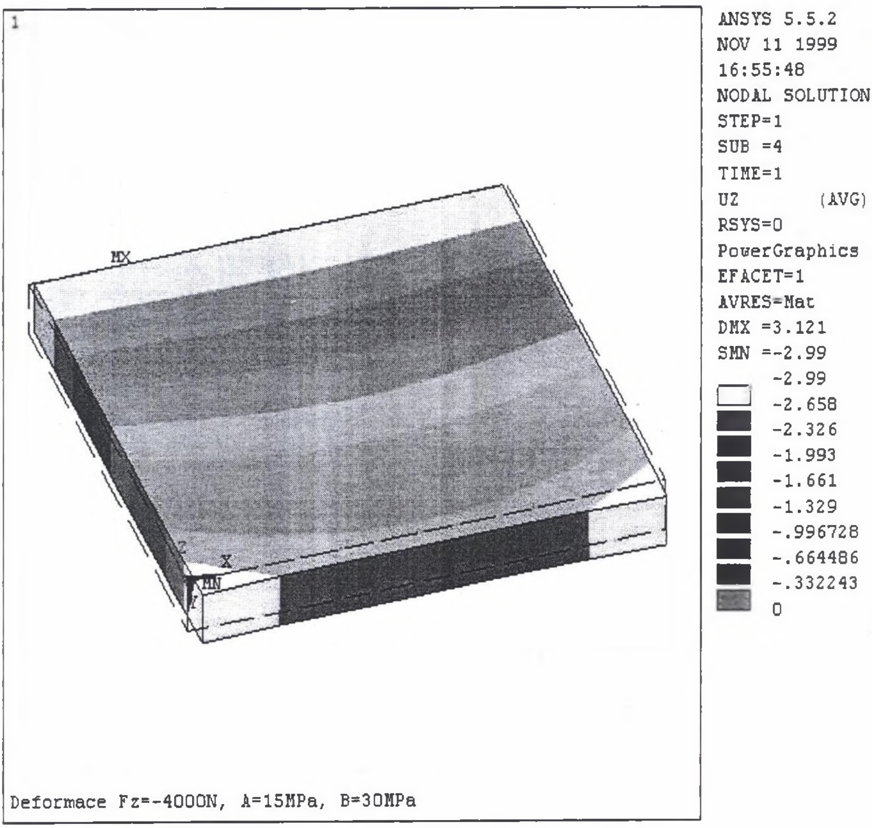

Fig.5. The distribution of deformation in CB, Fy=-4000N
Rys.5. Rozchodzenie się deformacji w CB, Fy = -4000N

- the control of elongation of the conveyor belt t = 6 mm with materials' constants (Mooney-Rivlin's model) A = 15 MPa and B = 30 MPa and A = 5 MPa and B = 40 MPa. For this 3D problem were use HYPER86 elements. One side of the model is firmly fixed (prevented movement), on the other side along the whole width of the model the tension force of -2 000 N or -4 000 N is evenly distributed. The simulation of stressing the model of part of the conveyor belt with the thickness t = 6 mm with material' constants A = 0.1 MPa and B = 1.6 MPa (Mooney-Rivlin's model).For this 3D problem were use HYPER86 elements. The concentrated force acted on the model in the middle between supports (rollers). Its values were as follows: F_y = -100 N, -200 N and -300 N. The calculations were carried out for two stress states (tensioned and free CB).

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Recenzent: Prof.dr hab.inż. Eugeniusz Świtoński